The Theory of Vortical Gravitational Fields

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This paper treats of vortical gravitational fields, a tensor of which is the rotor of the general covariant gravitational inertial force. The field equations for a vortical gravitational field (the Lorentz condition, the Maxwell-like equations, and the continuity equation) are deduced in an analogous fashion to electrodynamics. From the equations it is concluded that the main kind of vortical gravitational fields is "electric", determined by the non-stationarity of the acting gravitational inertial force. Such a field is a medium for traveling waves of the force (they are different to the weak deformation waves of the space metric considered in the theory of gravitational waves). Standing waves of the gravitational inertial force and their medium, a vortical gravitational field of the "magnetic" kind, are exotic, since a non-stationary rotation of a space body (the source of such a field) is a very rare phenomenon in the Universe.

1 The mathematical method

There are currently two methods for deducing a formula for the Newtonian gravitational force in General Relativity. The first method, introduced by Albert Einstein himself, has its basis in an arbitrary interpretation of Christoffel's symbols in the general covariant geodesic equations (the equation of motion of a free particle) in order to obtain a formula like that by Newton (see [1], for instance). The second method is due to Abraham Zelmanov, who developed it in the 1940's [2, 3]. This method determines the gravitational force in an exact mathematical way, without any suppositions, as a part of the gravitational inertial force derived from the non-commutativity of the differential operators invariant in an observer's spatial section. This formula results from Zelmanov's mathematical apparatus of chronometric invariants (physical observable quantities in General Relativity).

The essence of Zelmanov's mathematical apparatus [4] is that if an observer accompanies his reference body, his observable quantities are the projections of four-dimensional quantities upon his time line and the spatial section — *chronometrically invariant quantities*, via the projecting operators $b^{\alpha}=\frac{dx^{\alpha}}{ds}$ and $h_{\alpha\beta}=-g_{\alpha\beta}+b_{\alpha}b_{\beta}$, which fully define his real reference space (here b^{α} is his velocity relative to his real references). So the chr.inv.-projections of a world-vector Q^{α} are $b_{\alpha}Q^{\alpha}=\frac{Q_0}{\sqrt{g_{00}}}$ and $h^{i}_{\alpha}Q^{\alpha}=Q^{i}$, while the chr.inv.-projections of a 2nd rank world-tensor $Q^{\alpha\beta}$ are $b^{\alpha}b^{\beta}Q_{\alpha\beta}=\frac{Q_{00}}{g_{00}}$, $h^{i\alpha}b^{\beta}Q_{\alpha\beta}=\frac{Q^{i}_{0}}{\sqrt{g_{00}}}$, $h^{i}_{\alpha}h^{k}_{\beta}Q^{\alpha\beta}=Q^{ik}$. The principal physical observable properties of a space are derived from the fact that the chr.inv.-differential operators $\frac{^*\partial}{\partial t}=\frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{^*\partial}{\partial x^i}=\frac{\partial}{\partial x^i}+\frac{1}{c^2}v_i\frac{^*\partial}{\partial t}$ are non-commutative as $\frac{^*\partial^2}{\partial x^i\partial t}-\frac{^*\partial^2}{\partial t\,\partial x^i}=\frac{1}{c^2}F_i\frac{^*\partial}{\partial t}$ and $\frac{^*\partial^2}{\partial x^i\partial x^k}-\frac{^*\partial^2}{\partial x^k\partial x^i}=\frac{2}{c^2}A_{ik}\frac{^*\partial}{\partial t}$, and also that the chr.inv.-metric tensor $h_{ik}=-g_{ik}+b_i b_k$ may not be stationary. The principal physical observable characteristics are the chr.inv.-vector of the gravitational inertial force F_i, the chr.inv.-tensor of the angular velocities of the space rotation A_{ik}, and the chr.inv.-tensor of the rates of the space deformations D_{ik}:

$$F_i=\frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i}-\frac{\partial v_i}{\partial t}\right), \quad \mathrm{w}=c^2\left(1-\sqrt{g_{00}}\right), \tag{1}$$

$$A_{ik}=\frac{1}{2}\left(\frac{\partial v_k}{\partial x^i}-\frac{\partial v_i}{\partial x^k}\right)+\frac{1}{2c^2}\left(F_i v_k-F_k v_i\right), \tag{2}$$

$$D_{ik}=\frac{1}{2}\frac{^*\partial h_{ik}}{\partial t}, \quad D^{ik}=-\frac{1}{2}\frac{^*\partial h^{ik}}{\partial t}, \quad D=D^k_k=\frac{^*\partial\ln\sqrt{h}}{\partial t}, \tag{3}$$

where w is the gravitational potential, $v_i=-\frac{c\,g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation, $h_{ik}=-g_{ik}+\frac{1}{c^2}v_i v_k$ is the chr.inv.-metric tensor, $h=\det\|h_{ik}\|$, $h\,g_{00}=-g$, and $g=\det\|g_{\alpha\beta}\|$. The observable non-uniformity of the space is set up by the chr.inv.-Christoffel symbols

$$\Delta^i_{jk}=h^{im}\Delta_{jk,m}=\frac{1}{2}\,h^{im}\left(\frac{^*\partial h_{jm}}{\partial x^k}+\frac{^*\partial h_{km}}{\partial x^j}-\frac{^*\partial h_{jk}}{\partial x^m}\right), \tag{4}$$

which are constructed just like Christoffel's usual symbols $\Gamma^{\alpha}_{\mu\nu}=g^{\alpha\sigma}\Gamma_{\mu\nu,\sigma}$ using h_{ik} instead of $g_{\alpha\beta}$.

A four-dimensional generalization of the chr.inv.-quantities F_i, A_{ik}, and D_{ik} is [5]

$$F_{\alpha}=-2c^2 b^{\beta}a_{\beta\alpha}\,, \tag{5}$$

$$A_{\alpha\beta}=ch^{\mu}_{\alpha}h^{\nu}_{\beta}a_{\mu\nu}\,, \tag{6}$$

$$D_{\alpha\beta}=ch^{\mu}_{\alpha}h^{\nu}_{\beta}d_{\mu\nu}\,, \tag{7}$$

where

$$a_{\alpha\beta}=\frac{1}{2}\left(\nabla_{\alpha}b_{\beta}-\nabla_{\beta}b_{\alpha}\right), \quad d_{\alpha\beta}=\frac{1}{2}\left(\nabla_{\alpha}b_{\beta}+\nabla_{\beta}b_{\alpha}\right). \tag{8}$$

For instance, the chr.inv.-projections of F^{α} are

$$\varphi=b_{\alpha}F^{\alpha}=\frac{F_0}{\sqrt{g_{00}}}=0\,, \qquad q^i=h^i_{\alpha}F^{\alpha}=F^i. \tag{9}$$

Proceeding from the exact formula for the gravitational inertial force above, we can, for the first time, determine vortical gravitational fields.

