A Note on Geometric and Information Fusion Interpretation of Bell's Theorem and Quantum Measurement*

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In this paper we present four possible extensions of Bell's Theorem: Bayesian and Fuzzy Bayesian intrepretation, Information Fusion interpretation, Geometric interpretation, and the viewpoint of photon fluid as medium for quantum interaction.

1 Introduction

It is generally accepted that Bell's theorem [1] is quite exact to describe the linear hidden-variable interpretation of quantum measurement, and hence "quantum reality". Therefore null result of this proposition implies that no hidden-variable theory could provide good explanation of "quantum reality".

Nonetheless, after further thought we can find that Bell's theorem is nothing more than another kind of abstraction of quantum observation based on a set of assumptions and propositions [7]. Therefore, one should be careful before making further generalization on the null result from experiments which are "supposed" to verify Bell's theorem. For example, the most blatant assumption of Bell's theorem is that it takes into consideration only the classical statistical problem of chance of outcome A or outcome B, as result of adoption of Von Neumann's definition of "quantum logic". Another critic will be discussed here, i. e. that Bell's theorem is only a reformulation of statistical definition of correlation; therefore it is merely tautological [5].

Therefore in the present paper we will discuss a few plausible extension of Bell's theorem:

(a) Bayesian and Fuzzy Bayesian interpretation.

(b) Information Fusion interpretation. In particular, we propose a modified version of Bell's theorem, which takes into consideration this multivalued outcome, in particular using the information fusion Dezert-Smarandache Theory (DSmT) [2, 3, 4]. We suppose that in quantum reality the outcome of $P(A \cup B)$ and also $P(A \cap B)$ shall also be taken into consideration. This is where DSmT and Unification of Fusion Theories (UFT) could be found useful [2, 17].

(c) Geometric interpretation, using a known theorem connecting geometry and imaginary plane. In turn, this leads us to 8-dimensional extended-Minkowski metric.

(d) As an alternative to this geometric interpretation, we submit the viewpoint of photon fluid as medium for quantum interaction. This proposition leads us to Gross-Piteavskii equation which is commonly used to describe bose condensation phenomena. In turn we provide a route where Maxwell equations and Schrödinger equation could be deduced from Gross-Pitaevskii equation by using known algebra involving bi-quaternion number. In our opinion, this new proposition provides us a physical mechanism of quantum interaction, beyond conventional "quantum algebra" which hides causal explanation.

By discussing these various approaches, we use an expanded logic beyond "yes" or "no" type logic [3]. In other words, there could be new possibilities to describe quantum interaction: "both can be wrong", or "both can be right", as described in Table 1 below.

In Belnap's four-valued logic there are, besides Truth (T) and Falsehood (F), also Uncertainty (U) and Contradiction (C) but they are inter-related [30]. Belnap's logic is a particular case of Neutrosophic Logic (which considers three components: Truth, Falsehood, and Indeterminacy (I)) when indeterminacy is split into Uncertainty and Contradiction. In our article we have: Yes (Y), No (N), and Indeterminacy (I, which means: neither Yes nor No), but Indeterminacy is split into "both can be wrong" and "both can be right".

It could be expected that a combined interpretation represents multiple-facets of quantum reality. And hopefully it could bring better understanding on the physical mechanism beneath quantum measurement, beyond simple algebraic notions. Further experiments are of course recommended in order to verify or refute this proposition.

2 Bell's theorem. Bayesian and fuzzy Bayesian interpretation

Despite widespread belief of its ability to describe hidden-variables of quantum reality [1], it shall be noted that Bell's theorem starts with a set of assumptions inherent in its formulation. It is assumed that each pair of particles possesses a particular value of λ, and we define quantity $p(\lambda)$ so that probability of a pair being produced between λ and $\lambda + d\lambda$

*Note: The notion "hronir wave" introduced here was inspired from Borges' Tlon, Uqbar, Orbis Tertius.

Alternative	Bell's theorem	Implications	Special relativity
QM is nonlocal	Invalid	Causality breaks down; Observer determines the outcome	Is not always applicable
QM is local with hidden variable	Valid	Causality preserved; The moon is there even without observer	No interaction can exceed the speed of light
Both can be right	Valid, but there is a way to explain QM without violating Special Relativity	QM, special relativity and Maxwell electromagnetic theory can be unified. New worldview shall be used	Can be expanded using 8-dimensional Minkowski metric with imaginary plane
Both can be wrong	Invalid, and so Special Relativity is. We need a new theory	New nonlocal QM theory is required, involving quantum potential	Is not always applicable

