On Line-Elements and Radii: A Correction

Stephen J. Crothers

Queensland, Australia

E-mail: thenarmis@yahoo.com

Using a manifold with boundary various line-elements have been proposed as solutions to Einstein's gravitational field. It is from such line-elements that black holes, expansion of the Universe, and big bang cosmology have been alleged. However, it has been proved that black holes, expansion of the Universe, and big bang cosmology are not consistent with General Relativity. In a previous paper disproving the black hole theory, the writer made an error which, although minor and having no effect on the conclusion that black holes are inconsistent with General Relativity, is corrected herein for the record.

1 Introduction

In a previous paper [1] (see page 8 therein) the writer made the following claim:

"*the ratio* $\frac{\chi}{R_p} > 2\pi$ *for all finite* R_p"

where R_p is the proper radius and χ is the circumference of a great circle. This is not correct. In fact, the ratio $\frac{\chi}{R_p}$ is greater than 2π for some values of R_p and is less than 2π for other values of R_p. Furthermore, there is a value of χ for which $\frac{\chi}{R_p} = 2\pi$, thereby making $R_p = R_c$, where R_c is the radius of curvature. Thus, if the transitional value of the circumference of a great circle is χ_e, then

$$\chi < \chi_e \Rightarrow \frac{\chi}{R_p} > 2\pi,$$

$$\chi = \chi_e \Rightarrow \frac{\chi}{R_p} = 2\pi,$$

$$\chi > \chi_e \Rightarrow \frac{\chi}{R_p} < 2\pi.$$

2 Correction — details

Consider the general static vacuum line-element

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \quad (1)$$

$$A(r), B(r), C(r) > 0.$$

It has been shown in [1] that the solution to (1) is

$$ds^2 = \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right)dt^2 - \frac{1}{1 - \frac{\alpha}{\sqrt{C(r)}}}\, d\sqrt{C(r)}^{\,2} - C(r)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \quad (2)$$

$$\alpha < \sqrt{C(r)} < \infty,$$

where, using $c = G = 1$,

$$R_c = R_c(r) = \sqrt{C(r)} = \left(\left|r - r_0\right|^n + \alpha^n\right)^{\frac{1}{n}},$$

$$R_p = R_p(r) = \sqrt{R_c(r)\left(R_c(r) - \alpha\right)} + \alpha \left|\frac{R_c(r) + \sqrt{R_c(r) - \alpha}}{\sqrt{\alpha}}\right|, \quad (3)$$

$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_0,$$

and where r_0 and n are entirely arbitrary constants, and α is a function of the mass M of the source of the gravitational field: $\alpha = \alpha(M)$. Clearly, $\lim_{r\to r_0^\pm} R_p(r) = 0^+$ and also $\lim_{r\to r_0^\pm} R_c(r) = \alpha^+$ irrespective of the values of r_0 and n. Usually $\alpha = 2m \equiv 2GM/c^2$ by means of a comparision with the Newtonian potential, but this identification is rather dubious.

Setting $R_p = R_c$, one finds that this occurs only when

$$R_c \approx 1.467\,\alpha.$$

Then

$$\chi_e \approx 2.934\,\pi\,\alpha.$$

Thus, at χ_e the Euclidean relation $R_p = R_c$ holds. This means that when $\chi = \chi_e$ the line-element takes on a Euclidean character.

An analogous consideration applies for the case of a point-mass endowed with charge or with angular momentum or with both. In those cases α is replaced with the corresponding constant, as developed in [2].

3 Summary

The circumference of a great circle in Einstein's gravitational field is given by

$$\chi = 2\pi R_c,$$

$$2\pi\alpha < \chi < \infty.$$

In the case of the static vacuum field, the great circle with circumference $\chi = \chi_e \approx 2.934\,\pi\alpha$ takes on a Euclidean character in that $R_p = R_c \approx 1.467\,\alpha$ there, and so χ_e marks a transition from spacetime where $\frac{\chi}{R_p} < 2\pi$ to spacetime where $\frac{\chi}{R_p} > 2\pi$. Thus,

$$\lim_{r\to\infty^{\pm}} \frac{\chi}{R_p(r)} = 2\pi,$$

$$\lim_{r\to r_0^{\pm}} \frac{\chi}{R_p(r)} = \infty\,,$$

$$\lim_{\chi\to\chi_e^{\pm}} \frac{\chi}{R_p(r)} = 2\pi.$$

Similar considerations must be applied for a point-mass endowed with charge, angular momentum, or both, but with α replaced by the corresponding constant β in the expression for R_p [2],

$$\beta = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - \left(q^2 + a^2\cos^2\theta\right)}\,,$$

$$q^2 + a^2 < \frac{\alpha^2}{4}\,, \qquad a = \frac{2L}{\alpha}\,,$$

where q is the charge and L is the angular momentum, and so

$$R_c = R_c(r) = \left(\left|r - r_0\right|^n + \beta^n\right)^{\frac{1}{n}},$$

$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_0,$$

where both r_0 and n are entirely arbitrary constants.

Submitted on December 26, 2006
Accepted on January 06, 2007

References

1. Crothers S. J. On the geometry of the general solution for the vacuum field of the point-mass. *Progress in Physics*, 2005, v. 2, 3–14.
2. Crothers S. J. On the ramifications of the Schwarzschild space-time metric. *Progress in Physics*, 2005, v. 1, 74–80.