2 D'Alembert's equations of the force

It is a matter of fact that two bodies attract each other due to the transfer of the force of gravity. The force of gravity is absent in a homogeneous gravitational field, because the gradient of the gravitational potential w is zero everywhere therein. Therefore it is reasonable to consider the field of the vector potential F^α as a medium transferring gravitational attraction via waves of the force.

D'Alembert's equations of the vector field F^α without its inducing sources

$$\Box F^\alpha = 0 \tag{10}$$

are the equations of propagation of waves traveling in the field*. The equations have two chr.inv.-projections

$$b_\sigma \Box F^\sigma = 0\,, \qquad h^i_\sigma \Box F^\sigma = 0\,, \tag{11}$$

which are the same as

$$b_\sigma\, g^{\alpha\beta}\nabla_\alpha\nabla_\beta F^\sigma = 0\,, \qquad h^i_\sigma\, g^{\alpha\beta}\nabla_\alpha\nabla_\beta F^\sigma = 0\,. \tag{12}$$

These are the chr.inv.-d'Alembert equations for the field $F^\alpha = -2c^2 a_{\sigma\cdot}^{\cdot\alpha} b^\sigma$ without its-inducing sources. To obtain the equations in detailed form isn't an easy process. Helpful here is the fact that the chr.inv.-projection of F^α upon a time line is zero. Following this path, after some algebra, we obtain the chr.inv.-d'Alembert equations (11) in the final form

$$\left.\begin{aligned}
&\frac{1}{c^2}\frac{^*\partial}{\partial t}\left(F_k F^k\right) + \frac{1}{c^2}F_i\frac{^*\partial F^i}{\partial t} + D^k_m\frac{^*\partial F^m}{\partial x^k} + \\
&+ h^{ik}\frac{^*\partial}{\partial x^i}\left[(D_{kn}+A_{kn})F^n\right] - \frac{2}{c^2}A_{ik}F^iF^k + \\
&+ \frac{1}{c^2}F_mF^mD + \Delta^m_{kn}D^k_mF^n - \\
&- h^{ik}\Delta^m_{ik}\left(D_{mn}+A_{mn}\right)F^n = 0\,, \\
&\frac{1}{c^2}\frac{^*\partial^2 F^i}{\partial t^2} - h^{km}\frac{^*\partial^2 F^i}{\partial x^k\partial x^m} + \frac{1}{c^2}\left(D^i_k + A_{k\cdot}^{\cdot i}\right)\frac{^*\partial F^k}{\partial t} + \\
&+ \frac{1}{c^2}\frac{^*\partial}{\partial t}\left[\left(D^i_k + A_{k\cdot}^{\cdot i}\right)F^k\right] + \frac{1}{c^2}D\frac{^*\partial F^i}{\partial t} + \frac{1}{c^2}F^k\frac{^*\partial F^i}{\partial x^k} + \\
&+ \frac{1}{c^2}\left(D^i_n + A_{n\cdot}^{\cdot i}\right)F^nD - \frac{1}{c^2}\Delta^i_{km}F^kF^m + \frac{1}{c^4}F_kF^kF^i - \\
&- h^{km}\left\{\frac{^*\partial}{\partial x^k}\left(\Delta^i_{mn}F^n\right) + \left(\Delta^i_{kn}\Delta^n_{mp} - \Delta^n_{km}\Delta^i_{np}\right)F^p + \right. \\
&\left. + \Delta^i_{kn}\frac{^*\partial F^n}{\partial x^m} - \Delta^n_{km}\frac{^*\partial F^i}{\partial x^n}\right\} = 0\,.
\end{aligned}\right\} \tag{13}$$

*The waves travelling in the field of the gravitational inertial force aren't the same as the waves of the weak perturbations of the space metric, routinely considered in the theory of gravitational waves.

3 A vortical gravitational field. The field tensor and pseudo-tensor. The field invariants

We introduce the tensor of the field as a rotor of its four-dimensional vector potential F^α as well as Maxwell's tensor of electromagnetic fields, namely

$$F_{\alpha\beta} = \nabla_\alpha F_\beta - \nabla_\beta F_\alpha = \frac{\partial F_\beta}{\partial x^\alpha} - \frac{\partial F_\alpha}{\partial x^\beta}\,. \tag{14}$$

We will refer to $F_{\alpha\beta}$ (14) as the *tensor of a vortical gravitational field*, because this is actual a four-dimensional vortex of an acting gravitational inertial force F^α.

Taking into account that the chr.inv.-projections of the field potential $F^\alpha = -2c^2 a_{\sigma\cdot}^{\cdot\alpha} b^\sigma$ are $\frac{F_0}{\sqrt{g_{00}}} = 0$, $F^i = h^{ik}F_k$, we obtain the components of the field tensor $F_{\alpha\beta}$:

$$F_{00} = F^{00} = 0\,, \qquad F_{0i} = -\frac{1}{c}\sqrt{g_{00}}\,\frac{^*\partial F_i}{\partial t}\,, \tag{15}$$

$$F_{ik} = \frac{^*\partial F_i}{\partial x^k} - \frac{^*\partial F_k}{\partial x^i} + \frac{1}{c^2}\left(v_i\frac{^*\partial F_k}{\partial t} - v_k\frac{^*\partial F_i}{\partial t}\right), \tag{16}$$

$$F_{0\cdot}^{\cdot 0} = \frac{1}{c^2}v^k\frac{^*\partial F_k}{\partial t}, \qquad F_{0\cdot}^{\cdot i} = \frac{1}{c}\sqrt{g_{00}}\,h^{ik}\frac{^*\partial F_k}{\partial t}, \tag{17}$$

$$\begin{aligned}F_{k\cdot}^{\cdot 0} = \frac{1}{\sqrt{g_{00}}}\left[\frac{1}{c}\frac{^*\partial F_k}{\partial t} - \frac{1}{c^3}v_kv^m\frac{^*\partial F_m}{\partial t} + \right.\\ \left. + \frac{1}{c}v^m\left(\frac{^*\partial F_m}{\partial x^k} - \frac{^*\partial F_k}{\partial x^m}\right)\right],\end{aligned} \tag{18}$$

$$F_{k\cdot}^{\cdot i} = h^{im}\left(\frac{^*\partial F_m}{\partial x^k} - \frac{^*\partial F_k}{\partial x^m}\right) - \frac{1}{c^2}h^{im}v_k\frac{^*\partial F_m}{\partial t}\,, \tag{19}$$

$$\begin{aligned}F^{0k} = \frac{1}{\sqrt{g_{00}}}\left[\frac{1}{c}h^{km}\frac{^*\partial F_m}{\partial t} + \right.\\ \left. + \frac{1}{c}v^nh^{mk}\left(\frac{^*\partial F_n}{\partial x^m} - \frac{^*\partial F_m}{\partial x^n}\right)\right],\end{aligned} \tag{20}$$

$$F^{ik} = h^{im}h^{kn}\left(\frac{^*\partial F_m}{\partial x^n} - \frac{^*\partial F_n}{\partial x^m}\right). \tag{21}$$

