Photon Physics of Revised Electromagnetics

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Conventional theory, as based on Maxwell's equations and associated quantum electrodynamical concepts in the vacuum, includes the condition of zero electric field divergence. In applications to models of the individual photon and to dense light beams such a theory exhibits several discrepancies from experimental evidence. These include the absence of angular momentum (spin), and the lack of spatially limited geometry in the directions transverse to that of the propagation. The present revised theory includes on the other hand a nonzero electric field divergence, and this changes the field equations substantially. It results in an extended quantum electrodynamical approach, leading to nonzero spin and spatially limited geometry for photon models and light beams. The photon models thereby behave as an entirety, having both particle and wave properties and possessing wave-packet solutions which are reconcilable with the photoelectric effect, and with the dot-shaped marks and interference patterns on a screen by individual photons in a two-slit experiment.

1 Introduction

Conventional electromagnetic theory based on Maxwell's equations and quantum mechanics has been very successful in its applications to numerous problems in physics, and has sometimes manifested itself in an exceptionally good agreement with experiments. Nevertheless there exist areas within which these joint theories do not provide fully adequate descriptions of physical reality. As stated by Feynman [1], there are difficulties associated with the *ideas* of Maxwell's theory which are not solved by and not directly associated with quantum mechanics. Thus the classical theory of electromagnetism is in its conventional form an unsatisfactory theory of its own.

Maxwell's equations have served as a guiding line and basis for conventional quantum electrodynamics (QED) in which there is a vacuum state with a vanishing electric field divergence [2]. The quantized equations become equivalent to the classical field equations in which all field quantities are replaced by their expectations values [3]. According to Schiff [2] and Heitler [3], the Poynting vector further forms the basis for the quantized field momentum. Consequently, QED will also become subject to the shortcomings of a conventional field theory.

When applying such a theory to photon physics, it will lead to irrelevant results in a number of important cases. This occurs with the experimentally confirmed existence of angular momentum of individual photons and of light beams with a spatially limited cross-section, with the behaviour of individual photons as needle radiation in the photoelectric effect and in two-slit experiments, and with the particle-wave duality of the photon.

As a consequence of the revealed limitations, modified theories leading beyond Maxwell's equations have been elaborated by several authors. Among these there is an approach described in this paper [4–9]. It is based on a vacuum state that can give rise to local space charges and currents in addition to the displacement current. This changes the field equations in a substantial way, thus resulting in an extended quantum electrodynamical ("EQED") approach.

In the applications to photon physics the nonzero electric field divergence may appear as small, but it still comes out to have an essential effect on the end result. In other applications of the present theory, such as on an electron model [6, 7] not being treated here, the electric field divergence terms appear as large contributions already in the basic field equations.

2 Basis of present theory

The vacuum is not merely an empty space. There is a nonzero level of its ground state, the zero-point-energy, which derives from the quantum states of the harmonic oscillator [2]. An experimentally verified example of the related electromagnetic vacuum fluctuations is the Casimir effect [10]. Electron-positron pair formation due to an energetic photon also indicates that local positive and negative electric charges can be created out of an electrically neutral vacuum state. The basic physical concept of the present theory is therefore the appearance of a local charge density in such a state. In its turn, this becomes associated with a nonzero electric field divergence. The inclusion of the latter can as well be taken as a starting point of a corresponding field theory.

2.1 Lorentz invariant field equations

In presence of electric space charges and related current densities, a general form of the Proca-type equation

$$\Box A_\mu \equiv \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) A_\mu = \mu_0 J_\mu\,, \quad \mu = 1,2,3,4 \quad (1)$$

can be taken as a four-dimensional starting point of the present field equations, given in SI units. Here $A_\mu = (\mathbf{A}, i\phi/c)$, where $\mathbf{A}$ and ϕ are the magnetic vector potential and the electrostatic potential in three-space, and

$$J_\mu = (\mathbf{j},\, \mathrm{i}c\bar{\rho}) \tag{2}$$

is the four-current density. The right-hand member of equation (1) and the form (2) are now given a new interpretation, where $\bar{\rho}$ is the nonzero electric charge density in the vacuum, and $\mathbf{j}$ stands for an associated three-space current density. Maxwell's equations are recovered when the current density (2) disappears, and equation (1) reduces to the d'Alembert equation. The present charge and current densities should not become less conceivable than the conventional concepts of a nonzero curl of the magnetic field and an associated displacement current. All these concepts can be regarded as intrinsic properties of the electromagnetic field.

