Dual Phase Lag Heat Conduction and Thermoelastic Properties of a Semi-Infinite Medium Induced by Ultrashort Pulsed Laser

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In this work the uncopled thermoelastic model based on the Dual Phase Lag (DPL) heat conduction equation is used to investigate the thermoelastic properties of a semi-infinite medium induced by a homogeneously illuminating ultrashort pulsed laser heating. The exact solution for the temperature, the displacement and the stresses distributions obtained analytically using the separation of variables method (SVM) hybrid with the source term structure. The results are tested numerically for Cu as a target and presented graphically. The obtained results indicate that at very small time duration disturbance by the pulsed laser the behavior of the temperature, stress and the displacement distribution have wave like behaviour with finite speed.

1 Introduction

Heat transport and thermal stresses response of the medium at small scales becomes recently in the spot of interest due to application in micro-electronics [1] and biology [2, 3] and due to its wide applications in welding, cutting, drilling surface hardening, machining of brittle materials. Because of the unique capability of very high precision control of the ultrashort pulsed laser it is interesting to investigate the thermoelastic properties of the medium due to the ultrashort pulsed laser heating. The different models of thermoelasticity theory based on the equation of heat convection and the elasticity equations. The main categories of these models are the coupled thermoelasticity theory formulated by Biot [4], and the coupled thermoelasticity theory with one relaxation time [5], the two-temperature theory of thermoelasticity [6], the uncoupled classical linear theory of thermoelasticity based on Fourier'a law [7], the uncoupled thermoelasticity theory based on the Maxwell-Cattaneo modification of heat convection to include one time lag between heat flux and the temperature gradient [8, 9].

The coupled and uncoupled models have been used to solve some problems on the macroscale where the length and time scales are relatively large. The technological needs of a high precision control of the ultrashort pulsed laser applications processes at the microscales ($< 10^{-12}$ s), with high heating rates processes are not compatible with the Fourier's model of heat conduction because it implies to an infinite speed for heat propagation and infinite thermal flux on the boundaries. To overcome the deficiencies of Fourier's law in describing high rate heating processes the concept of wave nature of heat convection had been introduced [10]. Tzou [11, 12] had introduced another modification to Fourier law, by inventing two time lags, Dual Phase Lag (DPL), between the heat flux and the temperature gradient namely the heat flux time lag and the temperature gradient time lag. Therefore he had used the dual phase lag heat convection equation with the energy conservation law to obtain the dual phase lag model for heat convection.

The purpose of the present work is to study the induced thermoelastic waves in a homogeneous isotropic semi-infinite medium caused by an ultrashort pulsed laser heating exponentially decay, based on the dual phase lag modification of Fourier's law. The problem is formulated in the dimensionless form and then solved analytically by inventing a new sort of the separation of variables hybridized by the source structure function. The stress, the displacement and the temperature solutions are obtained and tested by a numerical study using the parameters of Cu as a target. The results performed and presented graphically and concluding remarks are given.

2 Problem formulation

In this investigation I considered a homogeneous isotropic semi-infinite medium with mass density ρ, specific heat c_E, thermal conductivity k, and thermal diffusivity $\alpha = \frac{k}{\rho c_E}$. The medium occupy the half space region $z \geqslant 0$ considering the Cartesian coordinates (x, y, z). the medium is assumed to be traction free, initially at uniform temperature T_0, and subjected to heating process by a ultrashort pulsed laser heat source its structure function; $g(z,t) = \frac{I_0(1-R)}{t_p \phi \sqrt{\pi}} e^{-\frac{z}{\varphi}} e^{-\left|\frac{t-t_p}{t_p}\right|}$, at the surface $z = 0$ as in Fig. 1. where the constants characterize this laser pulse are: I_0, the laser intensity, R the reflectivity of the irradiated surface of the medium, ϕ the absorption depth, and t_p the laser pulse duration. The Cartesian coordinates (x, y, z) are considered and z-axis pointing vertically into the medium. Therefore the governing equations are: The equation of motion in the absence of body forces