We see here two chr.inv.-projections of the field tensor $F_{\alpha\beta}$. We will refer to the time projection

$$E^i = \frac{F_{0\cdot}^{\cdot i}}{\sqrt{g_{00}}} = \frac{1}{c}h^{ik}\frac{^*\partial F_k}{\partial t}\,, \quad E_i = h_{ik}E^k = \frac{1}{c}\frac{^*\partial F_i}{\partial t} \tag{22}$$

as the "electric" observable component of the vortical gravitational field, while the spatial projection will be referred to as the "magnetic" observable component of the field

$$H^{ik} = F^{ik} = h^{im}h^{kn}\left(\frac{^*\partial F_m}{\partial x^n} - \frac{^*\partial F_n}{\partial x^m}\right), \tag{23}$$

$$H_{ik} = h_{im}h_{kn}H^{mn} = \frac{^*\partial F_i}{\partial x^k} - \frac{^*\partial F_k}{\partial x^i}\,, \tag{24}$$

which, after use of the 1st Zelmanov identity [2, 3] that links the spatial vortex of the gravitational inertial force to the non-stationary rotation of the observer's space

$$\frac{{}^*\partial A_{ik}}{\partial t}+\frac{1}{2}\left(\frac{{}^*\partial F_k}{\partial x^i}-\frac{{}^*\partial F_i}{\partial x^k}\right)=0\,, \qquad (25)$$

takes the form

$$H^{ik}=2h^{im}h^{kn}\frac{{}^*\partial A_{mn}}{\partial t}\,,\qquad H_{ik}=2\frac{{}^*\partial A_{ik}}{\partial t}\,. \qquad (26)$$

The "electric" observable component E^i of a vortical gravitational field manifests as the non-stationarity of the acting gravitational inertial force F^i. The "magnetic" observable component H_{ik} manifests as the presence of the spatial vortices of the force F^i or equivalently, as the non-stationarity of the space rotation A_{ik} (see formula 26). Thus, two kinds of vortical gravitational fields are possible:

1. Vortical gravitational fields of the "electric" kind ($H_{ik}=0,\ E^i\neq 0$). In this field we have no spatial vortices of the acting gravitational inertial force F^i, which is the same as a stationary space rotation. So a vortical field of this kind consists of only the "electric" component E^i (22) that is the non-stationarity of the force F^i. Note that a vortical gravitational field of the "electric" kind is permitted in both a non-holonomic (rotating) space, if its rotation is stationary, and also in a holonomic space since the zero rotation is the ultimate case of stationary rotations;
2. The "magnetic" kind of vortical gravitational fields is characterized by $E^i=0$ and $H_{ik}\neq 0$. Such a vortical field consists of only the "magnetic" components H_{ik}, which are the spatial vortices of the acting force F^i and the non-stationary rotation of the space. Therefore a vortical gravitational field of the "magnetic" kind is permitted only in a non-holonomic space. Because the d'Alembert equations (13), with the condition $E^i=0$, don't depend on time, a "magnetic" vortical gravitational field is a medium for *standing waves* of the gravitational inertial force.

In addition, we introduce the pseudotensor $F^{*\alpha\beta}$ of the field dual to the field tensor

$$F^{*\alpha\beta}=\frac{1}{2}\,E^{\alpha\beta\mu\nu}F_{\mu\nu}\,,\qquad F_{*\alpha\beta}=\frac{1}{2}\,E_{\alpha\beta\mu\nu}F^{\mu\nu}, \qquad (27)$$

where the four-dimensional completely antisymmetric discriminant tensors $E^{\alpha\beta\mu\nu}=\frac{e^{\alpha\beta\mu\nu}}{\sqrt{-g}}$ and $E_{\alpha\beta\mu\nu}=e_{\alpha\beta\mu\nu}\sqrt{-g}$ transform tensors into pseudotensors in the inhomogeneous anisotropic four-dimensional pseudo-Riemannian space*.

Using the components of the field tensor $F_{\alpha\beta}$, we obtain the chr.inv.-projections of the field pseudotensor $F^{*\alpha\beta}$:

$$H^{*i}=\frac{F_{0\cdot}^{*\cdot i}}{\sqrt{g_{00}}}=\frac{1}{2}\,\varepsilon^{ikm}\left(\frac{{}^*\partial F_k}{\partial x^m}-\frac{{}^*\partial F_m}{\partial x^k}\right), \qquad (28)$$

$$E^{*ik}=F^{*ik}=-\frac{1}{c}\,\varepsilon^{ikm}\frac{{}^*\partial F_m}{\partial t}\,, \qquad (29)$$

where $\varepsilon^{ikm}=b_0\,E^{0ikm}=\sqrt{g_{00}}\,E^{0ikm}=\frac{e^{ikm}}{\sqrt{h}}$ and $\varepsilon_{ikm}=b^0E_{0ikm}=\frac{E_{0ikm}}{\sqrt{g_{00}}}=e_{ikm}\sqrt{h}$ are the chr.inv.-discriminant tensors [2]. Taking into account the 1st Zelmanov identity (25) and the formulae for differentiating ε^{ikm} and ε_{ikm} [2]

$$\frac{{}^*\partial\varepsilon_{imn}}{\partial t}=\varepsilon_{imn}\,D\,,\qquad \frac{{}^*\partial\varepsilon^{imn}}{\partial t}=-\,\varepsilon^{imn}D\,, \qquad (30)$$

we write the "magnetic" component H^{*i} as follows

$$H^{*i}=\varepsilon^{ikm}\frac{{}^*\partial A_{km}}{\partial t}=2\left(\frac{{}^*\partial\Omega^{*i}}{\partial t}+\Omega^{*i}D\right), \qquad (31)$$

where $\Omega^{*i}=\frac{1}{2}\,\varepsilon^{ikm}A_{km}$ is the chr.inv.-pseudovector of the angular velocity of the space rotation, while the trace $D=h^{ik}D_{ik}=D_n^n$ of the tensor D_{ik} is the rate of the relative expansion of an elementary volume permeated by the field.