Physical experience further supports that also the present revised and extended field equations should remain Lorentz invariant. This implies that the current (2) has to transform as a four-vector, and its square then becomes invariant to a transform from one inertial frame K to another such frame K$'$. Thus

$$j^2 - c^2\bar{\rho}^2 = j'^2 - c^2\bar{\rho}'^2 = \text{const}\,. \tag{3}$$

In addition, the current density $\mathbf{j}$ should exist only when there is also a charge density $\bar{\rho}$, and this implies that the constant in equation (3) vanishes. Since $\mathbf{j}$ and $\bar{\rho}$ must behave as space and time parts of J_μ, the disappearance of this constant merely becomes analogous to the choice of origin for the space and time coordinates. Consequently the final form of he current density (2) becomes

$$\mathbf{j} = \bar{\rho}\,(\mathbf{C},\, \mathrm{i}c) \qquad \mathbf{C}^2 = c^2. \tag{4}$$

It is obvious that $\bar{\rho}$ as well as the velocity vector $\mathbf{C}$ vary from one inertial frame to another and do not become Lorentz invariant, whereas this is the case of J_μ.

It can be shown [6, 7] that there is a connection between the current density (4) and the electron theory by Dirac. A different form of the current density in equation (1) has further been introduced by de Broglie and Vigier [11] and applied by Evans and Vigier [12]. It explicitly includes a small nonzero photon rest mass.

The three-dimensional representation of the extended equations *in the vacuum* now becomes

$$\text{curl}\ \mathbf{B}/\mu_0 = \varepsilon_0\,(\text{div}\ \mathbf{E})\,\mathbf{C} + \varepsilon_0 \partial\mathbf{E}/\partial t \tag{5}$$

$$\text{curl}\ \mathbf{E} = -\partial\mathbf{B}/\partial t \tag{6}$$

$$\text{div}\ \mathbf{E} = \bar{\rho}/\varepsilon_0 \tag{7}$$

for the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$. Here the first term of the right-hand member of equation (5) and equation (7) are the new parts introduced. Thus, there is a change from a homogeneous to an inhomogeneous system of equations, a new degree of freedom is introduced by the nonzero electric field divergence, and the latter produces an asymmetry in the appearance of the electric and magnetic fields.

The presence in equations (5) and (7) of the dielectric constant ε_0 and the magnetic permeability μ_0 of the conventional vacuum may require further explanation. In the present approach the vacuum is considered not to include electrically polarized or magnetized atoms or molecules. In this respect equation (7) is the same as in the theory of plasmas which contain freely moving charged particles in a background of empty vacuum space.

A nonzero magnetic field divergence is not adopted in this theory, because this is so far a possible but not experimentally supported supposition which is here left as an open question.

Using vector identities, the basic equations (5)–(7) yield the local momentum equation

$$\text{div}^2\mathbf{S} = \bar{\rho}\,(\mathbf{E} + \mathbf{C}\times\mathbf{B}) + \frac{\partial}{\partial t}\,\mathbf{g} \tag{8}$$

and the local energy equation

$$-\text{div}\ \mathbf{S} = \bar{\rho}\,\mathbf{E}\cdot\mathbf{C} + \frac{\partial}{\partial t}\,w_f\,. \tag{9}$$

Here $\frac{1}{c^2}\,\mathbf{S}$ is the electromagnetic stress tensor,

$$\mathbf{g} = \varepsilon_0\,\mathbf{E}\times\mathbf{B} = \frac{1}{c^2}\,\mathbf{S} \tag{10}$$

can be interpreted as an electromagnetic momentum density with $\mathbf{S}$ denoting the Poynting vector, and

$$w_f = \frac{1}{2}\,\varepsilon_0\left(\mathbf{E}^2 + c^2\mathbf{B}^2\right) \tag{11}$$

representing the electromagnetic field energy density. The angular momentum density becomes

$$\mathbf{s} = \mathbf{r}\times\mathbf{S}/c^2 \tag{12}$$

where $\mathbf{r}$ is the radius vector from the origin.

Combination of equations (5) and (6) results in an extended wave equation for the electric field

$$\left(\frac{\partial^2}{\partial t^2} - c^2\,\nabla^2\right)\mathbf{E} + \left(c^2\,\nabla + \mathbf{C}\,\frac{\partial}{\partial t}\right)(\text{div}\ \mathbf{E}) = 0\,. \tag{13}$$

A divergence operation on equation (5) results in

$$\left(\frac{\partial}{\partial t} + \mathbf{C}\cdot\nabla\right)(\text{div}\ \mathbf{E}) = 0\,, \tag{14}$$

provided that div $\mathbf{C} = 0$.

2.2 Quantization of the revised theory

In the conventional QED formalism Maxwell's equations with a vanishing electric field divergence have been used as a basis, also including the Poynting vector in the representation of the quantized field momentum [2, 3]. The quantized equations then become equivalent to the classical ones in which the field quantities are replaced by their expectation values.

A similar situation also has to apply to the present revised equations. The resulting solutions are expected not to be too far from the truth, by representing the most probable trajectories. A rigorous extended quantum electrodynamical (EQED) formulation would imply that the field equations are quantized already from the outset. However, to simplify the analysis, we will instead solve the extended equations as they stand, and impose relevant quantum conditions afterward. For the considered photon models these conditions are given by the energy $h\nu$ related to the frequency ν, and by the angular momentum $h/2\pi$ of the individual photon with the property of a boson particle.