$$\sigma_{ji,j} = \rho \ddot{u}_i \qquad i, j = x, y, z\,, \tag{1}$$

where σ_{ij} is the stress tensor components, $u_i = (0, 0, w)$ are the displacement vector components. The constitutive relation

$$\sigma_{ij} = \left[\lambda\, div u_i - \gamma(T - T_0)\right]\delta_{ij} + 2\mu e_{ij} \tag{2}$$

by which the stress components are

$$\begin{aligned}\sigma_{xx} &= \sigma_{yy} = \lambda w_z - \gamma(T - T_0)\\ \sigma_{zz} &= (\lambda + 2\mu)w_z - \gamma(T - T_0)\\ \sigma_{xy} &= 0, \qquad \sigma_{xz} = 0, \qquad \sigma_{yz} = 0\,.\end{aligned} \tag{3}$$

The volume dilation e takes the form

$$e = e_{xx} + e_{yy} + e_{zz} = \frac{\partial w}{\partial z}\,. \tag{4}$$

Where the strain-displacement components e_{ij}, read;

$$\begin{aligned} e_{ij} &= \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad i, j = x, y, z\,,\\ e_{zz} &= \frac{\partial w}{\partial z}\,, \quad e_{xx} = e_{yy} = e_{xy} = e_{xz} = e_{yz} = 0\,,\end{aligned} \tag{5}$$

substituting from the constitutive relation into the equation of motion using the equation of motion we get:

- The displacement equation

$$(\lambda + 2\mu)\, w_{zz} - \gamma(T - T_0)_z = \rho\, \ddot{w}\,; \tag{6}$$

- The energy conservation

$$-\rho\, c_E \dot{T} = q_z\,. \tag{7}$$

Since the response of the medium to external heating effect comes later after the pulsed laser heating interacts with the medium surface then there is a time lag, and by using the dual phase lag modification of the Fourier's law as invented by Tzou;

$$\begin{aligned} q(z, t + \tau_q) &= -\,k\, T_z(z, t + \tau_T)\,,\\ q + \tau_q \dot{q} &= -\,k\, T_z - k\, \tau_T\, \dot{T}_z\,.\end{aligned} \tag{8}$$

Then the energy transport equation of hyperbolic type can be obtained by substituting in the energy conservation law and considering the laser heat source

$$\frac{\tau_q}{\alpha}\ddot{T} + \frac{1}{\alpha}\dot{T} = T_{zz} + \tau_T \dot{T}_{zz} - \frac{1}{\rho c_E}\, g(z,t) - \tau_q\, \dot{g}(z,t)\,. \tag{9}$$

This equation shows that the dual lagging should be considered for the processes whose characteristic time are scale comparable to τ_q and τ_T. It describes a heat propagation with finite speed. where τ_q is represents the effect of thermal inertia, it is the delay in heat flux and the associated conduction through the medium, and τ_T is represents the delay in the temperature gradient across the medium during which conduction occurs through its microstructure. For $\tau_T = 0$ one obtain the Maxwell-Cattaneo model, and Fourier law obtained if $\tau_T = \tau_q = 0$.

The boundary conditions are;

$$\begin{aligned} &-k\, T_z(z,t) = g(z,t)\,,\ w = 0\,,\ \sigma_{zz} = 0, \ \text{ at } \ z = 0\,,\\ &\sigma_{zz} = 0, \quad w = 0\,, \quad T = 0\,, \quad \text{as} \quad z \to \infty\,.\end{aligned} \tag{10}$$