Table 1: Going beyond classical logic view of QM

is $p\,(\lambda)\,d\lambda$. It is also assumed that this is normalized so that:

$$\int p\,(\lambda)\,d\lambda = 1\,. \tag{1}$$

Further analysis shows that the integral that measures the correlation between two spin components that are at an angle of $(\delta - \phi)$ with each other, is therefore equal to $C''(\delta - \phi)$. We can therefore write:

$$|C''(\phi) - C''(\delta)| - C''(\delta - \phi) \leqslant 1 \tag{2}$$

which is known as Bell's theorem, and it was supposed to represent any local hidden-variable theorem. But it shall be noted that actually this theorem cannot be tested completely because it assumes that all particle pairs have been detected. In other words, we find that a hidden assumption behind Bell's theorem is that it uses classical probability assertion [12], which may or may be not applicable to describe Quantum Measurement.

It is wothnoting here that the standard interpretation of Bell's theorem includes the use of Bayesian posterior probability [13]:

$$P(\alpha\,|\,x) = \frac{p\,(\alpha)\,p\,(x\,|\,\alpha)}{\sum_\beta p\,(\beta)\,p\,(x\,|\,\beta)}\,. \tag{3}$$

As we know Bayesian method is based on classical two-valued logic. In the meantime, it is known that the restriction of classical propositional calculus to a two-valued logic has created some interesting paradoxes. For example, the Barber of Seville has a rule that all and only those men who do not shave themselves are shaved by the barber. It turns out that the only way for this paradox to work is if the statement is both *true and false simultaneously* [14]. This brings us to *fuzzy Bayesian approach* [14] as an extension of (3):

$$P(s_i|\underline{M}) = \frac{p\,(\underline{M}\,|\,s_i)\,p\,(s_i)}{p\,(\underline{M})}\,, \tag{4}$$

where [14, p. 339]:

$$p\,(\underline{M}\,|\,s_i) = \sum_{k=1}^{r} p\,(x_k\,|\,s_i)\,\mu_{\underline{M}}(x_k)\,. \tag{5}$$

Nonetheless, it should also be noted here that there is shortcoming of this Bayesian approach. As Kracklauer points out, Bell's theorem is nothing but a reformulation of statistical definition of correlation [5]:

$$\mathrm{Corr}\,(A, B) = \frac{\langle |AB| \rangle - \langle A \rangle \langle B \rangle}{\sqrt{\langle A^2 \rangle \langle B^2 \rangle}}\,. \tag{6}$$

When $\langle A \rangle$ or $\langle B \rangle$ equals to zero and $\langle A^2 \rangle \langle B^2 \rangle = 1$ then equation (6) reduces to Bell's theorem. Therefore as such it could be considered as merely tautological [5].

3 Information fusion interpretation of Bell's theorem. DSmT modification

In the context of physical theory of information [8], Barrett has noted that "there ought to be a set theoretic language which applies directly to all quantum interactions". This is because the idea of a bit is itself straight out of *classical set theory*, the definitive and unambiguous assignment of an element of the set $\{0, 1\}$, and so the assignment of an information content of the photon itself is fraught with the same difficulties [8]. Similarly, the problem becomes more adverse because the fundamental basis of conventional statistal theories is the same classical set $\{0, 1\}$.

Not only that, there is also criticism over the use of Bayesian approach, i. e.: [13]

(a) In real world, neither class probabilities nor class densities are precisely known;

(b) This implies that one should adopt a parametric model for the class probabilities and class densities, and then use empirical data.

(c) Therefore, in the context where multiple sensors can be used, information fusion approach could be a better alternative to Bayes approach.

In other words, we should find an extension to standard proposition in statistical theory [8, p. 388]:

$$P(AB|C) = P(A|BC)P(B|C) \quad (7)$$
$$= P(B|AC)P(A|C) \quad (8)$$
$$P(A|B) + P(\bar{A}|B) = 1\,. \quad (9)$$

Such an extension is already known in the area of information fusion [2], known as Dempster-Shafer theory:

$$m(A) + m(B) + m(A \cup B) = 1\,. \quad (10)$$

Interestingly, Chapline [13] noted that neither Bayesian theory nor Dempster-Shafer could offer insight on how to minimize overall energy usage in the network. In the meantime, Dezert-Smarandache (DSmT) [2] introduced further improvement of Dempster-Shafer theory by taking into consideration chance to observe intersection between A and B:

$$m(A) + m(B) + m(A \cup B) + m(A \cap B) = 1\,. \quad (11)$$

Therefore, introducing this extension from equation (11) into equation (2), one finds a modified version of Bell's theorem in the form:

$$|C''(\phi) - C''(\delta)| - \\ - C''(\delta - \phi) + C''(\delta \cup \phi) + C''(\delta \cap \phi) \leqslant 1\,, \quad (12)$$

which could be called as modified Bell's theorem according to Dezert-Smarandache (DSmT) theory [2]. Its direct implications suggest that it could be useful to include more sensors in order to capture various possibilities beyond simple $\{0, 1\}$ result, which is typical in Bell's theorem.

