A Unified Theory of Interaction: Gravitation and Electrodynamics

Pieter Wagener

Department of Physics, Nelson Mandela Metropolitan University, Port Elizabeth, South Africa
E-mail: Pieter.Wagener@nmmu.ac.za

A theory is proposed from which the basic equations of gravitation and electromagnetism are derived from a single Lagrangian. The total energy of an atom can be expressed in a power series of the fine structure constant, α. Specific selections of these terms yield the relativistic correction to the Bohr values of the hydrogen spectrum and the Sommerfeld-Dirac equation for the fine structure spectrum of the hydrogen atom. Expressions for the classical electron radius and some of the Large Number Coincidences are derived. A Lorentz-type force equation is derived for both gravitation and electrodynamics. Electron spin is shown to be an effect of fourth order in α.

1 Introduction

In a previous article [2] in this journal we presented a classical Lagrangian characterizing the *dynamics* of gravitational interaction,

$$L = -\,m_0(c^2 + v^2)\exp R/r, \tag{1}$$

where we denote:

m_0 = *gravitational rest mass* of a test body moving at velocity $\mathbf{v}$ in the vicinity of a massive, central body of mass M,

γ = $1/\sqrt{1 - v^2/c^2}$,

R = $2GM/c^2$ is the Schwarzschild radius of the central body.

The following conservation equations follow:

$$E = mc^2 e^{R/r} = \text{total energy} = \text{constant}\,, \tag{2}$$

$$\mathbf{L} = e^{R/r}\mathbf{M} = \text{constant}, \tag{3}$$

$$L_z = M_z e^{R/r} = e^{R/r} m_0 r^2 \sin^2\theta\,\dot{\phi} \tag{4}$$

$$= z \text{ component of } \mathbf{L} = \text{constant},$$

where $m = m_0/\gamma^2$ and

$$\mathbf{M} = (\mathbf{r} \times m_0\mathbf{v}) \tag{5}$$

is the total angular momentum of the test body.

It was shown that the tests for perihelion precession and the bending of light by a massive body are satisfied by the equations of motion derived from the conservation equations.

The *kinematics* of the system is determined by assuming the local and instantaneous validity of special relativity (SR). This leads to an expression for gravitational redshift:

$$\nu = \nu_0\, e^{-R/2r}, \qquad (\nu_0 = \text{constant}), \tag{6}$$

which agrees with observation.

Electrodynamics is described by the theory of special relativity. If the motion of a particle is dynamically determined by the above Lagrangian, then a description of the kinematics of its motion in terms of special relativity should yield equations of motion analogous to those of electrodynamics. This, in principle, should allow the simultaneous manifestation of gravitation and electrodynamics in one model of interaction.

We follow this approach and show, amongst others, that electrical charge arises from a mathematical necessity for bound motion. Other expressions, such as the classical electron radius and expressions of the Large Number Hypothesis follow.

The total energy for the hydrogen atom can be expressed in terms of a power series of the fine structure constant, α. Summing the first four terms yields the Sommerfeld-Dirac expression for the total energy. For higher order terms the finite radius of the nucleus must be taken into account. This introduces a factor analogous to "electron spin".

Details of all calculations are given in the PhD thesis of the author [1].

2 Gravitation and Special Relativity

Einstein's title of his 1905 paper, *Zur Elektrodynamik bewegter Körper* indicates that electrodynamics and SR are interrelated, with SR giving an explanation for certain properties of electrodynamics. Red-shift is such a property, combining both gravitation and electromagnetism in a single formulation, and should provide us with a dynamical link between these two phenomena. To do this, we substitute the photoelectric effect,

$$h\nu = \tilde{m}c^2, \tag{7}$$

where $\tilde{m} = \gamma\tilde{m}_0$ and $\tilde{m}_0$ is the *electromagnetic rest mass* of a particle, into (6). This gives

$$\left.\begin{aligned} E &= \tilde{m}c^2 e^{R/2r} = \tilde{m}_0 c^2 \frac{e^{R/2r}}{\sqrt{1 - v^2/c^2}} = \tilde{E}e^{R/2r} \\ &= \tilde{m}_0 c^2 + \tilde{m}_0 v^2/2 + \tilde{m}_0 Rc^2/2r + \\ &+ \tilde{m}_0 Rv^2/4r + \ldots \end{aligned}\right\}, \tag{8}$$

where E is another constant of energy and $\tilde{E} = \tilde{m}c^2$ is the total energy of the theory of special relativity.