Calculating the invariants of a vortical gravitational field ($J_1=F_{\alpha\beta}F^{\alpha\beta}$ and $J_2=F_{\alpha\beta}F^{*\alpha\beta}$), we obtain

$$J_1=h^{im}h^{kn}\left(\frac{{}^*\partial F_i}{\partial x^k}-\frac{{}^*\partial F_k}{\partial x^i}\right)\left(\frac{{}^*\partial F_m}{\partial x^n}-\frac{{}^*\partial F_n}{\partial x^m}\right)-\\ -\frac{2}{c^2}\,h^{ik}\frac{{}^*\partial F_i}{\partial t}\frac{{}^*\partial F_k}{\partial t}, \qquad (32)$$

$$J_2=-\frac{2}{c}\,\varepsilon^{imn}\left(\frac{{}^*\partial F_m}{\partial x^n}-\frac{{}^*\partial F_n}{\partial x^m}\right)\frac{{}^*\partial F_i}{\partial t}\,, \qquad (33)$$

which, with the 1st Zelmanov identity (25), are

$$J_1=4h^{im}h^{kn}\frac{{}^*\partial A_{ik}}{\partial t}\frac{{}^*\partial A_{mn}}{\partial t}-\frac{2}{c^2}\,h^{ik}\frac{{}^*\partial F_i}{\partial t}\frac{{}^*\partial F_k}{\partial t}\,, \qquad (34)$$

$$J_2=-\frac{4}{c}\,\varepsilon^{imn}\frac{{}^*\partial A_{mn}}{\partial t}\frac{{}^*\partial F_i}{\partial t}=\\ =-\frac{8}{c}\left(\frac{{}^*\partial\Omega^{*i}}{\partial t}+\Omega^{*i}D\right)\frac{{}^*\partial F_i}{\partial t}\,. \qquad (35)$$

By the strong physical condition of isotropy, a field is isotropic if both invariants of the field are zeroes: $J_1=0$ means that the lengths of the "electric" and the "magnetic" components of the field are the same, while $J_2=0$ means that the components are orthogonal to each other. Owning the case of a vortical gravitational field, we see that such a field is isotropic if the common conditions are true

$$\left.\begin{aligned} &h^{im}h^{kn}\frac{{}^*\partial A_{ik}}{\partial t}\frac{{}^*\partial A_{mn}}{\partial t}=\frac{1}{2c^2}\,h^{ik}\frac{{}^*\partial F_i}{\partial t}\frac{{}^*\partial F_k}{\partial t}\\ &\frac{{}^*\partial A_{mn}}{\partial t}\frac{{}^*\partial F_i}{\partial t}=0 \end{aligned}\right\} \qquad (36)$$

*Here $e^{\alpha\beta\mu\nu}$ and $e_{\alpha\beta\mu\nu}$ are Levi-Civita's unit tensors: the four-dimensional completely antisymmetric unit tensors which transform tensors into pseudotensors in a Galilean reference frame in the four-dimensional pseudo-Euclidean space [1].

however their geometrical sense is not clear.

Thus the anisotropic field can only be a mixed vortical gravitational field bearing both the "electric" and the "magnetic" components. A strictly "electric" or "magnetic" vortical gravitational field is always spatially isotropic.

Taking the above into account, we arrive at the necessary and sufficient conditions for the existence of *standing waves of the gravitational inertial force*:

1. A vortical gravitational field of the strictly "magnetic" kind is the medium for standing waves of the gravitational inertial force;
2. Standing waves of the gravitational inertial force are permitted only in a non-stationary rotating space.

As soon as one of the conditions ceases, the acting gravitational inertial force changes: the standing waves of the force transform into traveling waves.

4 The field equations of a vortical gravitational field

It is known from the theory of fields that the field equations of a field of a four-dimensional vector-potential A^α is a system consisting of 10 equations in 10 unknowns:

- Lorentz's condition $\nabla_\sigma A^\sigma = 0$ states that the four-dimensional potential A^α remains unchanged;
- the continuity equation $\nabla_\sigma j^\sigma = 0$ states that the field-inducing sources ("charges" and "currents") can not be destroyed but merely re-distributed in the space;
- two groups ($\nabla_\sigma F^{\alpha\sigma} = \frac{4\pi}{c} j^\alpha$ and $\nabla_\sigma F^{*\alpha\sigma} = 0$) of the Maxwell-like equations, where the 1st group determines the "charge" and the "current" as the components of the four-dimensional current vector j^α of the field.

This system completely determines a vector field A^α and its sources in a pseudo-Riemannian space. We shall deduce the field equations for a vortical gravitational field as a field of the four-dimensional potential $F^\alpha = -2c^2 a_{\sigma\cdot}^{\cdot\alpha} b^\sigma$.

Writing the divergence $\nabla_\sigma F^\sigma = \frac{\partial F^\sigma}{\partial x^\sigma} + \Gamma^\sigma_{\sigma\mu} F^\mu$ in the chr.inv.-form [2, 3]

$$\nabla_\sigma F^\sigma = \frac{1}{c}\left(\frac{^*\partial\varphi}{\partial t} + \varphi D\right) + \frac{^*\partial q^i}{\partial x^i} + q^i \frac{^*\partial \ln\sqrt{h}}{\partial x^i} - \frac{1}{c^2} F_i q^i \quad (37)$$

where $\frac{^*\partial \ln\sqrt{h}}{\partial x^i} = \Delta^j_{ji}$ and $\frac{^*\partial q^i}{\partial x^i} + q^i \Delta^j_{ji} = {}^*\nabla_i q^i$, we obtain the *chr.inv.-Lorentz condition* in a vortical gravitational field

$$\frac{^*\partial F^i}{\partial x^i} + F^i \Delta^j_{ji} - \frac{1}{c^2} F_i F^i = 0\,. \quad (38)$$

To deduce the Maxwell-like equations for a vortical gravitational field, we collect together the chr.inv.-projections of the field tensor $F_{\alpha\beta}$ and the field pseudotensor $F^{*\alpha\beta}$. Expressing the necessary projections with the tensor of the rate of the space deformation D^{ik} to eliminate the free h^{ik} terms, we obtain

$$E^i = \frac{1}{c}\, h^{ik} \frac{^*\partial F_k}{\partial t} = \frac{1}{c}\frac{^*\partial F^i}{\partial t} + \frac{2}{c} F_k D^{ik}, \quad (39)$$

$$\begin{aligned} H^{ik} &= 2h^{im} h^{kn} \frac{^*\partial A_{mn}}{\partial t} = \\ &= 2\frac{^*\partial A^{ik}}{\partial t} + 4\left(A^{i\cdot}_{\cdot n} D^{kn} - A^{k\cdot}_{\cdot m} D^{im}\right), \end{aligned} \quad (40)$$

$$H^{*i} = \varepsilon^{imn} \frac{^*\partial A_{mn}}{\partial t} = 2\frac{^*\partial \Omega^{*i}}{\partial t} + 2\Omega^{*i} D\,, \quad (41)$$

$$E^{*ik} = -\frac{1}{c}\, \varepsilon^{ikm} \frac{^*\partial F_m}{\partial t}\,. \quad (42)$$