3 Axisymmetric model of the individual photon

When elaborating a model of the individual photon as a propagating boson, a wave or wave-packet with preserved and limited geometrical shape and undamped motion in a defined direction of space has to be taken as a starting point. This leads to cylindrical geometry and waves. A cylindrical frame (r, φ, z) becomes appropriate, with its z-axis in the direction of propagation. We further introduce a velocity vector of helical geometry

$$\mathbf{C} = c\,(0, \cos\alpha, \sin\alpha) \tag{15}$$

where the angle α is constant and $0 < \cos\alpha \ll 1$ for reasons to be clarified later. As will be shown, the component C_z is related to the wave propagation in the axial direction, and the component C_φ to the angular momentum and an associated small nonzero rest mass. Here we choose the positive values of $\sin\alpha$ and $\cos\alpha$, but have in general both signs representing the two directions of propagation and the two spin directions.

The wave equation (13) now leads to

$$\left(D_1 - \frac{1}{r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right) E_r - \frac{2}{r^2}\frac{\partial}{\partial\varphi} E_\varphi = \frac{\partial}{\partial r}(\mathrm{div}\ \mathbf{E}) \tag{16}$$

$$\begin{aligned}\left(D_1 - \frac{1}{r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right) E_\varphi - \frac{2}{r^2}\frac{\partial}{\partial\varphi} E_r = \\ = \left[\frac{1}{r}\frac{\partial}{\partial\varphi} + \frac{1}{c}(\cos\alpha)\frac{\partial}{\partial t}\right](\mathrm{div}\ \mathbf{E})\end{aligned} \tag{17}$$

$$\left(D_1 + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right) E_z = \left[\frac{\partial}{\partial z} + \frac{1}{c}(\sin\alpha)\frac{\partial}{\partial t}\right](\mathrm{div}\ \mathbf{E}) \tag{18}$$

where

$$D_1 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\,. \tag{19}$$

Equation (14) further becomes

$$\left(\frac{\partial}{\partial t} + c\cos\alpha\,\frac{1}{r}\frac{\partial}{\partial\varphi} + c\sin\alpha\,\frac{\partial}{\partial z}\right)(\mathrm{div}\ \mathbf{E}) = 0\,. \tag{20}$$

In this section restriction is made to purely axisymmetric normal modes for which $\partial/\partial\varphi = 0$, and where every quantity Q has the form $Q = \hat{Q}(r)\exp\left[\mathrm{i}\,(-\omega t + kz)\right]$ with a given angular frequency ω and wave number k.

3.1 Shortcomings of a conventional model

Turning first to a model based on Maxwell's equations, the system (16)–(18) with a vanishing electric field divergence results in the electric field components

$$\begin{aligned}\hat{E}_r &= k_{1r}\,r + k_{2r}/r \\ \hat{E}_\varphi &= k_{1\varphi}\,r + k_{2\varphi}/r \\ \hat{E}_z &= k_{1z}\ln r + k_{2z}\end{aligned} \tag{21}$$

and similar expressions for the magnetic field. The solutions for E_r and E_φ were first deduced by Thomson [13] who called attention to their divergent character, thus becoming unsuitable for a limited model.

However, an even more serious shortcoming arises from the requirement that the divergences of the fields have to vanish. Thus the second order equations (16)–(18) and their solutions (21) have to be checked with respect to the first order equations of an *identically* vanishing field divergence. This implies that

$$2k_{r_1} + \mathrm{i}k\,(k_{1z}\ln r + k_{2z}) \equiv 0 \tag{22}$$

for all k and r. Consequently E_z and k_{1r} have to vanish, only E_φ and k_{2r} remain nonzero, and similar results apply to the magnetic field. This implies that the wave becomes purely transverse, that the Poynting vector (10) has a component in the direction of propagation only, and that there is no spin in the axial direction.

3.2 Axisymmetric models in revised theory

For an axisymmetric normal mode, equation (20) of the revised theory yields the dispersion relation

$$\omega/k = c\sin\alpha = v\,, \qquad k^2 - \omega^2/c^2 = k^2\cos^2\alpha \tag{23}$$

where v stands for the phase velocity which becomes equal to the group velocity $\partial\omega/\partial k$. The parameter $\cos\alpha$ must be small here, such as not to get in conflict with experiments of the Michelson-Morley type. For $\cos\alpha \leqslant 10^{-4}$ the difference between v and c would thus become less than a change in the eight decimal of c. Equations (16), (17) and (23) further combine to

$$-k^2\cos^2\alpha\,E_r = \mathrm{i}\,k\,\frac{\partial E_z}{\partial r} \tag{24}$$

$$\begin{aligned}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} - k^2\cos^2\alpha\right) E_\varphi = \\ = -\,(\mathrm{tg}\ \alpha)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - k^2\cos^2\alpha\right) E_z\,.\end{aligned} \tag{25}$$