Let us compare this expansion with the expansion of (2) for the gravitational energy,

$$\frac{m_0c^2 - E}{2} = \frac{m_0v^2}{2} - \frac{GMm_0}{r} + \frac{m_0v^2R}{2r} - \\ - \frac{m_0c^2R^2}{4r^2} + \frac{m_0v^2R^2}{4r^2} + \ldots \tag{9}$$

The negative sign of the second right hand term in (9) ensures attractive, or bound, motion under gravitation. In order for the motion determined by (8) to be bounded, the third right hand term must similarly be negative and inversely proportional to r. To ensure this we let

$$\tilde{m}_0c^2 = -e^2/r_e, \tag{10}$$

where e^2 is an arbitrary constant and

$$r_e = R/2. \tag{11}$$

Eq.(8) can then be rewritten as

$$E = \tilde{m}c^2 e^{r_e/r}. \tag{12}$$

As we shall see for the hydrogen atom, e represents the electron charge, r_e represents the classical electron radius and (11) yields some of the numbers of Dirac's Large Number Hypothesis.

The choice of a positive sign in (10) gives repulsive motion. Such a freedom of choice is not possible for the gravitational energy of (9).

2.1 Hamiltonian formulation

Confirmation of the above conclusions can be found by examining the predictions for the hydrogen spectrum. We follow a classical approach based on the principles of action variables [3].

Using the identity $\gamma^2 = 1 + \gamma^2v^2/c^2$ to separate the kinetic and potential energies in (8), a corresponding Lagrangian can be found:

$$L = -\,\tilde{m}_0c^2\,\sqrt{1 - v^2/c^2}\,\exp(r_e/r). \tag{13}$$

We obtain the conjugate momenta:

$$p_r = \tilde{m}\dot{r}\,\exp(r_e/r)\,, \tag{14}$$

$$p_\theta = \tilde{m}r^2\dot{\theta}\,\exp(r_e/r)\,, \tag{15}$$

$$p_\phi = \tilde{m}r^2\sin^2\theta\,\dot{\phi}\,\exp(r_e/r)\,. \tag{16}$$

The associated Hamiltonian can be derived from the formula $H = \sum \dot{q}_ip_i - L$ as follows

$$H = \left[\tilde{m}_0^2c^4\exp(r_e/r) + \\ + c^2(p_r^2 + p_\theta^2/r^2 + p_\phi^2/r^2\sin^2\theta)\right]^{1/2}. \tag{17}$$

From the canonical equations

$$\dot{p}_i = \frac{\partial H}{\partial q_i}\,, \tag{18}$$

we find the following conservation equations:

$$L^2 \equiv M^2\exp(2r_e/r) = p_\theta^2 + p_\phi^2/\sin^2\theta\,, \tag{19}$$

$$L_z \equiv M_z\exp(r_e/r) = p_\phi\,, \tag{20}$$

where L^2 and L_z are constants and

$$\mathbf{M} = (\mathbf{r}\times\tilde{m}\mathbf{v}), \tag{21}$$

is the total angular momentum of the orbiting particle.

It should be noted that (12), (19), (20) and (21) have respectively the same forms as for the gravitational equations (2), (3), (4), (5), but with $m = m_0/\gamma^2$ replaced by $\tilde{m} = \gamma\tilde{m}_0$ and R by $r_e = R/2$.

3 The hydrogen spectrum

In order to determine an expression for the energy levels of the H-atom, two different approaches can be followed: (i) Analogously to the Wilson-Sommerfeld model, one can apply the procedures of action angle variables, or (ii) perturbation theory, where the contribution of each energy term is evaluated separately.

To generalize our discussion we shall, where appropriate, use a general potential $\Phi = Rc^2/2r = r_ec^2/r$.