After some algebra, we obtain the *chr.inv.-Maxwell-like equations* for a vortical gravitational field

$$\left.\begin{aligned} &\frac{1}{c}\frac{^*\partial^2 F^i}{\partial x^i \partial t} + \frac{2}{c}\frac{^*\partial}{\partial x^i}\left(F_k D^{ik}\right) + \frac{1}{c}\left(\frac{^*\partial F^i}{\partial t} + 2F_k D^{ik}\right)\Delta^j_{ji} - \\ &\qquad - \frac{2}{c} A_{ik}\left(\frac{^*\partial A^{ik}}{\partial t} + A^{i\cdot}_{\cdot n} D^{kn}\right) = 4\pi\rho \\ &2\frac{^*\partial^2 A^{ik}}{\partial x^k \partial t} - \frac{1}{c^2}\frac{^*\partial^2 F^i}{\partial t^2} + 4\frac{^*\partial}{\partial x^k}\left(A^{i\cdot}_{\cdot n} D^{kn} - A^{k\cdot}_{\cdot m} D^{im}\right) + \\ &+ 2\left(\Delta^j_{jk} - \frac{1}{c^2}F_k\right)\left\{\frac{^*\partial A^{ik}}{\partial t} + 2\left(A^{i\cdot}_{\cdot n} D^{kn} - A^{k\cdot}_{\cdot m} D^{im}\right)\right\} - \\ &- \frac{2}{c^2}\frac{^*\partial}{\partial t}\left(F_k D^{ik}\right) - \frac{1}{c^2}\left(\frac{^*\partial F^i}{\partial t} + 2F_k D^{ik}\right) D = \frac{4\pi}{c} j^i \end{aligned}\right\} \text{Group I,} \quad (43)$$

$$\left.\begin{aligned} &\frac{^*\partial^2 \Omega^{*i}}{\partial x^i \partial t} + \frac{^*\partial}{\partial x^i}\left(\Omega^{*i} D\right) + \frac{1}{c^2}\,\Omega^{*m}\frac{^*\partial F_m}{\partial t} + \\ &\qquad + \left(\frac{^*\partial \Omega^{*i}}{\partial t} + \Omega^{*i} D\right)\Delta^j_{ji} = 0 \\ &\varepsilon^{ikm}\frac{^*\partial^2 F_m}{\partial x^k \partial t} + \varepsilon^{ikm}\left(\Delta^j_{jk} - \frac{1}{c^2}F_k\right)\frac{^*\partial F_m}{\partial t} + 2\frac{^*\partial^2 \Omega^{*i}}{\partial t^2} + \\ &\qquad + 4D\frac{^*\partial \Omega^{*i}}{\partial t} + 2\left(\frac{^*\partial D}{\partial t} + D^2\right)\Omega^{*i} = 0 \end{aligned}\right\} \text{Group II.} \quad (44)$$

The *chr.inv.-continuity equation* $\nabla_\sigma j^\sigma = 0$ for a vortical gravitational field follows from the 1st group of the Maxwell-like equations, and is

$$\begin{aligned} &\frac{^*\partial^2}{\partial x^i \partial x^k}\left(\frac{^*\partial A^{ik}}{\partial t}\right) - \frac{1}{c^2}\left(\frac{^*\partial A^{ik}}{\partial t} + A^{i\cdot}_{\cdot n} D^{kn}\right)\left(A_{ik} D + \frac{^*\partial A_{ik}}{\partial t}\right) - \\ &- \frac{1}{c^2}\left[\frac{^*\partial^2 A^{ik}}{\partial t^2} + \frac{^*\partial}{\partial t}\left(A^{i\cdot}_{\cdot n} D^{nk}\right)\right] A_{ik} + \frac{1}{2c^2}\left(\frac{^*\partial F^i}{\partial t} + 2F_k D^{ik}\right)\times \\ &\times\left(\frac{^*\partial \Delta^j_{ji}}{\partial t} + \frac{D}{c^2}F_i - \frac{^*\partial D}{\partial x^i}\right) + 2\frac{^*\partial^2}{\partial x^i \partial x^k}\left(A^{i\cdot}_{\cdot n} D^{kn} - A^{k\cdot}_{\cdot m} D^{im}\right) + \\ &+ \left[\frac{^*\partial A^{ik}}{\partial t} + 2\left(A^{i\cdot}_{\cdot n} D^{kn} - A^{k\cdot}_{\cdot m} D^{im}\right)\right]\left[\frac{^*\partial}{\partial x^i}\left(\Delta^j_{jk} - \frac{1}{c^2}F_k\right) + \right. \\ &\qquad \left. + \left(\Delta^j_{ji} - \frac{1}{c^2}F_i\right)\left(\Delta^l_{lk} - \frac{1}{c^2}F_k\right)\right] = 0\,. \end{aligned} \quad (45)$$

To see a simpler sense of the obtained field equations, we take the field equations in a homogeneous space ($\Delta^i_{km} = 0$)

free of deformation ($D_{ik} = 0$)*. In such a space the chr.inv.-Maxwell-like equations obtained take the simplified form

$$\left.\begin{aligned} &\frac{1}{c}\frac{^*\partial^2 F^i}{\partial x^i \partial t} - \frac{2}{c}A_{ik}\frac{^*\partial A^{ik}}{\partial t} = 4\pi\rho \\ &2\frac{^*\partial^2 A^{ik}}{\partial x^k \partial t} - \frac{2}{c^2}F_k\frac{^*\partial A^{ik}}{\partial t} - \frac{1}{c^2}\frac{^*\partial^2 F^i}{\partial t^2} = \frac{4\pi}{c}\,j^i \end{aligned}\right\}\text{Group I,} \tag{46}$$

$$\left.\begin{aligned} &\frac{^*\partial^2 \Omega^{*i}}{\partial x^i \partial t} + \frac{1}{c^2}\,\Omega^{*m}\frac{^*\partial F_m}{\partial t} = 0 \\ &\varepsilon^{ikm}\frac{^*\partial^2 F_m}{\partial x^k \partial t} - \frac{1}{c^2}\,\varepsilon^{ikm}F_k\frac{^*\partial F_m}{\partial t} + 2\frac{^*\partial^2 \Omega^{*i}}{\partial t^2} = 0 \end{aligned}\right\}\text{Group II,} \tag{47}$$

where the field-inducing sources are

$$\rho = \frac{1}{4\pi c}\left(\frac{^*\partial^2 F^i}{\partial x^i \partial t} - 2A_{ik}\frac{^*\partial A^{ik}}{\partial t}\right), \tag{48}$$

$$j^i = \frac{c}{2\pi}\left(\frac{^*\partial^2 A^{ik}}{\partial x^k \partial t} - \frac{1}{c^2}F_k\frac{^*\partial A^{ik}}{\partial t} - \frac{1}{2c^2}\frac{^*\partial^2 F^i}{\partial t^2}\right), \tag{49}$$

and the chr.inv.-continuity equation (45) takes the form

$$\begin{aligned} &\frac{^*\partial^2}{\partial x^i \partial x^k}\left(\frac{^*\partial A^{ik}}{\partial t}\right) - \frac{1}{c^2}A_{ik}\frac{^*\partial^2 A^{ik}}{\partial t^2} - \frac{1}{c^2}\frac{^*\partial A_{ik}}{\partial t}\frac{^*\partial A^{ik}}{\partial t} - \\ &\qquad - \frac{1}{c^2}\left(\frac{^*\partial F_k}{\partial x^i} - \frac{1}{c^2}F_i F_k\right)\frac{^*\partial A^{ik}}{\partial t} = 0\,. \end{aligned} \tag{50}$$

