Numerical Solution of Radial Biquaternion Klein-Gordon Equation

Vic Christianto* and Florentin Smarandache†

E-mail: admin@sciprint.org

†Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA
E-mail: smarand@unm.edu

In the preceding article we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we present numerical/computer solution of radial biquaternionic KGE (radial BQKGE); which differs appreciably from conventional Yukawa potential. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In the preceding article [1] we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we presented here for the first time a numerical/computer solution of radial biquaternionic KGE (radial BQKGE); which differs appreciably from conventional Yukawa potential.

This biquaternionic effect may be useful in particular to explore new effects in the context of low-energy reaction (LENR) [2]. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 Radial biquaternionic KGE (radial BQKGE)

In our preceding paper [1], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows:

\[
\left( \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \varphi(x, t) = -m^2 \varphi(x, t),
\]

(1)

or this equation can be rewritten as:

\[
(\diamond \diamond + m^2) \varphi(x, t) = 0,
\]

(2)

provided we use this definition:

\[
\diamond = \nabla^q + i \nabla^q = \left( -i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z} \right) + \\
+ i \left( -i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial \bar{x}} + e_2 \frac{\partial}{\partial \bar{y}} + e_3 \frac{\partial}{\partial \bar{z}} \right),
\]

(3)

where \(e_1, e_2, e_3\) are quaternion imaginary units obeying (with ordinary quaternion symbols: \(e_1 = i, e_2 = j, e_3 = k\)):

\[
\begin{align*}
\bar{e}^2 &= j^2 = k^2 = -1, & i\bar{e} &= -j = k, \\
jk &= -kj = i, & ki &= -ik = j.
\end{align*}
\]

(4)

and quaternion Nabla operator is defined as [1]:

\[
\nabla^q = -i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z}.
\]

(5)

(Note that (3) and (5) included partial time-differentiation.)

In the meantime, the standard Klein-Gordon equation usually reads [3, 4]:

\[
\left( \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \varphi(x, t) = -m^2 \varphi(x, t).
\]

(6)

Now we can introduce polar coordinates by using the following transformation:

\[
\nabla = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) - \frac{\partial^2}{\partial \vartheta^2}.
\]

(7)

Therefore, by substituting (7) into (6), the radial Klein-Gordon equation reads — by neglecting partial-time differentiation — as follows [3, 5]:

\[
\left( \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) - \frac{\ell(\ell + 1)}{r^2} + m^2 \right) \varphi(x, t) = 0,
\]

(8)

and for \(\ell = 0\), then we get [5]:

\[
\left( \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + m^2 \right) \varphi(x, t) = 0.
\]

(9)

The same method can be applied to equation (2) for radial biquaternionic KGE (BQKGE), which for the 1-dimensional situation, one gets instead of (8):

\[
\left( \frac{\partial}{\partial \vartheta} \left( \frac{\partial}{\partial \vartheta} \right) - i \frac{\partial}{\partial \vartheta} \left( \frac{\partial}{\partial \vartheta} \right) + m^2 \right) \varphi(x, t) = 0.
\]

(10)

In the next Section we will discuss numerical/computer solution of equation (10) and compare it with standard solution of equation (9) using Maxima software package [6]. It can be shown that equation (10) yields potential which differs appreciably from standard Yukawa potential. For clarity, all solutions were computed in 1-D only.
3 Numerical solution of radial biquaternionic Klein-Gordon equation

Numerical solution of the standard radial Klein-Gordon equation (9) is given by:

\[(%i1) \text{diff}(y,t,2)-\text{diff}(y,r,2)+m^2y;\]
\[(%o1) y = %k_1 \cdot \exp(mr) + %k_2 \cdot \exp(-mr) \quad (11)\]

In the meantime, numerical solution of equation (10) for radial biquaternionic KGE (BQKGE), is given by:

\[(%i2) \text{diff}(y,t,2)- (i+1)\text{diff}(y,r,2)+m^2y;\]
\[(%o2) y = %k_1 \cdot \exp(imr) + %k_2 \cdot \exp(-irm) \quad (12)\]

Therefore, we conclude that numerical solution of radial biquaternionic extension of Klein-Gordon equation yields different result compared to the solution of standard Klein-Gordon equation; and it differs appreciably from the well-known Yukawa potential [3, 7]:

\[u(r) = -\frac{g^2}{r} e^{-mr}. \quad (13)\]

Meanwhile, Comay puts forth argument that the Yukawa lagrangian density has theoretical inconsistency within itself [3].

Interestingly one can find argument that biquaternion Klein-Gordon equation is nothing more than quadratic form of (modified) Dirac equation [8], therefore BQKGE described herein, i.e. equation (12), can be considered as a plausible solution to the problem described in [3]. For other numerical solutions to KGE, see for instance [4]. Nonetheless, we recommend further observation [9] in order to refute or verify this proposition of new type of potential derived from biquaternion Klein-Gordon equation.

Acknowledgement

VC would like to dedicate this article for RFF.

Submitted on November 12, 2007
Accepted on November 30, 2007

References