A *generating function*

$$G_0 \cdot G = E_z + (\cot\alpha) E_\varphi\,, \quad G = R(\rho)\, \mathrm{e}^{\mathrm{i}(-\omega t + kz)} \quad (26)$$

can now be found which has the amplitude G_0, a normalized dimensionless part G, the normalized coordinate $\rho = r/r_0$, and the characteristic radius r_0 of the configuration represented by the radial function R. The function (26) generates the solutions

$$E_r = -\mathrm{i} G_0 \left(\theta \cos^2\alpha\right)^{-1} \frac{\partial}{\partial\rho}\left[\left(1-\rho^2 D\right) G\right] \quad (27)$$

$$E_\varphi = G_0\, (\mathrm{tg}\,\alpha)\, \rho^2 D G \quad (28)$$

$$E_z = G_0 \left(1-\rho^2 D\right) G \quad (29)$$

where

$$D = D_\rho - \theta^2 \cos^2\alpha\,, \qquad D_\rho = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} \quad (30)$$

and $\theta = k r_0$. The solutions (27)–(29) are reconfirmed by insertion into equations (16)–(18). The magnetic field components are derived from the induction law (6). The helical-like structure of the obtained solution, with its axial field components, is similar but not identical to that deduced by Evans and Vigier [12].

From the normal modes a wave packet solution is now formed which has finite extensions, and a narrow line width as required by experimental observation. We are free to rewrite the amplitude factor (26) as $G_0 = g_0 \cos^2\alpha$. The wave packet has the amplitude

$$A_k = \left(\frac{k}{k_0^2}\right) \mathrm{e}^{-z_0^2 (k-k_0)^2} \quad (31)$$

in the interval $\mathrm{d}k$ being centered around the main wave number k_0. Here $2z_0$ represents the axial length of the packet. With $z = \bar z - vt$ and the notation

$$\bar E_0 = E_0(\bar z) = \left(\frac{g_0}{k_0 r_0}\right)\left(\frac{\sqrt{\pi}}{k_0 z_0}\right) \exp\left[-\left(\frac{\bar z}{2z_0}\right)^2 + \mathrm{i} k_0 \bar z\right] \quad (32)$$

the spectral averages of the field components are

$$\bar E_r = -\mathrm{i} E_0 \left[R_5 + (\theta_0')^2 R_1\right] \quad (33)$$

$$\bar E_\varphi = E_0 \theta_0 (\sin\alpha)(\cos\alpha) \left[R_3 - (\theta_0')^2 R_1\right] \quad (34)$$

$$\bar E_z = E_0 \theta_0 (\cos^2\alpha) \left[R_4 + (\theta_0')^2 R_1\right] \quad (35)$$

where $\theta_0 = k_0 r_0$, $\theta_0' = \theta_0 \cos\alpha$ and

$$R_1 = \rho^2 R\,, \quad R_3 = \rho^2 D_\rho R\,, \quad R_4 = \left(1-\rho^2 D_\rho\right) R\,, \quad (36)$$

$$R_5 = \frac{d}{d\rho}\left[\left(1-\rho^2 D_\rho\right) R\right], \quad R_7 = \left(\frac{d}{d\rho} + \frac{1}{\rho}\right)\left(\rho^2 R\right). \quad (37)$$

The packet components $\left(\bar E_\varphi, \bar E_z, \bar B_r\right)$ are in phase with the generating function (26). The components $\left(\bar E_r, \bar B_\varphi, \bar B_z\right)$ are ninety degrees out of phase with it. We now choose the real part of the function (26), i. e. $G = R(\rho)\cos k\bar z$, which is symmetric with respect to the axial center $\bar z = 0$ of the moving wave packet. Then the real part of the form (32) is adopted, from which $\left(\bar E_\varphi, \bar E_z, \bar B_r\right)$ become symmetric and $\left(\bar E_r, \bar B_\varphi, \bar B_z\right)$ antisymmetric. Under these conditions the integrated electric charge and magnetic moment vanish.

The electromagnetic field energy density (11) defines an equivalent total mass

$$m = \frac{1}{c^2}\int w_f\, dV \cong \frac{2\pi\varepsilon_0}{c^2}\int_{-\infty}^{+\infty} r\, E_r^2\, dr d\bar z \quad (38)$$

to lowest order. Integration and quantization yields

$$m = a_0 W_m \cong \frac{h\nu_0}{c^2}\,, \qquad W_m = \int \rho\, R_5^2\, d\rho\,, \quad (39)$$

where

$$a_0 = \varepsilon_0\, \pi^{5/2} \sqrt{2}\, z_0 \left(g_0/c k_0^2 z_0\right)^2 \equiv 2 a_0^* g_0^2 \quad (40)$$

and $\nu_0 = c/\lambda_0$. Here the slightly reduced phase and group velocity (23) can be associated with a very small nonzero photon rest mass $m_0 = m\cos\alpha$.