The obtained field equations describe the main properties of vortical gravitational fields:

1. The chr.inv.-Lorentz condition (38) shows the inhomogeneity of a vortical gravitational field depends on the value of the acting gravitational inertial force F^i and also the space inhomogeneity Δ^j_{ji} in the direction the force acts;
2. The 1st group of the chr.inv.-Maxwell-like equations (43) manifests the origin of the field-inducing sources called "charges" ρ and "currents" j^i. The "charge" ρ is derived from the inhomogeneous oscillations of the acting force F^i and also the non-stationary rotation of the space (to within the space inhomogeneity and deformation withheld). The "currents" j^i are derived from the non-stationary rotation of the space, the spatial inhomogeneity of the non-stationarity, and the non-stationary oscillations of the force F^i (to within the same approximation);
3. The 2nd group of the chr.inv.-Maxwell-like equations (44) manifests the properties of the "magnetic" component H^{*i} of the field. The oscillations of the acting force F^i is the main factor making the "magnetic" component distributed inhomogeneously in the space. If there is no acting force ($F^i = 0$) and the space is free of deformation ($D_{ik} = 0$), the "magnetic" component is stationary.
4. The chr.inv.-continuity equation (50) manifests in the fact that the "charges" and the "currents" inducing a vortical gravitational field, being located in a non-deforming homogeneous space, remain unchanged while the space rotation remains stationary.

*Such a space has no waves of the space metric (waves the space deformation), however waves of the gravitational inertial force are permitted therein.

Properties of waves travelling in a field of a gravitational inertial force reveal themselves when we equate the field sources ρ and j^i to zero in the field equations (because a free field is a wave):

$$\frac{^*\partial^2 F^i}{\partial x^i \partial t} = 2A_{ik}\frac{^*\partial A^{ik}}{\partial t}\,, \tag{51}$$

$$\frac{^*\partial^2 A^{ik}}{\partial x^k \partial t} = \frac{1}{c^2}F_k\frac{^*\partial A^{ik}}{\partial t} + \frac{1}{2c^2}\frac{^*\partial^2 F^i}{\partial t^2}\,, \tag{52}$$

which lead us to the following conclusions:

1. The inhomogeneous oscillations of the gravitational inertial force F^i, acting in a free vortical gravitational field, is derived mainly from the non-stationary rotation of the space;
2. The inhomogeneity of the non-stationary rotations of a space, filled with a free vortical gravitational field, is derived mainly from the non-stationarity of the oscillations of the force and also the absolute values of the force and the angular acceleration of the space.

The foregoing results show that numerous properties of vortical gravitational fields manifest only if such a field is due strictly to the "electric" or the "magnetic" kind. This fact forces us to study these two kinds of vortical gravitational fields separately.

5 A vortical gravitational field of the "electric" kind

We shall consider a vortical gravitational field strictly of the "electric" kind, which is characterized as follows

$$H_{ik} = \frac{^*\partial F_i}{\partial x^k} - \frac{^*\partial F_k}{\partial x^i} = 2\frac{^*\partial A_{ik}}{\partial t} = 0\,, \tag{53}$$

$$H^{ik} = 2h^{im}h^{kn}\frac{^*\partial A_{mn}}{\partial t} = 0\,, \tag{54}$$

$$E_i = \frac{1}{c}\frac{^*\partial F_i}{\partial t} \neq 0\,, \tag{55}$$

$$E^i = \frac{1}{c}\,h^{ik}\frac{^*\partial F_k}{\partial t} = \frac{1}{c}\frac{^*\partial F^i}{\partial t} + \frac{2}{c}F_k D^{ik} \neq 0\,, \tag{56}$$

$$H^{*i} = \varepsilon^{imn}\frac{^*\partial A_{mn}}{\partial t} = 2\frac{^*\partial \Omega^{*i}}{\partial t} + 2\Omega^{*i}D = 0\,, \tag{57}$$

$$E^{*ik} = -\frac{1}{c}\,\varepsilon^{ikm}\frac{^*\partial F_m}{\partial t} \neq 0\,. \tag{58}$$

We are actually considering a stationary rotating space (if it rotates) filled with the field of a non-stationary gravitational inertial force without spatial vortices of the force. This is the main kind of vortical gravitational fields, because a non-stationary rotation of a space body is very rare (see the "magnetic" kind of fields in the next Section).

In this case the chr.inv.-Lorentz condition doesn't change to the general formula (38), because the condition does not have the components of the field tensor $F_{\alpha\beta}$.

The field invariants $J_1 = F_{\alpha\beta}F^{\alpha\beta}$ and $J_2 = F_{\alpha\beta}F^{*\alpha\beta}$ (34, 35) in this case are

$$J_1 = -\frac{2}{c^2}\, h^{ik}\frac{{}^*\partial F_i}{\partial t}\frac{{}^*\partial F_k}{\partial t}\,, \qquad J_2 = 0\,. \tag{59}$$

The chr.inv.-Maxwell-like equations for a vortical gravitational field strictly of the "electric" kind are

$$\left.\begin{aligned} &{}^*\nabla_i\, E^i = 4\pi\rho \\ &\frac{1}{c}\left(\frac{{}^*\partial E^i}{\partial t} + E^i D\right) = -\frac{4\pi}{c}\, j^i \end{aligned}\right\}\ \text{Group I,} \tag{60}$$

$$\left.\begin{aligned} &E^{*ik}A_{ik} = 0 \\ &{}^*\nabla_k\, E^{*ik} - \frac{1}{c^2}F_k E^{*ik} = 0 \end{aligned}\right\}\ \text{Group II,} \tag{61}$$

and, after E^i and E^{*ik} are substituted, take the form

$$\left.\begin{aligned} &\frac{1}{c}\frac{{}^*\partial^2 F^i}{\partial x^i\partial t} + \frac{1}{c}\left(\frac{{}^*\partial F^i}{\partial t} + 2F_k D^{ik}\right)\Delta^j_{ji} + \\ &\qquad + \frac{2}{c}\frac{{}^*\partial}{\partial x^i}\left(F_k D^{ik}\right) = 4\pi\rho \\ &\frac{1}{c^2}\frac{{}^*\partial^2 F^i}{\partial t^2} + \frac{2}{c^2}\frac{{}^*\partial}{\partial t}\left(F_k D^{ik}\right) + \\ &\qquad + \frac{1}{c^2}\left(\frac{{}^*\partial F^i}{\partial t} + 2F_k D^{ik}\right)D = -\frac{4\pi}{c}\, j^i \end{aligned}\right\}\ \text{Group I,} \tag{62}$$

$$\left.\begin{aligned} &\frac{1}{c^2}\,\Omega^{*m}\frac{{}^*\partial F_m}{\partial t} = 0 \\ &\varepsilon^{ikm}\frac{{}^*\partial^2 F_m}{\partial x^k\partial t} + \varepsilon^{ikm}\left(\Delta^j_{jk} - \frac{1}{c^2}F_k\right)\frac{{}^*\partial F_m}{\partial t} = 0 \end{aligned}\right\}\ \text{Group II.} \tag{63}$$