3.1 Method of action angle variables

The theory of action angle variables originated in the description of periodic motion in planetary mechanics [4, Ch.9]. From that theory Wilson and Sommerfeld postulated the quantum condition:

> For any physical system in which the coordinates are periodic functions of time, there exists a quantum condition for each coordinate. These quantum conditions are
>
> $$J_i = \oint p_i\,dq_i = n_ih\,, \tag{22}$$
>
> where q_i is one of the coordinates, p_i is the momentum associated with that coordinate, n_i is a quantum number which takes on integral values, and the integral is taken over one period of the coordinate q_i.

Applying these quantization rules to the conjugate momenta of (14), (15) and (16) gives [3]

$$L_z = M_z\exp(r_e/r) = n_\phi\hbar\,, \tag{23}$$

$$L = M\exp(r_e/r) = (n_\theta + n_\phi)\,\hbar = k\hbar\,, \tag{24}$$

$$\oint\left[E^2/c^2 - \tilde{m}_0^2c^2\exp(r_e/r) - k^2\hbar^2/r^2\right]^{\frac{1}{2}}dr = n_rh\,, \tag{25}$$

where n_θ, n_ϕ, k and n_r have the values 0, 1, 2, ...

To determine the atomic spectrum we need to evaluate the integral of (25). Because of the finite radius of the nucleus we choose an arbitrary effective nuclear radius of gr_e. The potential term in the exponentials is then written as

$$\exp\left(\frac{2\Phi}{c^2}\right) = \exp\left(\frac{2r_e}{r - gr_e}\right), \tag{26}$$

so that

$$\exp(2\Phi/c^2) = \\ = 1 + 2\,\frac{r_e}{r} + 2\,\frac{r_e^2}{r^2}\,(g+1) + 3\,\frac{r_e^3}{r^3}\,g\,(g+1) + \ldots \tag{27}$$

For convenience we also define a parameter f such that

$$f = 2(g+1). \tag{28}$$

We shall subsequently see that the value of g, or f, is related to the concept of electron spin.

Approximating (27) to second order in r_e/r, substituting this approximation in (25) and integrating gives

$$E_m^2 = 1 - \frac{\alpha^2}{\left[n - k + \sqrt{k^2 + f\alpha^2}\right]^2}, \tag{29}$$

where $E_m = E/\tilde{m}_0 c^2$, $n = n_r + k$ and $\alpha = e^2/\hbar c$ is the fine structure constant. This expression is simplified by expanding to fourth order in α:

$$E_m \cong 1 - \frac{\alpha^2}{2n^2}\left[1 + \frac{\alpha^2}{n}\left(\frac{1}{4n} - \frac{f}{k}\right)\right]. \tag{30}$$

The corresponding Sommerfeld/Dirac expressions are respectively

$$E_m^2 = \left(1 + \frac{\alpha^2}{\left[n - k + \sqrt{k^2 - \alpha^2}\right]^2}\right)^{-1} \tag{31}$$

and

$$E_m \cong 1 - \frac{\alpha^2}{2n^2}\left[1 + \frac{\alpha^2}{n}\left(\frac{1}{k} - \frac{3}{4n}\right)\right], \tag{32}$$

where $k = j + \frac{1}{2}$ for the Dirac expression, and $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots \ldots \frac{(n-1)}{2}$.

The difference between the energy given by our model E_W, as given by (30), and that of the Sommerfeld-Dirac model, E_D, as given by (32), is

$$(E_D - E_W)/\tilde{m}_0 c^2 = \frac{\alpha^4}{2n^3}\left[\frac{1}{k}(f+1) - \frac{1}{n}\right]. \tag{33}$$

We shall show below that this difference corresponds to the energy associated with the "spin-orbit" interaction of our model.

4 Perturbation method

We use this method as applied by Born and others [3, Ch. 4].