Turning finally to the momentum balance, all integrated volume forces in equation (8) vanish on account of the symmetry properties, and expression (12) gives

$$s = \int s_z\, dV = -2\pi\varepsilon_0 \int_{-\infty}^{+\infty}\int r^2 \bar E_r \bar B_z\, dr d\bar z\,. \quad (41)$$

It reduces to the quantum condition

$$s = a_0 r_0 c (\cos\alpha) W_s = \frac{h}{2\pi}\,, \quad W_s = -\int \rho^2 R_5 R_7\, d\rho\,. \quad (42)$$

So far the radial function R has not been specified. We first consider the case where it is finite at the axis $\rho = 0$ and tends to zero at large ρ, as in the form

$$R(\rho) = \rho^\gamma \mathrm{e}^{-\rho} \quad (43)$$

where $\gamma > 0$. In principle, the factor in front of the exponential would have to be replaced by a power series of ρ, but since we will proceed to the limit of large γ, only one dominating term becomes sufficient. The exponential factor in (43) is further included to secure the convergence of any moment with R. The function (43) has a sharply defined maximum at the radius $\hat r = \gamma r_0$. Combination of relations (39) and (42) finally results in an effective photon diameter

$$2\hat r = \frac{\lambda_0}{\pi\cos\alpha} \quad (44)$$

being independent of γ and the exponential factor in equation (43).

We next consider the alternative of a radial function R which *diverges* at the axis, i. e.

$$R(\rho) = \rho^{-\gamma} \mathrm{e}^{-\rho}. \quad (45)$$

Here $\hat{r} = r_0$ can be taken as an effective radius. To obtain *finite* integrated values of the total mass m and spin s, small lower limits ρ_m and ρ_s are now introduced in the integrals of W_m and W_s. We further introduce

$$r_0 = c_r \cdot \varepsilon \, , \qquad g_0 = c_g \cdot \varepsilon^{\beta} \tag{46}$$

such as to make the characteristic radius r_0 and the factor g_0 shrink to small but nonzero values as the lower limits ρ_m and ρ_s approach zero. In equations (46), ε is an independent smallness parameter, $0 < \varepsilon \ll 1$, and c_r, c_g and β stand for positive constants. Equations (40), (39) and (44) combine to

$$m = a_0^* \gamma^5 c_g^2 \left(\varepsilon^{2\beta}/\rho_m^{2\gamma}\right) \cong h/\lambda_0 c \, , \tag{47}$$

$$s = a_0^* \gamma^5 c_g^2 c_r c (\cos\alpha) \left(\varepsilon^{2\beta+1}/\rho_m^{2\gamma-1}\right) = h/2\pi \, . \tag{48}$$

To obtain finite m and s it is then necessary that

$$\rho_m = \varepsilon^{\beta/\gamma}, \qquad \rho_s = \varepsilon^{(2\beta+1)(2\gamma-1)}. \tag{49}$$

We are here free to choose $\beta = \gamma \gg 1$ by which $\rho_s \cong \rho_m = \varepsilon$. This leads to a similar set of geometrical configurations which have a shape being independent of ρ_m, ρ_s and ε in the range of small ε. From equations (47) and (48) the effective photon diameter finally becomes

$$2\hat{r} = \frac{\varepsilon \lambda_0}{\pi \cos\alpha} \tag{50}$$

which is independent of γ, β and the exponential factor.

3.3 Summary on the photon model

- Conventional theory results in a model of the individual photon which has no spin, and is not reconcilable with a limited geometrical shape.
- The present axisymmetric wave packet model is radially polarized, does not consist of purely transverse elementary waves as in conventional theory, has a nonzero spin and an associated very small rest mass, and a helical-like field structure.
- The spatial dimensions of the present model are limited. The solutions are reconcilable with the concepts of needle radiation proposed by Einstein. This provides an explanation of the photoelectric effect in which a photon knocks out an electron from an atom, and of the dot-shaped marks on a screen in two-slit experiments on individual photons as reported by Tsuchiya et al. [14]. As an example with $\cos\alpha = 10^{-4}$ and $\lambda_0 = 3 \times 10^{-7}$m, equation (44) yields a diameter $2\hat{r} = 10^{-3}$m, and equation (50) results in $2\hat{r} \leqslant 10^{-7}$m when $\varepsilon \leqslant \cos\alpha$ for needle-like radiation.
- The present individual photon model is relevant in respect to particle-wave dualism. A subdivision into a particle and an associated pilot wave is not necessary, because the rest mass merely constitutes an integrating part of the total field energy. The wave packet behaves as an entirety, having particle and wave properties at the same time. There is a particle like behaviour such as by needle radiation and a nonzero rest mass, and a wave-like behaviour in terms of interference between cylindrical waves. The rest mass may make possible "photon oscillations" between different modes [8], such as those of the results (44) and (50).

4 Screw-shaped light

In a review by Battersby [15] new results have been reported on twisted light in which the energy travels along a corkscrew-shaped path. These discoveries are expected to become important in communication and microbiology.

In this section, equations (16)–(18) will be applied to screw-shaped waves with the factor

$$\exp\Big[\mathrm{i}(-\omega t + \bar{m}\varphi + kz)\Big] = \exp\left(\mathrm{i}\theta_m\right) \tag{51}$$

and $\bar{m}$ as a positive or negative integer. Since the analysis is similar to that of Section 3.2, we shall leave out its details.