The chr.inv.-continuity equation for such a field, in the general case of a deforming inhomogeneous space, takes the following form

$$\left(\frac{{}^*\partial F^i}{\partial t} + 2F_k D^{ik}\right)\left(\frac{{}^*\partial \Delta^j_{ji}}{\partial t} - \frac{{}^*\partial D}{\partial x^i} + \frac{D}{c^2}F_i\right) = 0\,, \tag{64}$$

and becomes the identity "zero equal to zero" in the absence of space inhomogeneity and deformation. In fact, the chr.inv.-continuity equation implies that one of the conditions

$$\frac{{}^*\partial F^i}{\partial t} = -2F_k D^{ik}, \qquad \frac{{}^*\partial \Delta^j_{ji}}{\partial t} = \frac{{}^*\partial D}{\partial x^i} - \frac{D}{c^2}F_i \tag{65}$$

or both, are true in such a vortical gravitational field.

The chr.inv.-Maxwell-like equations (62, 63) in a non-deforming homogeneous space become much simpler

$$\left.\begin{aligned} &\frac{1}{c}\frac{{}^*\partial^2 F^i}{\partial x^i\partial t} = 4\pi\rho \\ &\frac{1}{c^2}\frac{{}^*\partial^2 F^i}{\partial t^2} = -\frac{4\pi}{c}\, j^i \end{aligned}\right\}\ \text{Group I,} \tag{66}$$

$$\left.\begin{aligned} &\frac{1}{c^2}\,\Omega^{*m}\frac{{}^*\partial F_m}{\partial t} = 0 \\ &\varepsilon^{ikm}\frac{{}^*\partial^2 F_m}{\partial x^k\partial t} - \frac{1}{c^2}\,\varepsilon^{ikm}F_k\frac{{}^*\partial F_m}{\partial t} = 0 \end{aligned}\right\}\ \text{Group II.} \tag{67}$$

The field equations obtained specify the properties for vortical gravitational fields of the "electric" kind:

1. The field-inducing sources ρ and j^i are derived mainly from the inhomogeneous oscillations of the acting gravitational inertial force F^i (the "charges" ρ) and the non-stationarity of the oscillations (the "currents" j^i);
2. Such a field is permitted in a rotating space $\Omega^{*i} \neq 0$, if the space is inhomogeneous ($\Delta^i_{kn} \neq 0$) and deforming ($D_{ik} \neq 0$). The field is permitted in a non-deforming homogeneous space, if the space is holonomic ($\Omega^{*i} = 0$);
3. Waves of the acting force F^i travelling in such a field are permitted in the case where the oscillations of the force are homogeneous and stable;
4. The sources ρ and q^i inducing such a field remain constant in a non-deforming homogeneous space.

6 A vortical gravitational field of the "magnetic" kind

A vortical gravitational field strictly of the "magnetic" kind is characterized by its own observable components

$$H_{ik} = \frac{{}^*\partial F_i}{\partial x^k} - \frac{{}^*\partial F_k}{\partial x^i} = 2\frac{{}^*\partial A_{ik}}{\partial t} \neq 0\,, \tag{68}$$

$$H^{ik} = 2h^{im}h^{kn}\frac{{}^*\partial A_{mn}}{\partial t} \neq 0\,, \tag{69}$$

$$E_i = \frac{1}{c}\frac{{}^*\partial F_i}{\partial t} = 0\,, \tag{70}$$

$$E^i = \frac{1}{c}\, h^{ik}\frac{{}^*\partial F_k}{\partial t} = \frac{1}{c}\frac{{}^*\partial F^i}{\partial t} + \frac{2}{c}\, F_k D^{ik} = 0\,, \tag{71}$$

$$H^{*i} = \varepsilon^{imn}\frac{{}^*\partial A_{mn}}{\partial t} = 2\frac{{}^*\partial \Omega^{*i}}{\partial t} + 2\Omega^{*i}D \neq 0\,, \tag{72}$$

$$E^{*ik} = -\frac{1}{c}\,\varepsilon^{ikm}\frac{{}^*\partial F_m}{\partial t} = 0\,. \tag{73}$$

Actually, in such a case, we have a non-stationary rotating space filled with the spatial vortices of a stationary gravitational inertial force F_i. Such kinds of vortical gravitational fields are exotic compared to those of the "electric" kind, because a non-stationary rotation of a bulky space body (planet, star, galaxy) — the generator of such a field — is a very rare phenomenon in the Universe.

In this case the chr.inv.-Lorentz condition doesn't change to the general formula (38) or for a vortical gravitational field of the "electric" kind, because the condition has no components of the field tensor $F_{\alpha\beta}$.

The field invariants (34, 35) in the case are

$$J_1 = 4h^{im}h^{kn}\frac{{}^*\partial A_{ik}}{\partial t}\frac{{}^*\partial A_{mn}}{\partial t}\,, \qquad J_2 = 0\,. \tag{74}$$

The chr.inv.-Maxwell-like equations for a vortical gravitational field strictly of the "magnetic" kind are

$$\left.\begin{aligned} &\frac{1}{c}\,H^{ik}A_{ik} = -4\pi\rho \\ &{}^*\nabla_k\,H^{ik} - \frac{1}{c^2}F_k H^{ik} = \frac{4\pi}{c}\,j^i \end{aligned}\right\}\ \text{Group I,} \tag{75}$$

$$\left.\begin{aligned} &{}^*\nabla_i\,H^{*i} = 0 \\ &\frac{{}^*\partial H^{*i}}{\partial t} + H^{*i}D = 0 \end{aligned}\right\}\ \text{Group II,} \tag{76}$$

which, after substituting for H^{ik} and H^{*i}, are

$$\left.\begin{aligned} &\frac{1}{c}\,A^{ik}\frac{{}^*\partial A_{ik}}{\partial t} = -\,2\pi\rho \\ &\frac{{}^*\partial^2 A^{ik}}{\partial x^k\partial t} + 2\frac{{}^*\partial}{\partial x^k}\left(A^{i\cdot}_{\cdot n}D^{kn} - A^{k\cdot}_{\cdot m}D^{im}\right) + \left(\Delta^j_{jk} - \frac{1}{c^2}F_k\right)\times \\ &\quad\times\left\{\frac{{}^*\partial A^{ik}}{\partial t} + 2\left(A^{i\cdot}_{\cdot n}D^{kn} - A^{k\cdot}_{\cdot m}D^{im}\right)\right\} = \frac{2\pi}{c}\,j^i \end{aligned}\right\}\ \text{Group I,} \tag{77}$$