To apply the perturbation method we need to express the energy $\tilde{E}$ in terms of the momentum:

$$E = (p^2c^2 + \tilde{m}_0^2 c^4)^{1/2}\exp(\Phi/c^2), \tag{34}$$

where $\mathbf{p} = \tilde{m}\mathbf{v}$. Again, taking the finite radius of the nucleus into account, we choose for the potential,

$$\exp(\Phi/c^2) = \exp\bigl(r_e/(r - gr_e)\bigr), \tag{35}$$

so that the potential term can be written as

$$\exp(\Phi/c^2) = 1 + \frac{r_e}{r} + w\frac{r_e^2}{r^2} + \left(w^2 - \frac{1}{4}\right)\frac{r_e^3}{r^3} + \ldots, \tag{36}$$

where

$$w = (g + 1/2) = (f-1)/2. \tag{37}$$

With this form for the potential, and using $\tilde{m}_0 c^2 r_e = -e^2$, (34) can be expanded as

$$\begin{aligned} E \;=\;& \underbrace{\tilde{m}_0 c^2}_{E_0} + \underbrace{\frac{p^2}{2\tilde{m}_0} - \frac{e^2}{r}}_{E_1} - \underbrace{\frac{p^4}{8\tilde{m}_0^3 c^2}}_{E_2} + \underbrace{\frac{p^2 r_e}{2\tilde{m}_0 r}}_{E_3} + \\ &+ \underbrace{w\,\frac{r_e^2\tilde{m}_0 c^2}{r^2}}_{E_4} + \underbrace{w\,\frac{p^2 r_e^2}{2\tilde{m}_0 r^2}}_{E_5} - \underbrace{\frac{p^4 r_e}{8\tilde{m}_0^3 c^2 r}}_{E_6} + \\ &+ \underbrace{\tilde{m}_0 c^2\left(w^2 - \frac{1}{4}\right)\frac{r_e^3}{r^3}}_{E_7} + \ldots \end{aligned} \tag{38}$$

Applying the unperturbed Bohr theory to each braced term, we find the following quantized expressions:

4.1 E_0: rest mass energy

The first term on the right is the rest mass energy, which we denote by E_0:

$$E_0 = \tilde{m}_0 c^2. \tag{39}$$

4.2 E_1: Bohr energy

The next two terms represent the unperturbed Coulomb energy of the hydrogen atom, which we indicate by E_1:

$$E_1 = p^2/2\tilde{m}_0 - e^2/r. \tag{40}$$

According to the method of the Bohr theory,

$$E_1 = -R_e/n^2, \qquad n = 1, 2, \ldots \tag{41}$$

where

$$\left.\begin{aligned} R_e &= R_y hc = e^2/2a_0 = \alpha^2\tilde{m}_0 c^2/2 \\ a_0 &= \text{Bohr radius} = \hbar^2/\tilde{m}_0 e^2 \\ R_y &= \text{Rydberg constant} = 2\pi^2 e^4\tilde{m}_0/ch^3 = \alpha/4\pi a_0 \end{aligned}\right\}. \tag{42}$$

4.3 E_2: relativistic correction

The third term is denoted by E_2. It can be shown that [1]

$$E_2 = -p^4/8\tilde{m}_0^3c^2, \tag{43}$$

$$= -\frac{\alpha^2 R_e}{n^3}\left[\frac{1}{k}-\frac{3}{4n}\right]. \tag{44}$$

This is the "relativistic correction" of the Bohr-Sommerfeld model [3, §33]. This energy term is similar to that contained in the Dirac expression of (32). The sum of E_0, E_1 and E_2 gives an expression identical to that of Sommerfeld and similar to that of Dirac.

It is well-known that Sommerfeld's result was fortuitous as the effect of spin-orbit coupling was ignored in his model. This effect is incorporated in the Dirac model. In our model we shall see below that E_3 is an orbit-interaction term and that E_4 is related to 'electron spin'. These two terms, missing in the Sommerfeld model, can now be added to $E_0+E_1+E_2$ of the Sommerfeld energy expression.

4.4 E_3: orbital magnetic energy

We denote the fourth term by E_3:

$$E_3 = p^2 r_e/2\tilde{m}_0 r\,. \tag{45}$$

Applying the unperturbed Bohr theory, we find from (40):

$$E_3 = (E_1 + e^2/r)r_e/r$$
$$= r_e(E_1/r + e^2/r^2)\,. \tag{46}$$

Using (41) and the average values [3, p144],

$$\overline{1/r} = 1/n^2 a_0\,, \tag{47}$$

$$\overline{1/r^2} = 1/a_0^2 n^3 k\,, \quad k = 1,2,\ldots n \tag{48}$$

as well as

$$r_e/a_0 = \alpha^2, \tag{49}$$

we get

$$E_3 = \frac{\alpha^2 R_e}{n^3}\left(\frac{2}{k}-\frac{1}{n}\right) = \frac{\alpha^4\tilde{m}_0c^2}{2n^3}\left(\frac{2}{k}-\frac{1}{n}\right). \tag{50}$$

The physical interpretation of E_3 is that it is the energy due to the magnetic interaction of an electron moving in orbit about a proton. This can be seen as follows.

