Key Notes on a Geometric Theory of Fields

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The role of potentials and sources in electromagnetic and gravitational fields is investigated. A critical analysis leads to the result that sources have to be replaced by integration constants. The existence of spatial boundaries gives reasons for this step. Potentials gain physical relevance first with it. The common view, that fields are “generated” by sources, appears as not tenable. Fields do exist by their own. These insights as well as results from numerical simulations force the conclusion that a Riemannian-geometrical background of electromagnetism and even quantum phenomena cannot be excluded. Nature could differ from abstract geometry in a way that distances and intervals never become infinitesimally small.

1 Introduction

In Physics a unified theory including all phenomena of nature is considered as the greatest challenge. All attempts founded on the present definition of matter have manifest to fail. It will require a redefinition of this term.

The traditional view consists on the assumption that matter “generates” fields. All effort aims at the description of this matter, detached from fields, at least from gravitation. This single-edged view led to the known problems and cannot bring more than stagnation. One had to unify different methods being used for handling of different physical situations. Also new mathematical procedures cannot help to master this unsolvable problem.

The traditional mathematical description puts the matter on the right-hand-side of partial differential equations, while the left-hand-side contains differential terms of the field quantities. However, practice demonstrates that only field quantities are measurable, never any form of matter terms. If we consider the practice impartially, the right-hand-sides of the field equations have to become zero. That means, there are no sources of fields.

There are severe caveats in physics against this conclusion. However, it will be demonstrated that any infinities like singular points are physically irrelevant. Connecting electromagnetism to gravitation without obstacles is only possible avoiding sources.

In this paper, solutions of known linear field equations (electromagnetism and gravitation) with and without sources are compared, in which, integration constants from source-free equations take the role of sources. Mass, spin, charge, magnetic momentum are first integration constants. The nonlinear case will validate the linear basic approach. Boundaries, introduced to solve linear source-free equations, reveal to be geometric limits in the space-time, described by nonlinear equations. This fact makes any artifacts unnecessary. The theory can be managed with exclusively classical mathematical methods.

These insights are not familiar in physics, because the present standard is the Quantum Field Theory [1, 2], in which the most known part, the Standard Model, is told to be very successful and precise [3, 4]. The existence of subatomic particles has been deduced from scattering experiments [3]. The field term, used in these theories, differs considerably from the classical field term. Actually, these theories are founded on building block models which more seem to aim at a phenomenology of a “particle zoo” than a description of nature based on first principles. In order to describe the interactions between particles respectively sub-particles, it needs the introduction of virtual particles like the Higgs, which have not been experimentally verified to date.* By principle, the subatomic particles cannot be observed directly. — Are the limits of classical methods really so narrow, that they would justify these less strict methods of natural philosophy?

The mathematical methods are more and more advanced (for example introducing several “gauge fields”) according to the requirements by the building block models. However, these methods approach to limits [3, 4]. Gravitation must be handled external to the model and appears as an external force. The deeper reason is that the standard model is based on Special Relativity while gravitation is the principal item of General Relativity. These differences are inherent and do not lead to a comprehensive model which reflects the fact that gravitation and electromagnetism have analogous properties. Pursuing theories like string theory (quoted by [4]) do not really close this gap. Any predictions or conjectures are not validated, as demonstrated for example in [6].

The central question of modern physics is: How to quantize field theory? [4] In view of the looming limits, another question is proposed instead: Which quantities have discrete values? — In order to answer this alternative question, we

*Manfred Geilhaupt claims to “provide” a kind of “Higgs field” in his theory, called GR+QTD (General Relativity + Quantum Thermodynamics) by him [5]. It were a step beyond virtual particles “because they possess restmass itself due to TD principles. Second it also seems to be obvious that the fine structure constant of space fundamentally can be derived by GR but not without precursor extended by QTD” [5].

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have to go back to the roots. That are Maxwell’s theory and General Theory of Relativity as Einstein himself taught in his Four Lectures [7]. The simple approach of these basics should be a specific benefit, and a low standard by no means. We have to take notice of any proportions of forces (how extreme these may ever be), and to accept the direct consequences like the non-existence of sources (as explained in this paper) and the non-applicability of building block models. We have to compare not forces but the fields with respect to metrics.* The following lines will make General Relativity provide the basis which can describe all real forces of nature.