Further generalization of DSmT theory (11) is known as Unification of Fusion Theories [15, 16, 17]:

$$m(A) + m(B) + m(A \cup B) + m(A \cap B) + \\ + m(\bar{A}) + m(\bar{B}) + m(\bar{A} \cup \bar{B}) + m(\bar{A} \cap \bar{B}) = 1\,, \quad (13)$$

where $\bar{A}$ is the complement of A and $\bar{B}$ is the complement of B (if we consider the set theory).

(But if we consider the logical theory then $\bar{A}$ is the negation of A and $\bar{B}$ is the negation of B. The set theory and logical theory in this example are equivalent, hence doesn't matter which one we use from them.) In equation (13) above we have a complement/negation for A. We might define the $\bar{A}$ as the entangle of particle A. Hence we could expect to further extend Bell's inequality considering UFT; nonetheless we leave this further generalization for the reader.

Of course, new experimental design is recommended in order to verify and to find various implications of this new proposition.

4 An alternative geometric interpretation of Bell-type measurement. Gross-Pitaevskii equation and the "hronir wave"

Apart from the aforementioned Bayesian interpretation of Bell's theorem, we can consider the problem from purely geometric viewpoint. As we know, there is linkage between geometry and algebra with imaginary plane [18]:

$$x + iy = \rho e^{i\phi}. \quad (14)$$

Therefore one could expect to come up with geometrical explanation of quantum interaction, provided we could generalize the metric using imaginary plane:

$$X + iX' = \rho e^{i\phi}. \quad (15)$$

Interestingly, Amoroso and Rauscher [19] have proposed exactly the same idea, i. e. generalizing Minkowski metric to become 8-dimensional metric which can be represented as:

$$Z^{\mu} = X^{\mu}_{re} + iX^{\mu}_{im} = \rho e^{i\phi}. \quad (16)$$

A characteristic result of this 8-dimensional metric is that "space separation" vanishes, and quantum-type interaction could happen in no time.

Another viewpoint could be introduced in this regard, i. e. that the wave nature of photon arises from "photon fluid" medium, which serves to enable photon-photon interaction. It has been argued that this photon-fluid medium could be described using Gross-Pitaevskii equation [20]. In turns, we could expect to "derive" Schrödinger wave equation from the Gross-Pitaevskii equation.

It will be shown, that we could derive Schrödinger wave equation from Gross-Pitaevskii equation. Interestingly, a new term similar to equation (14) arises here, which then we propose to call it "hronir wave". Therefore one could expect that this "hronir wave" plays the role of "invisible light" as postulated by Maxwell long-time ago.

Consider the well-known Gross-Pitaevskii equation in the context of superfluidity or superconductivity [21]:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi + \left(V(x) - \gamma\,|\Psi|^{p-1}\right)\Psi, \quad (17)$$

where $p < 2N/(N-2)$ if $N \geqslant 3$. In physical problems, the equation for $p = 3$ is known as Gross-Pitaevskii equation. This equation (17) has standing wave solution quite similar to Schrödinger equation, in the form:

$$\Psi(x,t) = e^{-iEt/\hbar} \cdot u(x)\,. \quad (18)$$

Substituting equation (18) into equation (17) yields:

$$-\frac{\hbar^2}{2m}\,\Delta u + \left(V(x) - E\right) u = |u|^{p-1}\,u\,, \quad (19)$$

which is nothing but time-independent linear form of Schrödinger equation, except for term $|u|^{p-1}$ [21]. In case the right-hand side of this equation is negligible, equation (19) reduces to standard Schrödinger equation. Using Maclaurin series expansion, we get for (18):

$$\Psi(x,t) = \left(1 - \frac{iEt}{\hbar} + \frac{\left(\frac{iEt}{\hbar}\right)^2}{2!} + \frac{\left(-\frac{iEt}{\hbar}\right)^3}{3!} + \ldots\right) \cdot u(x)\,. \quad (20)$$

Therefore we can say that standing wave solution of Gross-Pitaevskii equation (18) is similar to standing wave solution of Schrödinger equation (u), except for nonlinear term which comes from Maclaurin series expansion (20). By neglecting third and other higher order terms of equation (20), one gets an approximation:

$$\Psi(x,t) = \left[1 - iEt/\hbar\right] \cdot u(x)\,. \tag{21}$$

Note that this equation (21) is very near to hyperbolic form $z = x + iy$ [18]. Therefore one could conclude that standing wave solution of Gross-Pitaevskii equation is merely an extension from ordinary solution of Schrödinger equation into Cauchy (imaginary) plane. In other words, there shall be "hronir wave" part of Schrödinger equation in order to describe Gross-Pitaevskii equation. We will use this result in the subsequent section, but first we consider how to derive bi-quaternion from Schrödinger equation.