Introducing the dimensionless transformations $z^* = \frac{z}{\sqrt{\alpha\tau_q}}$, $w^* = \frac{w}{\sqrt{\alpha\tau_q}}$, $\sigma^*_{ij} = \frac{\sigma_{ij}}{\mu}$, $t^* = \frac{t}{\tau_q}$, $t^*_p = \frac{t_p}{\tau_q}$, $\varphi^* = \frac{\varphi}{\sqrt{\alpha\tau_q}}$, $\tau^* = \frac{\tau_T}{\tau_q}$, $\theta_0\theta^* = T - T_0$, $\lambda^*_1 = \frac{\lambda}{\mu}$, $\lambda^*_2 = \frac{\lambda+2\mu}{\mu}$, $\gamma_0 = \frac{\gamma\theta_0}{\mu}$, $\theta_0 = \frac{I_0(1-R)}{k}\sqrt{\frac{\alpha}{\pi\tau_q}}$, substituting in the governing equations and in boundary conditions of the problem by the above dimensionless transformations and then omitting the $(^*)$ from the resulting equations we obtain the dimensionless set of the governing equations and boundary conditions:

- The dimensionless temperature equation

$$\ddot{\theta} + \dot{\theta} = \theta_{zz} + \tau\dot{\theta}_{zz} + \left(\frac{1 - t_p}{t_p^2\,\varphi}\right) e^{-\frac{z}{\varphi}}\, e^{-\left|\frac{t-2t_p}{t_p}\right|}\,; \tag{11}$$

- The dimensionless displacement equation

$$w_{zz} - B^2\ddot{w} = G\theta_z\,, \tag{12}$$

where $B^2 = \frac{\rho\alpha}{\tau_q t_p^2(\lambda+2\mu)}$ and $G = \frac{\gamma_0\theta_0}{(\lambda+2\mu)}$;

- The dimensionless stresses equations

$$\begin{aligned}\sigma_{zz} &= \lambda_2 w_z - \gamma_0\,\theta\,,\\ \sigma_{xx} &= \sigma_{yy} = \lambda_1 w_z - \gamma_0\,\theta\,;\end{aligned} \tag{13}$$

- Dimensionless boundary conditions

$$\begin{aligned} &w = 0\,, \quad \sigma_{zz} = 0\,, \quad \text{at} \quad z = 0\,,\\ &\theta_z(z,t) = -\frac{1}{k\sqrt{\alpha\tau_q}}\, e^{-\left|\frac{t-2t_p}{t_p}\right|}, \quad \text{at} \quad z = 0\,,\\ &\sigma_{zz} = 0\,, \quad w = 0\,, \quad T = 0\,, \quad \text{as} \quad z \to \infty\,.\end{aligned} \tag{14}$$

3 Solution of the problem

In this section I introduced the hybrid separation of variables method (HSVM) to get the solution of equations (11) and (12). Using this method one can construct the analytic solution for some type of nonhomogeneous partial differential equations (or system). Its idea based on using the structure of the nonhomogeneous term to invent the form of separation of variables. Therefore the PDE (or system) will reduced to ODE (or system) which can be solved. To illustrate the (HSVM) we use it to solve the problem in this paper. Introducing the following separation of variables based on the structure of the source function, which represents the inhomogeneous term,

$$\theta(z,t) = Z(z)\, e^{-\left|\frac{t-2t_p}{t_p}\right|}, \quad w(z,t) = W(z)\, e^{-\left|\frac{t-2t_p}{t_p}\right|} \tag{15}$$

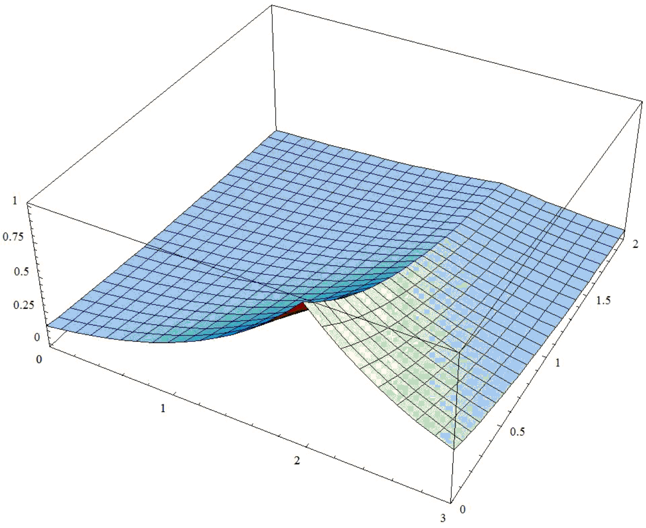

Fig. 1: The strcture function of the ultrashort pulsed laser of exponentially decay.