2 Electromagnetism

As known, electromagnetic fields in the vacuum can be described by Maxwell’s equations, with tensor notation

\[ F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 , \]  
\[ F^{ia}_{\ jb} = S^{i} \]  

where \( S \) is the vector of source terms. With Eq. (1), the field tensor is identically representable from a vector potential \( \mathcal{A} \) with

\[ F_{ij} = A_{i,j} - A_{j,i} . \]  

The six independent components of the field tensor are reduced to four components of the vector potential. These four components can be put in the four equations (2).

If one changes the vector potential for the gradient of an arbitrary scalar

\[ A_{i} \implies A_{i} + \psi_{j} , \]  

field tensor and source \( S \) (currents and charges) do not change. These quantities are told to be gauge-invariant [9].

The vector potential has been introduced to solve equations (2). It is at first an auxiliary quantity. Reasons for possible physical relevance are mentioned later. However, the Aharonov-Bohm effect (for example) does not give evidence for the physical relevance of vector potential and gauge, as Bruhn [10] demonstrated.

2.1 The Poisson equation

In order to get more close solutions, one can apply the Lorenz convention (see [9])

\[ \Box A = -S , \]  

with the retarded potential

\[ A = \frac{1}{4\pi} \int \frac{S(r_{0},\ r) \cdot \ r}{|r - r_{0}|} \, dV_{0} \]  

as solution (without spatial boundaries).

Time-independent solutions

\[ A = \frac{1}{4\pi} \int \frac{S(r_{0})}{|r - r_{0}|} \, dV_{0} \]  

can be decomposed into several multipoles. As well, the term \( 1/|r - r_{0}| \) is developed in series. The vector potential results in

\[ A = \frac{1}{4\pi} \sum_{i=0}^{\infty} \frac{1}{i+1} \int r_{0}^{i} P_{i} \left( \frac{r \cdot r_{0}}{r r_{0}} \right) \cdot S(r_{0}) \, dV_{0} \]  

with \( r = |r|, \ r_{0} = |r_{0}|. \) \( P_{i} \) are Legendre’s polynoms (Wunsch [11]).

Introducing spherical coordinates with

\[ x = r \sin \theta \sin \varphi , \ y = r \sin \theta \cos \varphi , \ z = r \cos \theta , \]  

in which

\[ x^{1} = r , \ x^{2} = \theta , \ x^{3} = \varphi , \ x^{4} = jct \]  

(with \( \vec{r} = -1 \)), the argument is

\[ \frac{r \cdot r_{0}}{r r_{0}} = \sin \theta \sin \varphi \cos(\varphi - \varphi_{0}) + \cos \theta \cos \varphi_{0} . \]  

By this, the fixed volume integrals become functions of \( \theta \) and \( \varphi \). Rotationally symmetric ansatzes

\[ \rho(r_{0}, \theta_{0}, \varphi_{0}) = \rho(r_{0}, \theta_{0}) \]  

(charge density), and

\[ J_{\mu}(r_{0}, \theta_{0}, \varphi_{0}) = J_{\mu}(r_{0}, \theta_{0}, \varphi) \cdot \cos(\varphi - \varphi_{0}) \]  

(current density) lead to momenta that will be compared with the solutions from wave equations. The calculation of the first momenta, i.e., charge and magnetic momentum, is demonstrated in [12]. As well, the charge follows directly as a first approximation of the volume integral from Eq. (8). The magnetic momentum is calculated with a current loop model, see [12].

2.2 The wave equation

The wave equation follows from the Poisson equation if the sources vanish, i.e.,

\[ \Box A = 0 . \]  

*See more Section 6.1

†The tensor equations have been normalized, see Kästner [8] and appendix.

Bruhn explains these basics with traditional notation.

†This condition is mostly met, but it is not ensured.