Substituting $\mathbf{p}=\tilde{m}\mathbf{v}$ and $r_e = -e^2/\tilde{m}_0c^2$ into (45) gives

$$E_3 = -\frac{e^2v^2}{2rc^2}\left(\frac{\tilde{m}}{\tilde{m}_0}\right)^2$$
$$\approx -\frac{e^2v^2}{2rc^2} \quad \text{in the non-relativistic limit.} \tag{51}$$

It corresponds to the classical form of the magnetic energy due to orbital motion, as given by (70) below:

4.5 E_4: "electron spin"

$$E_4 = w r_e^2 \tilde{m}_0 c^2/r^2, \quad = w e^4/\tilde{m}_0 c^2 r^2. \tag{52}$$

Applying (48) gives

$$E_4 = \frac{w2\alpha^2 R_e}{n^3 k} = w\alpha^4\tilde{m}_0c^2\frac{1}{n^3k}\,. \tag{53}$$

We consider the significance of the factor w. We note that the potential energy expression (36) can be truncated after the quadratic term in r_e/r by letting $w^2-\frac{1}{4}=0$. As such, truncation can be considered as the limit to the resolution of the apparatus used for spectral observation. With this condition, we find that

$$w = \pm\frac{1}{2} \tag{54}$$

gives the spectrum due to all interactions up to second degree in r/r_e. Therefore, from (42) and (53):

$$E_4 = \pm\frac{1}{2}\frac{e^8\tilde{m}_0}{\hbar^4c^2}\frac{1}{n^3k}\,. \tag{55}$$

The above expression for E_4 corresponds to the quantum mechanical result for the energy due to electron spin. Except for the quantum numbers, Eisberg and Resnick [6, Example 8–3] find a similar result for the energy due to spin-orbit interaction.

The equivalence of (55) to the result of Eisberg and Resnick also confirms the implicit value $g_s = 2$ in E_4.

In this study E_4 corresponds to the energy due to quantum mechanical spin only. Combining E_3 and E_4 gives the corresponding total spin-orbit energy.

For $k=1$ the expression for E_4 is equal to the Darwin term of the Dirac theory. In the Dirac theory the Darwin term has to be introduced separately for $\ell=0$ states, whereas in our model E_4 already provides for $\ell=0$ through the degeneracy ($\ell=0,1$) associated with the $k=1$ level.

In summary, 'electron spin' represents a second order contribution r_e^2/r^2 to the total energy of the atom.

The above reasoning also applies to higher orders of approximation. Expanding (35) to fourth degree in r_e/r gives:

$$\exp(\Phi/c^2) = 1+\frac{r_e}{r}+\frac{r_e^2}{r^2}w+\frac{r_e^3}{r^3}(w^2-\frac{1}{2})+ + \frac{r_e^4}{r^4}\left(w^2-\frac{1}{4}\right)w+\ldots \tag{56}$$

The coefficient of r_e^4/r^4 is zero if $(w^2-\frac{1}{4})\,w=0$, or

$$w = \frac{1}{2},\ -\frac{1}{2},\ 0. \tag{57}$$

A next higher resolution to r_e^3/r^3 therefore introduces an additional value of $w=0$, giving a triplet symmetrical about this value.

For a comprehensive survey of the conceptual developments surrounding electron spin we refer to the text by Tomonaga [7].