It is known that solutions of Riccati equation are logarithmic derivatives of solutions of Schrödinger equation, and *vice versa* [22]:

$$u'' + vu = 0\,. \tag{22}$$

Bi-quaternion of differentiable function of $x = (x_1, x_2, x_3)$ is defined as [22]:

$$Dq = -\mathrm{div}\,(q) + \mathrm{grad}\,(q_0) + \mathrm{rot}\,(q)\,. \tag{23}$$

By using alternative representation of Schrödinger equation [22]:

$$\left[-\Delta + u\right] f = 0\,, \tag{24}$$

where f is twice differentiable, and introducing quaternion equation:

$$Dq + q^2 = -u\,. \tag{25}$$

Then we could find q, where q is purely vectorial differentiable bi-quaternion valued function [22].

We note that solutions of (24) are related to (25) as follows [22]:

- For any nonvanishing solution f of (24), its logarithmic derivative:

  $$q = \frac{Df}{f}\,, \tag{26}$$

  is a solution of equation (25), and *vice versa* [22].

Furthermore, we also note that for an arbitrary scalar twice differentiable function f, the following equality is permitted [22]:

$$\left[-\Delta + u\right] f = \left[D + M^h\right]\left[D - M^h\right] f\,, \tag{27}$$

provided h is solution of equation (25).

Therefore we can summarize that given a particular solution of Schrödinger equation (24), the general solution reduces to the first order equation [22, p. 9]:

$$\left[D + M^h\right] F = 0\,, \tag{28}$$

where

$$h = \frac{D\sqrt{\varepsilon}}{\varepsilon}\,. \tag{29}$$

Interestingly, equation (28) is equivalent to **Maxwell equations**. [22] Now we can generalize our result from the preceding section, in the form of the following conjecture:

Conjecture 1 *Given a particular solution of Schrödinger equation (24), then the approximate solution of Gross-Pitaevskii equation (17) reduces to the first order equation:*

$$\left[1 - iEt/\hbar\right]\left[D + M^h\right] F = 0\,. \tag{30}$$

Therefore we can conclude here that there is neat linkage between Schrödinger equation, Maxwell equation, Riccati equation via biquaternion expression [22, 23, 24]. And approximate solution of Gross-Pitaevskii equation is similar to solution of Schrödinger equation, except that it exhibits a new term called here "*the hronir wave*" (30).

Our proposition is that considering equation (30) has imaginary plane wave, therefore it could be expected to provided "physical mechanism" of quantum interaction, in the same sense of equation (14). Further experiments are of course recommended in order to verify or refute this

5 Some astrophysical implications of Gross-Pitaevskii description

Interestingly, Moffat [25, p. 9] has also used Gross-Pitaevskii in his "phion condensate fluid" to describe CMB spectrum. Therefore we could expect that this equation will also yield interesting results in cosmological scale.

Furthermore, it is well-known that Gross-Pitaevskii equation could exhibit *topologically* non-trivial vortex solutions [26, 27], which can be expressed as quantized vortices:

$$\oint p \bullet dr = N_v\, 2\pi\hbar\,. \tag{31}$$

Therefore an implication of Gross-Pitaevskii equation [25] is that topologically quantized vortex could exhibit in astrophysical scale. In this context we submit the viewpoint that this proposition indeed has been observed in the form of Tifft's quantization [28, 29]. The following description supports this assertion of topological quantized vortices in astrophysical scale.

We start with standard definition of Hubble law [28]:

$$z = \frac{\delta\lambda}{\lambda} = \frac{Hr}{c} \tag{32}$$

or

$$r = \frac{c}{H}\,z\,. \tag{33}$$

Now we suppose that the major parts of redshift data could be explained via Doppler shift effect, therefore [28]:

$$z = \frac{\delta\lambda}{\lambda} = \frac{v}{c}\,. \tag{34}$$

In order to interpret Tifft's observation of quantized redshift corresponding to quantized velocity 36.6 km/sec and

72.2 km/sec, then we could write from equation (34):

$$\frac{\delta v}{c} = \delta z = \delta\left(\frac{\delta\lambda}{\lambda}\right). \tag{35}$$

Or from equation (33) we get:

$$\delta r = \frac{c}{H}\,\delta z\,. \tag{36}$$

In other words, we submit the viewpoint that Tifft's observation of quantized redshift implies a quantized distance between galaxies [28], which could be expressed in the form:

$$r_n = r_0 + n\,(\delta r)\,. \tag{35a}$$

It is proposed here that this equation of quantized distance (5) is resulted from topological quantized vortices (31), and agrees with Gross-Pitaevskii (quantum phion condensate) description of CMB spectrum [25]. Nonetheless, further observation is recommended in order to verify the above proposition.