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2.2.1 The plane wave

A known solution is the plane wave, for propagation in direction of $\mathbf{x}^\perp$ (with Cartesian coordinates, without gravitation)

$$A_2 = A_2(ct - \mathbf{x}^\perp).$$

(16)

One can take $A_3$ instead of $A_2$. However, $A_1$ and $A_4$ are irrelevant for the Lorenz convention, because this takes

$$A_4' = |A_1'|,$$

(17)
in which the apostrophe means the total derivative with respect to $ct - \mathbf{x}^\perp$. The component $F_{41}$ is always zero for that reason, and $F_{33}$ vanishes anyway. It is the reason for the very fact that longitudinal electromagnetic waves (also called scalar waves) do not exist. The Lorenz convention is the prerequisite of the wave equation.

This solution is not physical, and has to be discussed in context with gravitation. A special kind of boundary could make plane waves physical. A possible context with Planck’s constant is discussed in [17].

2.2.2 The spherical wave

The central symmetrical ansatz can be written for any scalar potential, and components treated by this means,

$$c^2 \frac{\partial^2}{\partial r^2} (r \phi) = \frac{\partial^2}{\partial \theta^2} (r \phi)$$

(18)

with the solution

$$r \phi = B(ct \mp r)$$

(19)

(Reichardt [13]), in which only the minus sign might be relevant here.

Transforming to the potential itself becomes problematic at $r = 0$. We shall see that this critical point proves to be physically irrelevant. Aware of this, one could take this solution as element of the retarded potential according to Eq. (7).

A spherical boundary around $r = 0$ does not change this solution at and outside of the boundary, and eliminates the mathematical problem. The solution is linked with the potential of the boundary then.

*Since the boundary is part of the field, the question for cause and effect becomes irrelevant.*

2.2.3 Time-independent solutions

Static solutions of the wave equation require the existence of spatial boundaries. That may be ideal conductors in electric fields, or hard bodies in sound fields. These problems are known as “marginal-problems” (for example [14, 15]). The values of integration constants in the solutions are linked with the potentials of the boundaries against infinity.

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The wave equations for several components become for rotational symmetry with spherical coordinates

$$\frac{\partial^2 A_4}{\partial r^2} + \frac{2}{r} \frac{\partial A_4}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A_4}{\partial \theta^2} + \frac{1}{r^2} \frac{\partial A_4}{\partial \varphi} \cot \theta = 0$$

(20)

(electric potential) and

$$\frac{\partial^2 A_3}{\partial r^2} + \frac{1}{r^2} \frac{\partial A_3}{\partial \theta^2} - \frac{1}{r^2} \frac{\partial A_3}{\partial \varphi} \cot \theta = 0$$

(21)

(magnetic vector potential). The magnetic vector potential consists of only one component in direction of the azimuth

$$A_3 = A_\varphi \tau \sin \theta$$

(22)
in which $A_\varphi$ means the physical component.

The differently looking equations (20) and (21) follow from coordinate transformation.

Developments of series with ansatzes

$$A_4 = \sum_{i,k} a_{i[k]} r^i \cos^k \theta$$

(23)

lead, by means of comparison of the coefficients, to the performing laws

$$0 = a_{i[k]} r^{i+1} - (k+1) a_{i[k+2]}$$

(24)

Physically meaningful are only the cases $i < 0$ and $k > 0$.

With this, the series become

$$A_4 = \frac{a_{i[-1,0]}}{r} + \frac{a_{i[-2,1]}}{r^2} \cos \theta + \frac{a_{i[-3,2]}}{r^3} \left( -\frac{1}{3} + \cos^2 \theta \right) + \ldots ,$$

$$A_\varphi = \sin \theta \left( \frac{a_{i-1,2}}{r^2} + \frac{a_{i-2,3}}{r^3} \sin \theta + \frac{a_{i-3,4}}{r^4} \left( -\frac{4}{5} + \sin^2 \theta \right) + \ldots \right) .$$

(25)

*\(1\)Who insists on sources may take these regions as source. Lastly the connection of electromagnetism with gravitation will show, that this step is illogical.

*\(2\)On physical components see Kästner [8].
A comparison of these solutions with static solutions of the Poisson equation results for the first integration constants in
\[ a_{[4]}-1,0 = -\frac{\mu_0}{4\pi} Q \]
(26)
(charge) and
\[ a_{[3]}-1,1 = -\frac{e_0}{4\pi} M \]
(27)
(magnetic momentum).