4.6 E_5: radiative reaction

$$E_5 = w \frac{p^2 r_e^2}{2\tilde{m}_0 r^2} \tag{58}$$

$$= \pm \frac{1}{2} \alpha^4 R_e \left[\frac{1}{n^5 k} - \frac{2}{n^3 k^3} \right]. \tag{59}$$

Substituting $\mathbf{p} = \tilde{m}\mathbf{v}$ in (58) gives

$$E_5 = \pm \frac{1}{2} \frac{v^2 e^4}{2\tilde{m}_0 c^4 r^2} \left(\frac{\tilde{m}}{\tilde{m}_0} \right)^2. \tag{60}$$

In the non-relativistic limit, $\tilde{m} \approx \tilde{m}_0$, the above term corresponds to the last RHS term of (69), i.e. the classical energy resulting from radiative reaction. Its value is too small ($\sim 10^{-8}$ eV) to affect the values of the fine-spectrum.

4.7 Summary

$$E_0 = m_0 c^2 \quad : \text{ rest mass energy,}$$

$$E_1 = -\frac{R_e}{n^2} \quad : \text{ Bohr energy,}$$

$$E_2 = -\frac{\alpha^2 R_e}{n^3} \left[\frac{1}{k} - \frac{3}{4n} \right] \quad : \text{ relativistic correction,}$$

$$E_3 = \frac{\alpha^2 R_e}{n^3} \left[\frac{2}{k} - \frac{1}{n} \right] \quad : \text{ orbital magnetic energy,}$$

$$E_4 = w \frac{2\alpha^2 R_e}{n^3 k} \quad : \text{ electron spin energy,}$$

$$E_5 = w\alpha^4 R_e \left[\frac{1}{n^5 k} - \frac{2}{n^3 k^3} \right] \quad : \text{ Radiative reaction,}$$

where $w = \pm\frac{1}{2}$.

The sum of the energy terms $\sum E_i = E_0 + E_1 + E_2 + E_3 + E_4 + E_5$ is:

$$\sum E_i / \tilde{m}_0 c^2 = 1 - \frac{\alpha^2}{2n^2} \left[1 - \frac{\alpha^2}{n} \left(\frac{f}{k} - \frac{1}{4n} \right) \right], \tag{61}$$

which, as expected, is the same as (30).

Each term in (38) can be related to a standard electrodynamic effect. It is significant that although (38) does not explicitly contain any vector quantities, such as the vector potential $\mathbf{A}$, this potential is implicit, as shown in the discussion of E_3 and the comparison with (69).

An explanation for the difference (33) between the spectrum of the proposed model and that of Dirac-Sommerfeld can be seen as follows:

Consider the sum

$$E_3 + E_4 = \frac{\alpha^2 R_e}{n^3} \left[\frac{2}{k} (w+1) - \frac{1}{n} \right] \tag{62}$$

or, since $w = (f-1)/2$,

$$E_3 + E_4 = \frac{\alpha^2 R_e}{n^3} \left[\frac{1}{k} (f+1) - \frac{1}{n} \right]. \tag{63}$$

The above equation corresponds to (33), the difference between the Sommerfeld-Dirac expression and that of our model. The expression (30) therefore already incorporates the spin-orbit interaction.

The energy $E_3 + E_4$ therefore represents a perturbation to the Sommerfeld-Dirac values. The only candidate for this perturbation is the Lamb-shift. For the (n, k) = (2,1) level and for $w = -0.5$ the value of $E_3 + E_4$ is 4.52 831 78 $e-5$ ev. The Lamb-shift for this level is 4.37 380 19 $e-6$ eV, which is an order 10 smaller. It would be overly ambitious to find the observed Lamb-shift from the present simple model. At this degree of spectral resolution one would have to look at a modification of the effective nuclear radius to $r - a_1 r_e - a_2 r_e^2 - \ldots$

4.8 Comparison with classical electromagnetic energy

In order to compare the results of this study with those of conventional electromagnetic theory, we give a brief summary of the energy relations of classical electrodynamic theory.

The Hamiltonian describing the interaction of an electron with fields $\mathbf{H}$ and $\mathbf{E}$ is given by [8, p. 124]

$$H_{\text{classical}} = e\,\Phi + \left(\mathbf{p} - \frac{e}{c}\mathbf{A} \right)^2 / 2\tilde{m}\,, \tag{64}$$

where Φ and $\mathbf{A}$ are respectively the electrostatic and vector potentials of the system.

It is important to note that $\mathbf{A}$ and Φ do not merely represent the external fields in which the particle moves, but also the particle's own fields. This implies that the force of radiative reaction is automatically included.