4.1 Shortcomings of the conventional analysis

With Maxwell's equations the system (16)–(18) reduces to

$$\left[D_\rho - \frac{\left(1+\bar{m}^2\right)}{\rho^2}\right](E_r, \mathrm{i}E_\varphi) - \frac{2\bar{m}^2}{\rho^2}\left(\mathrm{i}E_\varphi, E_r\right) = 0 \, , \tag{52}$$

$$\left[D_\rho - \frac{\bar{m}^2}{\rho^2}\right] E_z = 0 \, . \tag{53}$$

For nonzero values of $\bar{m}$, equations (52) combine to

$$\hat{E}_r = c_{1r}\,\rho^{1\pm\bar{m}} + c_{2r}\,\rho^{-(1\pm\bar{m})} = \pm\mathrm{i}\hat{E}_\varphi \tag{54}$$

when $1 \pm \bar{m} \neq 0$ and

$$\hat{E}_r = c_{1r0} + c_{2r0} \ln\rho = \pm\mathrm{i}\hat{E}_\varphi \tag{55}$$

when $1 \pm \bar{m} = 0$. Further equation (53) gives

$$\hat{E}_z = c_{1z}\,\rho^{\bar{m}} + c_{2z}\,\rho^{-\bar{m}}. \tag{56}$$

As in Section 3.1 these results become divergent.

An even more serious shortcoming is again due to an identically vanishing electric and magnetic field divergence. This makes the axial components E_z and B_z disappear, thus resulting in a vanishing spin.

4.2 Twisted modes in revised theory

For nonzero values of $\bar{m}$, the second term in equation (20) introduces complications. This problem is approached by limiting the analysis to sufficient small $\cos\alpha$, and the dispersion relation to be approximated by relations (23). From equation (18) can be seen that E_z is of the order of $\cos^2\alpha$ as compared to E_r and E_φ when $\bar{m} \neq 0$. Equation (16) then takes the approximate form

$$E_r \cong -\left(\frac{r}{\bar{m}}\right)\left[1-k^2\left(\cos^2\alpha\right)\left(\frac{r}{\bar{m}}\right)^2\right]\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)(\mathrm{i}E_\varphi)\,. \quad (57)$$

When inserting this relation into equation (17), the latter is identically satisfied up to the order $\cos^2\alpha$. The component $\mathrm{i}E_\varphi$ can be used as a generating function

$$\mathrm{i}E_\varphi = G_0 G \qquad G = R(\rho)\,\mathrm{e}^{\mathrm{i}\theta_m}\,. \quad (58)$$

The analysis proceeds in forming a wave packet of narrow line width, as given in detail elsewhere [9]. The radial forms (43) and (45) lead to effective diameters for which a factor $|\bar{m}|^{3/2}$ has to be added in the denominators of expressions (44) and (50). These diameters also apply to radially polarized dense light beams, because the mass and angular momentum are both proportional to the same number of photons.

5 Boundary conditions and spin of light beams

A light beam of low photon density can merely be regarded as a stream of non-interacting photons. At high photon densities a unidirectional beam of limited cross-section becomes more complex. The observed angular momentum of such a linearly or elliptically polarized beam has been proposed to be due to transverse spatial derivatives at its boundary [3, 16]. The angular momentum which would have existed for the individual photons in the beam core have been imagined to be substituted by the momentum generated in the boundary region. However, the detailed explanation is so far not clear.

In this section a dense light beam is considered where the individual photons in the beam core overlap each other, such as to form a plane classical electromagnetic (EM) wave as conceived in earlier considerations [7, 8]. Outside the beam there is a vacuum region. The main purpose is to analyze the boundary conditions and the angular momentum of this system.

5.1 Definitions of beam conditions

A beam is considered having an arbitrary cross-section of large size as compared to its characteristic wave lengths. The analysis of a general case with elliptically polarized modes of various wave lengths can be subdivided into a study on each of the included elementary and linearly polarized modes of a specific wave length. A further simplification is provided by the narrow boundary region where the boundary conditions can be applied separately to every small local segment. The analysis is then limited to one linearly polarized core wave. In its turn, this wave can be subdivided into two waves of the same frequency, but having electric field vectors which are perpendicular and parallel to the local segment.

The following analysis starts with an investigation in terms of Maxwell's equations. It then proceeds by the revised theory, first on a flat-shaped configuration with main electric field vectors being either perpendicular or parallel to the boundary. Finally a simplified study is undertaken on a beam of circular cross-section.