Integration constants take the role of the sources. In more complex solutions, the $1/r$ field from point charges (for example) is assumed only for a large radius.

3 Gravitation

Another kind of potential can be derived from Einstein’s [7] gravitation equations
\[ R_{ik} - \frac{1}{2} g_{ik} R = -\kappa T_{ik} , \]
(28)
or
\[ R_{ik} = -\kappa \left( T_{ik} - \frac{1}{2} g_{ik} T \right) = -\kappa T_{ik}^* \]
(29)
with $T = T_a^a$. These equations indicate the relations of the Ricci tensor with energy and momentum components. The Ricci tensor is a purely geometrical quantity of the spacetime. It contains differential terms of metrics components.

One can approximate metrics, with Cartesian coordinates, as
\[ g_{ik} = \delta_{(ik)} + \gamma_{(ik)} \quad \text{with} \quad |\gamma_{(ik)}| \ll 1 . \]
(30)
The $\gamma_{(ik)}$ are “physical components” of metrics and have the character of a potential.

The arbitrary conditions
\[ 0 = \frac{\partial \gamma_{(ia)}}{\partial x^a} - \frac{1}{2} \frac{\partial \gamma_{(aa)}}{\partial x^a} \]
(31)
may be the analogy of the Lorentz convention. These lead to Poisson equations
\[ \Box \gamma_{(ik)} = 2\kappa T_{ik}^* , \]
(32)
with retarded potentials as solution
\[ \gamma_{(ik)} = -\frac{\kappa}{2\pi} \int \frac{T_{ik}^* (r_0, ct - |r - r_0|)}{|r - r_0|} \, dV_0 . \]
(33)

Using the energy-momentum tensor of the distributed mass
\[ T_{ik} = \sigma \frac{dx_i}{ds} \frac{dx_k}{ds} , \]
(34)
in which $\sigma$ be the mass density, static solutions result approximately in
\[ \gamma_{(11)} = \gamma_{(22)} = \gamma_{(33)} = -\frac{\kappa}{4\pi} \int \frac{\sigma (r_0)}{|r - r_0|} \, dV_0 , \]
(35)

The rest zero (Einstein [7]). This approximation is not more sufficient for the calculation of the spin.

The actual field quantity might be the curvature vector (Eisenhart [19]) of the world-line described by the test body
\[ k_i = \frac{dx_i}{ds} \left( \frac{dx^i}{ds} \right)_{,\alpha} = \frac{d^2 x^i}{ds^2} + \left\{ a^b_{,i} \right\} \frac{dx^a}{ds} \frac{dx^b}{ds} , \]
(37)
because it acts as a force to the body by its mass.

With distributed mass, the force density becomes
\[ K^i = T^{a}_{\alpha,\cdot} = \sigma k^i . \]
(38)
The force balance* is given only with $\sigma = 0$, unless one uses discrete masses. These are integration constants from $\Box \gamma_{(44)} = 0$. In this case, force balance is obtained with the equations of geodesics [19]
\[ k_i = 0 . \]
(39)

The curvature vector also contains accelerated motion, this is the most simple interpretation of the equivalence principle. The equations of geodesics become equations of motion with it.

The wave equations are analogous to those of electromagnetism, that means also analogous series and analogous integration constants (using spherical coordinates)
\[ a_{[4]}-1,0 = -\frac{\kappa m}{4\pi} \]
(40)
(mass) and
\[ a_{[3]}-1,2 = \frac{\kappa s}{4\pi c} \]
(41)
(with the force density
\[ K^i = T^{a}_{\alpha,\cdot} = \sigma k^i . \]
(43)

*Respectively energy conservation, mathematically expressed with the Bianchi identities [19] in Einstein’s equations.

Force balance is only given with $S^i = 0$. Using this energy-momentum tensor means, there is no choice: The sources must vanish, with them the divergences of the field tensor

$$F^a_{\alpha \beta} = 0.$$  

(44)

Einstein stated this already in his Four Lectures [7]. This step is possible, as explained.