Concluding remarks

In the present paper we review a few extension of Bell's theorem which could take into consideration chance to observe outcome beyond classical statistical theory, in particular using the information fusion theory. A new geometrical interpretation of quantum interaction has been considered, using Gross-Pitaevskii equation. Interestingly, Moffat [25] also considered this equation in the context of cosmology.

It is recommended to conduct further experiments in order to verify and also to explore various implications of this new proposition, including perhaps for the quantum computation theory [8, 13].

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References

1. Shimony A. http://plato.stanford.edu/entries/bell-theorem/ ; http://plato.stanford.edu/entries/kochen-specker/.
2. Smarandache F. and Dezert J. Advances and applications of of DSmT for information fusion. American Research Press, Rehoboth, 2004.
3. Smarandache F. *Bulletin of Pure and Applied Sciences*, Ser. Physics, 2003, v. 22D, No. 1, 13–25.
4. Smarandache F. and Christianto V. Multivalued logic, neutrosophy and Schrödinger equation. Hexis, Phoenix, 2005.
5. Kracklauer A. La theorie de Bell, est-elle la plus grande meprise de l'histoire de la physique? *Annales de la Fondation Louis de Broglie*, v. 25, 2000, 193.
6. Aharonov Y. *et al.* arXiv: quant-ph/0311155.
7. Rosu H. C. arXiv: gr-qc/9411035.
8. Zurek W. (ed.), Complexity, entropy and the physics of information. Santa Fe Inst. Studies, Addison-Wesley, 1990.
9. Schrieffer J. R. Macroscopic quantum phenomena from pairing in superconductors. Lecture, December 11, 1972.
10. Anandan J. S. In: *Quantum Coherence and Reality*. Columbia SC, World Sci., 1994; arXiv: gr-qc/9504002.
11. Goldstein S. Quantum theory without observers — part one. *Physics Today*, March 1998, 42–46.
12. Pitowski I. arXiv: quant-ph/0510095.
13. Chapline G. arXiv: adap-org/9906002; quant-ph/9912019; Granik A. and Chapline G. arXiv: quant-ph/0302013; http://www.whatsnextnetwork.com/technology/index.php/2006/03/.
14. Ross T. J. fuzzy logic with engineering applications. McGraw-Hill, 1995, 196–197, 334–341.
15. Smarandache F. Unification of Fusion Theories (UFT). *Intern. J. of Applied Math. & Statistics*, 2004, v. 2, 1–14.
16. Smarandache F. An in-depth look at information fusion rules and unification of fusion theories. Invited speech at NASA Langley Research Center, Hampton, VA, USA, Nov 5, 2004.
17. Smarandache F. Unification of the fusion theory (UFT). Invited speech at NATO Advance Study Institute, Albena, Bulgaria, May 16–27, 2005.
18. Gogberashvili M. arXiv: hep-th/0212251.
19. Rauscher E. A. and Amoroso R. *Intern. J. of Comp. Anticipatory Systems*, 2006.
20. Chiao R. *et al.*. arXiv: physics/0309065.
21. Dinu T. L. arXiv: math.AP/0511184.
22. Kravchenko V. arXiv: math.AP/0408172.
23. Lipavsky P. *et al.* arXiv: cond-mat/0111214.
24. De Haas E. P. *Proc. of the Intern. Conf. PIRT-2005*, Moscow, MGTU Publ., 2005.
25. Moffat J. arXiv: astro-ph/0602607.
26. Smarandache F. and Christianto V. *Progress in Physics*, 2006, v. 2, 63–67.
27. Fischer U. arXiv: cond-mat/9907457; cond-mat/ 0004339.
28. Russell Humphreys D. Our galaxy is the centre of the universe, "quantized" red shifts show. *TJ Archive*, 2002, v. 16(2), 95–104; http://answersingenesis.org/tj/v16/i2/galaxy.asp.
29. Setterfield B. http://www.journaloftheoretics.com.
30. Belnap N. A useful four-valued logic, modern uses of multiple-valued logic. (D. Reidel, editor), 8–37, 1977.