5.2 Shortcomings of the conventional analysis

We consider a beam which propagates in the z-direction of a frame (x, y, z) and where every field quantity Q has the form $\hat{Q}(x, y)\exp\left[\mathrm{i}\,(-\omega t + kz)\right]$. The conventional limit of the field equation (13) then reduces to

$$\left[k_0^2 - \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right](\mathbf{E},\mathbf{B}) = 0 \quad (59)$$

where $k_0^2 = k^2 - \left(\frac{\omega}{c}\right)^2$ can have any value. A separable form $X(x)\cdot Y(y)$ of each component then leads to

$$k_0^2 = k_{0x}^2 + k_{0y}^2\,, \quad X''/X = k_{0x}^2\,, \quad Y''/Y = k_{0y}^2\,, \quad (60)$$

where k_{0x}^2 and k_{0y}^2 can have any sign and value. The solution for the electric field becomes

$$\begin{aligned}\bar{E}_\nu = &\left[a_{\nu 1}\exp(k_{0x}x) + a_{\nu 2}\exp(-k_{0x}x)\right]\cdot\\ &\cdot\left[b_{\nu 1}\exp(k_{0y}y) + b_{\nu 2}\exp(-k_{0y}y)\right]\end{aligned} \quad (61)$$

with $\nu = x, y, z$ and an analogous form for the magnetic field. The divergences have to vanish identically. With the solution (61), this leads to a purely transverse wave with zero spin as shown by equation (12). Further one should either have $E_x = E_y = 0$ and $B_x = B_y = 0$, or $k_{0x} = k_{0y} = k_0 = 0$ and $\omega^2 = k^2c^2$. There are no transverse derivatives in an exact solution.

The alternative has also be taken into account where k_0 is zero already from the beginning. Then

$$\bar{E}_\nu = (c_{\nu 1}x + c_{\nu 2})\,(d_{\nu 1}y + d_{\nu 2})\,. \quad (62)$$

With these solutions inserted into the condition of vanishing field divergence

$$\bar{E}_x = c_0\,x + c_1\,y + c_2\,,\ \ \bar{E}_y = c_3\,x - c_0\,y + c_4\,,\ \ \bar{E}_z = 0\,. \quad (63)$$

All the obtained solutions thus have a vanishing spin, and are not reconcilable with a beam of spatially limited cross-section.

5.3 Revised equations of flat-shaped geometry

We now proceed to a revised analysis of flat-shaped beam geometry. With z still in the direction of propagation and x along the normal of the boundaries of a slab-like beam, all field quantities become independent of y. The velocity vector is given by a form similar to (15), having a small component C_y along the boundary and a large component C_z in the direction of propagation.

Now equation (14) yields the same dispersion relation as (23), and the three component equations reduce to

$$E_x = -\frac{\mathrm{i}}{k\cos^2\alpha}\frac{\partial E_z}{\partial x}\,, \tag{64}$$

$$\left(k^2\cos^2\alpha - \frac{\partial^2}{\partial x^2}\right)\left(E_y + \frac{\sin\alpha}{\cos\alpha}E_z\right) = 0\,. \tag{65}$$

Consequently E_z can be considered as a generating function of E_x and E_y. One solution of equation (65) is found where E_y has the same spatial profile as E_z and

$$E_y = -\frac{\sin\alpha}{\cos\alpha}\,E_z\,. \tag{66}$$

5.4 Two special cases of flat-shaped geometry

A flat-shaped (slab-like) beam is now considered which has a core region $-a < x < a$ and two narrow boundary regions, $-(a+b) < x < -a$ and $a < x < a+b$, with thickness $d = b - a$. With the frame chosen in Section 5.3, we first consider the case where E_x is the main electric component. Within the core a homogeneous linearly polarized EM wave is assumed to exist, having the constant components E_{0x} and B_{0y}. In the boundary region an axial field component E_z is chosen which increases linearly with x, from zero at $x = a$, and in such a way that E_x of equation (64) becomes matched to E_{0x} at $x = a$. In the same region the field E_z further passes a maximum, and then drops to zero at the vacuum interface $x = a + b$. The resulting field E_x is reversed in the boundary layer, having a maximum strength of the order of E_{0x}. With $E_{0x} = \mathrm{O}(1)$ in respect to the smallness parameter $\cos\alpha$, equations (64) and (66) show that $E_z = \mathrm{O}(\cos^2\alpha)$ and $E_y = \mathrm{O}(\cos\alpha)$. Here B_y is of zero order and matches B_{0y} at the edge of the core. The components of the Poynting vector are $S_x = 0$ and

$$S_y \cong c\,(\cos\alpha)\,\varepsilon_0\,E_x^2\,, \qquad S_z \cong c\,(\sin\alpha)\,\varepsilon_0\,E_x^2\,. \tag{67}$$

Thus there is a primary flow of momentum S_z in the direction of propagation, and a secondary flow S_y along the boundary, but no flow across it. The field energy density finally becomes $w_f \cong \varepsilon_0 E_x^2$.