The necessity of this energy-momentum tensor to have just this form is also derived by Montesinos and Flores [21] based on Noether’s theorem [22], but only without sources.

Numerical simulations according to source-free Einstein-Maxwell equations [18] demonstrate that the areas around possible formal singularities do not exist at all. Also known analytic solutions of Einstein’s equations like the isotropic Schwarzschild solution [7],[19] indicate this. The event horizon here is the boundary. In general, a geometric boundary is given when physical components of metrics take an absolute value of 1. It is a kind of horizon in any case. We have to suppose it at the conjectural radius of the particle respectively nucleus, for chaos from the non-linear field equations (see next section).

However, any additional terms or extended methods cannot really repair the inconsistencies from the sources. For $T = 0$ and $R = 0$, Einstein’s equations now result in

$$R_{ik} = \kappa \left( \frac{1}{4} g_{ik} F_{ab} F^{ab} - F_{ia} F^{ia} \right).$$  

(45)

Equations (1), (44), and (45) involve a special Riemannian geometry of the space-time, as explained in [12] and [20]. The field tensor becomes a curve parameter of the world-lines like the curvature vector.

5 On numerical simulations

The precedingly explained insights are supported by numerical simulations according to equations (3), (44), and (45).

Recent robust results can be seen at [23], including the Pascal code of the used program, and a program visualizing these results.

Algorithms and simulation techniques are discussed in [18], as well as the method of approximating the partial differential equations by discrete ones. The principle consists in going from the known (e.g. the distant field of a point charge) to the unknown. In this paper, two visualized samples are shown.

The particle quantities like mass, spin, charge, magnetic momentum are integration constants from mentioned tensor equations, and are inserted as parameters into the initial conditions. The initial conditions start from point charges, or analogous functions for the other integration constants respectively, and are assumed only for great radius. The non-linearities are absolutely negligible at this place.

The number of iterations during the computation up to terminating the actual test means a degree of stability of the solution, and is marked in the graphs as a more or less fat “point”. The reference point (according to literature [24]) is displayed as small circle.

In tests only with mass and charge (remaining parameters zero), masses of preferably small nuclei emerge significantly, together with the right charge at the Helium nucleus, Figure 1. Unfortunately, the procedure is too inaccurate for the electron mass. In return, the other parameters emerge very significantly, see Figure 2.

Above mentioned stability could have to do with chaos. The author had to take notice of the fact, that the numerical solutions are fundamentally different from analytic solutions. Any singularities from analytic solutions are always replaced by boundaries, which can be interpreted as geometrical limits.

The non-linear equations (which behave chaotically) lead

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*Concrete initial conditions see [23], also [18].

†The masses of proton and deuteron are in a sense an add-on of the Helium nucleus tests.
always to these geometrical boundaries, which are 1) finite and 2) outside of possible singular points. Areas with singular points do not exist, i.e. are irrelevant.

One could understand this fundamental contrast by the fact that the differences in time and length are never made zero in a numerical way. The results, exclusively achieved this way, support the view that one has to assume a discrete space-time that does not give reasons for action at a distance. The continuum is only defined with action from point to point, independently on distance or interval between adjacent points.

In order to correctly depict nature, it is apparently necessary to take into consideration the deviations, appearing during the calculation with finite differences. In nature apparently these deviations do not vanish with the transition to very small differences.

Konrad Zuse asked the question, if the possibility to arbitrarily subdivide quantities is “conceivable at all” in nature [25]. Common imagination of a consequent quantization leads to the problem of privileged coordinates, or a privileged frame [25]. Nature has never indicated it. However, it is successful practice in electrical engineering to adapt the coordinates to the actual problem (Wunsch [11]). Linear equations showed to be insensitive to the selection of coordinates. It requires intense research work to prove the chaotic behaviour of the non-linear equations dependent on the coordinates. The author was so fortunate to see the mentioned correlations with spherical coordinates. As well, the correlations became highly significant when the raster distances were the same tangentially as well as radially (dr = r dθ) just at the conjectural particle radius.

6 Concluding remarks

If the obtained insights are right, all quantum phenomena should be understandable by them. At this place, tunnel effects are mentioned. This example is supplemented with very brief but essential remarks on causality.