The corresponding classical Lagrangian is

$$L_{\text{classical}} = \frac{p^2}{2\tilde{m}} - e\,\Phi + \frac{e}{c}\,\mathbf{A}\cdot\mathbf{v}. \tag{65}$$

For an electron moving under the influence of a magnetic field,

$$\mathbf{H} = e\,(\mathbf{v} \times \mathbf{r})/cr^3, \tag{66}$$

a vector potential $\mathbf{A}$ can be found as

$$\mathbf{A} = \frac{1}{2}\,(\mathbf{H} \times \mathbf{r}) = e\mathbf{v}/2cr\,. \tag{67}$$

Substituting this expression for $\mathbf{A}$ and using $\mathbf{p} = \tilde{m}\mathbf{v}$, yields

$$\left(\mathbf{p} - \frac{e}{c}\,\mathbf{A} \right)^2 = p^2 - \frac{e^2 v^2 \tilde{m}}{c^2 r} + \frac{e^4 v^2}{4c^4 r^2}\,. \tag{68}$$

Since the Hamiltonian of (64) does not contain t explicitly, we may equate it to the total energy. Consequently, substituting (68), and $e\,\Phi = -\,e^2/r$, in (64) gives the classical energy

$$E_{\text{classical}} = -\frac{e^2}{r} + \frac{p^2}{2\tilde{m}} - \frac{e^2 v^2}{2c^2 r} + \frac{e^4 v^2}{8\tilde{m}c^4 r^2}\,. \tag{69}$$

The third RHS term is the magnetic energy due to the orbital motion of the electron:

$$E_{\text{orbital}} = \boldsymbol{\mu}_\ell \cdot \mathbf{H} = -\frac{g_\ell\, e^2 v^2}{2rc^2}, \tag{70}$$

where $\boldsymbol{\mu}_\ell$ = magnetic moment, g_ℓ = Landé g factor = 1, and **M** and **H** are parallel to one another. This energy corresponds to that of E_3 above.

The fourth RHS term of (69) represents radiative reaction, which corresponds to our E_5 as given by (60).

The standard relativistic Hamiltonian is given by:

$$H_{\text{relativistic}} = \left[(\mathbf{p} - q\mathbf{A}/c)^2 c^2 + \tilde{m}_0^2 c^4\right]^{\frac{1}{2}} + q\,\Phi\,. \tag{71}$$

The Hamiltonians of (64) and (71) must be compared to ours of (17).

It is well-known that the Bohr model for the atom fails because of radiative reaction; in our model this loss is compensated for by the additional and associated potential term, E_4, This term can also be interpreted as a modification of Coulomb's law. It is significant that this energy term can also be interpreted as arising from electron spin.

It is also significant that the Sommerfeld relativistic correction term, E_2, does not appear in either (69) or (71).

We can consider the electromagnetic energy arising from the Hamiltonians of (64) and (71) as approximations to that of our Hamiltonian of (17).

We also note that the energy derived from the Hamiltonian of (64), which is normally derived from a Lagrangian containing the vector potential **A**, appears as an approximation to our model, which does not explicitly contain a vector potential. A vector potential arises in our theory because of the variation of mass according to (12).

5 The large number coincidences

Dirac postulated that the large dimensionless ratios ($\sim 10^{40}$) of certain universal constants underlie a fundamental relationship between them. A theoretical explanation for these ratios has not yet been found, but it became known as Dirac's Large Number Hypothesis (LNH).[9] Some of these relations are derivable from (11).

Taking R as the Schwarzschild radius of the proton, $R_p = 2GM_p/c^2$, we rewrite (11) as

$$-\frac{e^2}{\tilde{m}_0 c^2} = \frac{GM_p}{c^2}$$

$$\text{or}\quad -\frac{e^2}{GM_p\tilde{m}_0} = 1. \tag{72}$$

Defining the relationship between the *gravitational mass* Mp and the *electromagnetic rest mass* $\tilde{m}_{0p}$ of the proton as

$$M_p = N_D \tilde{m}_{0p}\,, \tag{73}$$

where N_D is a dimensionless number, we can write (72) as

$$-\frac{e^2}{G\tilde{M}_{0p}\tilde{m}_0} = N_D\,, \tag{74}$$

which, if the absolute value is taken, is the basic relationship of the LNH.