$$\left.\begin{aligned} &\frac{{}^*\partial^2\Omega^{*i}}{\partial x^i\partial t} + \frac{{}^*\partial}{\partial x^i}\left(\Omega^{*i}D\right) + \left(\frac{{}^*\partial\Omega^{*i}}{\partial t} + \Omega^{*i}D\right)\Delta^j_{ji} = 0 \\ &\frac{{}^*\partial^2\Omega^{*i}}{\partial t^2} + \frac{{}^*\partial}{\partial t}\left(\Omega^{*i}D\right) + \left(\frac{{}^*\partial\Omega^{*i}}{\partial t} + \Omega^{*i}D\right)D = 0 \end{aligned}\right\}\ \text{Group II.} \tag{78}$$

The chr.inv.-continuity equation for such a field, in a deforming inhomogeneous space, is

$$\begin{aligned} &\frac{{}^*\partial^2}{\partial x^i\partial x^k}\left(\frac{{}^*\partial A^{ik}}{\partial t}\right) - \frac{1}{c^2}A^{ik}\frac{{}^*\partial^2 A_{ik}}{\partial t^2} - \frac{1}{c^2}\left(\frac{{}^*\partial A^{ik}}{\partial t} + A^{ik}D\right)\times \\ &\quad\times\frac{{}^*\partial A_{ik}}{\partial t} + 2\frac{{}^*\partial^2}{\partial x^i\partial x^k}\left(A^{i\cdot}_{\cdot n}D^{kn} - A^{k\cdot}_{\cdot m}D^{im}\right) + \left\{\frac{{}^*\partial A^{ik}}{\partial t} + \right. \\ &\quad\left. + 2\left(A^{i\cdot}_{\cdot n}D^{kn} - A^{k\cdot}_{\cdot m}D^{im}\right)\right\}\left\{\left(\frac{{}^*\partial\Delta^j_{jk}}{\partial x^i} - \frac{1}{c^2}\frac{{}^*\partial F_k}{\partial x^i} + \right.\right. \\ &\quad\left.\left. + \left(\Delta^j_{jk} - \frac{1}{c^2}F_k\right)\left(\Delta^l_{li} - \frac{1}{c^2}F_i\right)\right\} = 0\,. \end{aligned} \tag{79}$$

If the space is homogeneous and free of deformation, the continuity equation becomes

$$\begin{aligned} &\frac{{}^*\partial^2}{\partial x^i\partial x^k}\left(\frac{{}^*\partial A^{ik}}{\partial t}\right) - \frac{1}{c^2}A^{ik}\frac{{}^*\partial^2 A_{ik}}{\partial t^2} - \\ &\quad - \frac{1}{c^2}\left(\frac{{}^*\partial A_{ik}}{\partial t} + \frac{{}^*\partial F_k}{\partial x^i} - \frac{1}{c^2}F_iF_k\right)\frac{{}^*\partial A^{ik}}{\partial t} = 0\,. \end{aligned} \tag{80}$$

In such a case (a homogeneous space free of deformation) the chr.inv.-Maxwell-like equations (77, 78) become

$$\left.\begin{aligned} &\frac{1}{c}\,A^{ik}\frac{{}^*\partial A_{ik}}{\partial t} = -2\pi\rho \\ &\frac{{}^*\partial^2 A^{ik}}{\partial x^k\partial t} - \frac{1}{c^2}F_k\frac{{}^*\partial A^{ik}}{\partial t} = \frac{2\pi}{c}\,j^i \end{aligned}\right\}\ \text{Group I,} \tag{81}$$

$$\left.\begin{aligned} &\frac{{}^*\partial^2\Omega^{*i}}{\partial x^i\partial t} = 0 \\ &\frac{{}^*\partial^2\Omega^{*i}}{\partial t^2} = 0 \end{aligned}\right\}\ \text{Group II.} \tag{82}$$

The obtained field equations characterizing a vortical gravitational field of the "magnetic" kind specify the properties of such kinds of fields:

1. The field-inducing "charges" ρ are derived mainly from the non-stationary rotation of the space, while the field "currents" j^i are derived mainly from the non-stationarity and its spatial inhomogeneity;
2. Such a field is permitted in a non-deforming homogeneous space, if the space rotates homogeneously at a constant acceleration;
3. Waves in such a field are standing waves of the acting gravitational inertial force. The waves are permitted only in a space which is inhomogeneous ($\Delta^i_{kn} \neq 0$) and deforming ($D_{ik} \neq 0$);
4. The sources ρ and j^i inducing such a field remain unchanged in a non-deforming homogeneous space where $F^i \neq 0$.

7 Conclusions

According the foregoing results, we conclude that the main kind of vortical gravitational fields is "electric", derived from a non-stationary gravitational inertial force and, in part, the space deformation. Such a field is a medium for traveling waves of the gravitational inertial force. Standing waves of a gravitational inertial force are permitted in a vortical gravitational field of the "magnetic" kind (spatial vortices of a gravitational inertial force or, that is the same, a non-stationary rotation of the space). Standing waves of the gravitational inertial force and their medium, a vortical gravitational field of the "magnetic" kind, are exotic, due to a non-stationary rotation of a bulky space body (the source of such a field) is a very rare phenomenon in the Universe.

It is a matter of fact that gravitational attraction is an everyday reality, so the traveling waves of the gravitational inertial force transferring the attraction should be incontrovertible. I think that the satellite experiment, propounded in [6], would detect the travelling waves since the amplitudes of the lunar or the solar flow waves should be perceptible.

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References

1. Landau L. D. and Lifshitz E. M. The classical theory of fields. Butterworth–Heinemann, 2003, 428 pages (4th edition).
2. Zelmanov A. L. Chronometric invariants. Dissertation thesis, 1944. American Research Press, Rehoboth (NM), 2006.
3. Zelmanov A. L. Chronometric invariants and co-moving coordinates in the general relativity theory. *Doklady Acad. Nauk USSR*, 1956, v. 107(6), 815 – 818.
4. Rabounski D. Zelmanov's anthropic principle and the infinite relativity principle. *Progress in Physics*, 2005, v. 1, 35 –37.
5. Zelmanov A. L. Orthometric form of monad formalism and its relations to chronometric and kinemetric invariants. *Doklady Acad. Nauk USSR*, 1976, v. 227 (1), 78–81.
6. Rabounski D. A new method to measure the speed of gravitation. *Progress in Physics*, 2005, v. 1, 3–6; The speed of gravitation. *Proc. of the Intern. Meeting PIRT-2005*, Moscow, 2005, 106–111.