6.1 On tunnel effects

Equations (1), (44), and (45) allow structures, in which a finite distance (as the outer observer sees it) can locally become zero, but metrics does not become singular. That were a real tunnel with an “inner” length of zero. An event at the one side is “instantaneously” seen at the other side. A known effect, that could be interpreted this way, is the EPR effect [26, 27]. Such tunnels might arise by accident*

This view is supported with changes of metrics by electromagnetism. Distances are locally shortened (at electric fields in direction of the field strength), what can lead to a feedback. Trump and van de Graaf have measured the flashover in the vacuum, dependent on the distance of the electrodes (Kapcov [28]). As well, the product of voltage and field strength was nearly constant

$$U \cdot E \approx 10^{18} \text{V}^2 \text{m}^{-1}.$$ (46)

That means

$$\frac{\partial g_{11}}{\partial r} \approx -2 \times 10^{-44} \text{m}^{-1}.$$ (47)

One will not see these tiny changes, but they are apparently enough to release lightning etc.

On the whole, the influence of gravitation prevails, so that the space-time is macroscopically stable. Table 1 shows the arithmetical deviations of metrics at a radius of $10^{-16}$ m, that is roughly the conjectural radius of nuclei.

<table>
<thead>
<tr>
<th></th>
<th>proton</th>
<th>free electron</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{11}$</td>
<td>$2.48 \times 10^{-30}$</td>
<td>$1.30 \times 10^{-42}$</td>
</tr>
<tr>
<td>$\gamma_{11}$</td>
<td>$-1.85 \times 10^{-43}$</td>
<td>$-1.85 \times 10^{-42}$</td>
</tr>
<tr>
<td>$\gamma_{34}^j$</td>
<td>$1.20 \times 10^{-40}$</td>
<td>$1.20 \times 10^{-40}$</td>
</tr>
<tr>
<td>$\gamma_{34}^j$</td>
<td>$-5.57 \times 10^{-43}$</td>
<td>$-3.6 \times 10^{-40}$</td>
</tr>
<tr>
<td>$\gamma_{38}^j$</td>
<td>$-1.64 \times 10^{-43}$</td>
<td>$-6.84 \times 10^{-38}$</td>
</tr>
</tbody>
</table>

Table 1: The arithmetical deviations of metrics at $10^{-16}$ m.

The influence by mass decreases with $1/r$, however, that by charge and spin with $1/r^2$, and that by magnetic momentum with $1/r^4$.

6.2 On causality

Firstly, equations (3), (44), and (45) provide 10 independent equations for 14 components $g_{ik}$, $A_i$. With it, causality is not given in principle. It is false to claim, a geometric approach would imply causality. Geometry has nothing to do with causality, because causality has not been geometrically defined at all.

If we see something causal, it comes from approximations by wave equations, as precedingly explained. These provide close solutions.

Appendix

“Classical” electric and magnetic fields in the vacuum are joined to an antisymmetric tensor of 2nd rank

$$D = \varepsilon_0 E = \text{im} \frac{\hbar}{2} \begin{pmatrix} P_{(14)} \\ P_{(24)} \\ P_{(34)} \end{pmatrix},$$

$$B = \mu_0 \mathcal{H} = \varepsilon_0 \frac{\hbar}{2} \begin{pmatrix} P_{(23)} \\ P_{(31)} \\ P_{(12)} \end{pmatrix}.$$ (48)

*See also the joke with Mozart’s Fortieth symphony by Nimtz.
Current density and charge density result in a source vector $\mathbf{S}$

$$\mathbf{J} = e\mathbf{u} - \frac{2}{3} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}, \quad \mathbf{\rho} = -j\mathbf{u} - \frac{2}{3} S_4.$$ (49)

The indices in parentheses stand for physical components. See also Kästner [8].

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References

18. An obsolete report is to find at http://bruchholz.psf.net or http://UlrichBruchholz.homepage.t-online.de; See also a German-language textbook, Chapter 4.
23. Recent visible data are in the packages http://www.bruchholz-acoustics.de/robust.tar.gz; http://UlrichBruchholz.homepage.t-online.de/HomepageClassic01/robust.tar.bz2