6 Lorentz force

The force equation for a particle, mass $\tilde{m}$ and velocity $\mathbf{v}$ is found by applying the Euler-Lagrange equations to (13). This gives

$$\dot{\mathbf{p}} = \hat{\mathbf{r}}\,\frac{\tilde{m}r_e c^2}{r^2} + \frac{\tilde{m}r_e}{r^3}\,\mathbf{v}\times(\mathbf{v}\times\mathbf{r})\,. \tag{75}$$

Defining

$$\mathbf{E} = \hat{\mathbf{r}}\,\frac{r_e c^2}{r^2}, \tag{76}$$

$$\mathbf{H} = \frac{r_e \mathbf{v}\times\mathbf{r}}{r^3}\,, \tag{77}$$

we can write (75) as

$$\text{Electromagnetic } \dot{\mathbf{p}} = \tilde{m}\left[\mathbf{E} + \mathbf{v}\times\mathbf{H}\right]. \tag{78}$$

For $v \ll c$, $\tilde{m}r_e c^2 \to \tilde{m}_0 r_e c^2 = e^2$ and then (75) approaches the classical Lorentz form.

7 Unifying gravitation and electromagnetism

Equation (16) of reference [1] can be combined with (78) in one formulation:

$$\dot{\mathbf{p}} = \tilde{m}\left[k\mathbf{E} + \mathbf{v}\times\mathbf{H}\right], \tag{79}$$

where for

Gravitation : $k = -1$,
Electromagnetism : $k = 1$.

The same equation gives either planetary or atomic motion, where the vectors **E** and **H** are respectively given by

$$\mathbf{E} = \hat{\mathbf{r}}\,\frac{GM}{r^2} = \hat{\mathbf{r}}\,\frac{r_e c^2}{r^2}\,, \tag{80}$$

$$\mathbf{H} = \frac{GM(\mathbf{v}\times\mathbf{r})}{c^2 r^3} = \frac{r_e \mathbf{v}\times\mathbf{r}}{r^3}\,. \tag{81}$$

8 Summary

Gravitation	Electromagnetism
$R = 2GM/c^2$	$r_e = R/2$
m_0	$\tilde{m}_0 = m_0/N$
$L = -m_0(c^2 + v^2)e^{R/r}$	$L = -(\tilde{m}_0 c^2/\gamma)e^{r_e/r}$
$E = mc^2 e^{R/r}$	$E = \tilde{m}c^2 e^{r_e/r}$
$m = m_0/\gamma^2$	$\tilde{m} = \gamma\tilde{m}_0$
$\mathbf{L}^2 = \mathbf{M}^2 e^{2R/r} =$ constant	$\mathbf{L}^2 = \mathbf{M}^2 e^{2r_e/r} =$ constant
$L_z = M_z e^{R/r} =$ constant	$L_z = M_z e^{r_e/r} =$ constant
$\mathbf{M} = (\mathbf{r} \times m_0\mathbf{v})$	$\mathbf{M} = (\mathbf{r} \times \tilde{m}_0\mathbf{v})$
$\dot{\mathbf{p}} = m\mathbf{E} + m_0\mathbf{v} \times \mathbf{H}$	$\dot{\mathbf{p}} = \tilde{m}[\mathbf{E} + \mathbf{v} \times \mathbf{H}]$
$\mathbf{p} = m_0\mathbf{v}$	$\mathbf{p} = \tilde{m}\mathbf{v}$
$\mathbf{E} = -\hat{\mathbf{r}}GM/r^2$	$\mathbf{E} = \hat{\mathbf{r}}r_e c^2/r^2$
$\mathbf{H} = GM(\mathbf{v} \times \mathbf{r})/r^3c^2$	$\mathbf{H} = r_e(\mathbf{v} \times \mathbf{r})/r^3$

9 Nuclear force

In a subsequent article we shall show that equations for the nuclear force, such as the Yukawa potential, can be derived by considering the forms of both the energy equations (2) and (8) at $r \approx R/2 = r_e$.

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