Turning to the second case where E_y is the main electric component and is parallel to the boundary, there is an EM core wave with the components E_{0y} and B_{0x}. In a small range of x near $x = a$ the axial field E_z is assumed to be constant, and $E_x = 0$. Relation (66) then makes it possible to match E_y to E_{0y} at $x = a$. Moreover, the field E_z is chosen to decrease towards zero when approaching the outer boundary $x = a + b$. According to equation (64) the field E_x increases from zero at $x = a$ to a maximum, and then drops towards zero when approaching the outer boundary at $x = a + b$. Combination of equations (64) and (66) yields

$$|E_x/E_y| = \lambda/2\pi L_{cy}\cos\alpha \tag{68}$$

where $\lambda = 2\pi/k$ and L_{cy} stands for the characteristic length of the derivative of E_y. As an example with $\lambda/L_{cy} = 10^{-4}$ and $\cos\alpha = 10^{-4}$, equation (68) gives a ratio of about 0.16. The Poynting vector components become $S_x = 0$ and

$$S_y = c\,(\cos\alpha)\,\varepsilon_0\,E_y^2\left[1 + \sin^2\alpha\,(E_x/E_y)^2\right]/\sin^2\alpha\,, \tag{69}$$

$$S_z = c\,\varepsilon_0\,E_y^2\left[1 + \sin^2\alpha\,(E_x/E_y)^2\right]/\sin^2\alpha\,. \tag{70}$$

The energy density is $w_f \cong \varepsilon_0 E_y^2$ as long as $E_x^2 \ll E_y^2$.

5.5 Simplified analysis on the spin of a beam

A simplified analysis is performed on a beam of circular cross-section. The frame is redefined for a linearly polarized EM core wave $\mathbf{E}_0 = (E_0, 0, 0)$ and $\mathbf{B}_0 = (0, B_0, 0)$. With the angle θ between the y-direction and the radial direction, the electric components are

$$E_{0\perp} = E_0\sin\theta\,, \qquad E_{0\parallel} = E_0\cos\theta \tag{71}$$

in the perpendicular and parallel directions of the boundary. The solutions of Section 5.4 are now matched to these core components at the inner surface of the boundary layer. The energy density is $w_f = \varepsilon_0\,\mathbf{E}^2$ where $\mathbf{E}^2 = \mathbf{E}_0^2$ at the edge of the beam core.

With the numerical example of Section 5.4 as a reference where $E_x^2 \ll E_y^2$, the Poynting vector components in the transverse direction now add up to

$$S_t = c(\cos\alpha)\,\varepsilon_0\,\mathbf{E}^2\,. \tag{72}$$

The energy density of the beam core can be written as

$$\varepsilon E_0^2 = n_p hc/\lambda \tag{73}$$

where n_p is the number of equivalent photons per unit volume. With the spin $h/2\pi$ of each photon, the core contains a total angular momentum per unit length

$$s_c = r_0^2 n_p h/2 = \varepsilon_0 E_0^2 \lambda r_0^2/2c \tag{74}$$

with r_0 standing for the core radius. From equations (12) and (72) the angular momentum generated per axial length in the boundary layer becomes on the other hand

$$s_b = 2\pi(\cos\alpha)\,\varepsilon\,E_0^2\,f_E\,r_0^2\,d/c \tag{75}$$

where d is the thickness of the boundary layer and $f_E < 1$ is a profile factor of $\mathbf{E}^2$ across the layer. Thus

$$\frac{s_b}{s_c} = \frac{4\pi(\cos\alpha)f_E\,d}{\lambda}\,. \tag{76}$$

Here $s_b = s_c$ when the equivalent angular momentum of the core is compensated by that generated in the boundary layer. As an example, for $\lambda = 3\times10^{-7}$m, $f_E = 0.2$ and $d = 10^{-3}$m this becomes possible when $\cos\alpha = 10^{-4}$.

5.6 Summary of the analysis on a dense light beam

- Conventional theory leads to a vanishing spin, and is not reconcilable with a beam of limited extensions in its transverse directions. A limited cross-section can only appear in an approximate solution when the characteristic lengths of the transverse derivatives are much larger than the included wavelengths.
- The present revised theory leads to a Poynting vector with a primary component in the direction of propagation, and a secondary component in the transverse directions which generates a spin.
- The angular momentum represented by the spin of the photons in the beam core is substituted by a real spin generated in the boundary layer.
- Even large transverse spatial derivatives and a corresponding limited beam cross-section can exist according to the revised theory.

6 Conclusions

Conventional theory which is based on Maxwell's equations and the associated quantum electrodynamical concepts in the vacuum state includes the condition of zero electric field divergence. When being applied to the physics of the individual photon and of dense light beams, such a theory exhibits a number of discrepancies from experimental evidence. These shortcomings include the absence of spin and of spatially limited geometry in the directions which are transverse to that of the propagation.

The present revised theory on the vacuum state is based on a nonzero electric field divergence which introduces an additional degree of freedom into the field equations, thereby changing the latter and their solutions substantially as compared to the conventional ones. The resulting extended quantum electrodynamics (EQED) makes it possible for both individual photons and for dense light beams to possess a nonzero spin, and to have a spatially limited geometry in the transverse directions. Moreover the individual photon models behave as an entirety in having both particle and wave properties. There are wave-packet solutions with the character of needle radiation which become reconcilable with the photoelectric effect, and with the dot-shaped marks and interference patterns due to individual photons in two-slit experiments.

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