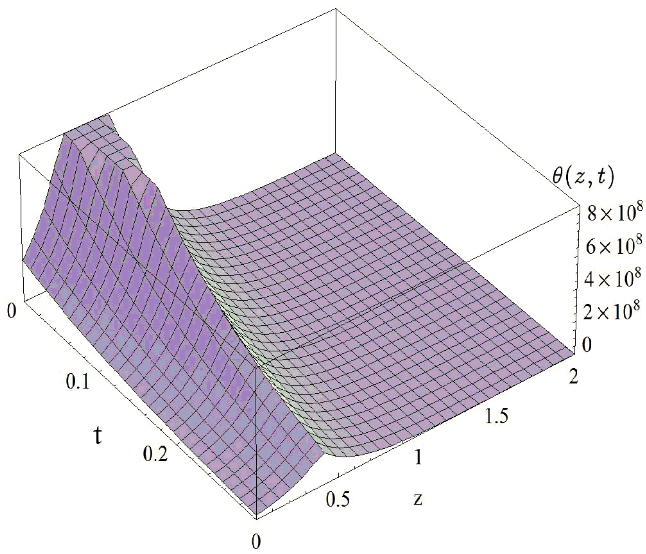

Fig. 2: The dimensionless temperature distribution.

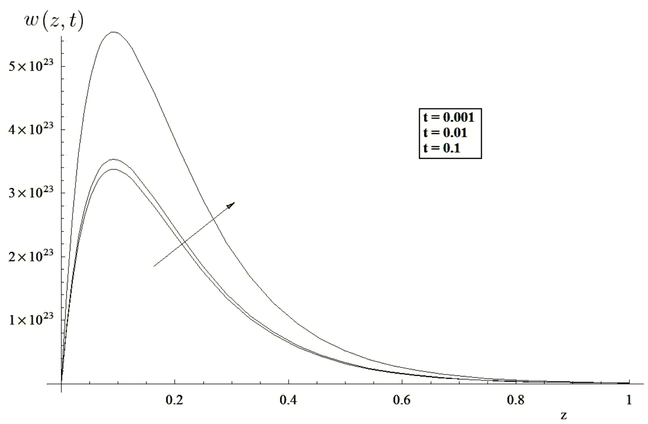

Fig. 3: The dimensionless w-displacement distribution.

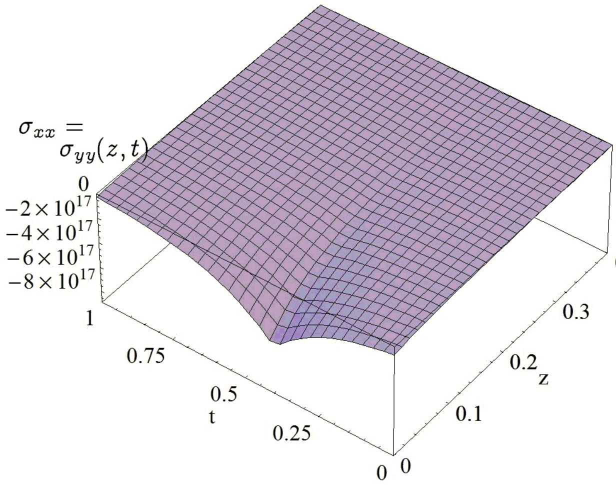

Fig. 4: The dimensionless stresses $\sigma_{xx} = \sigma_{yy}$ distributions.

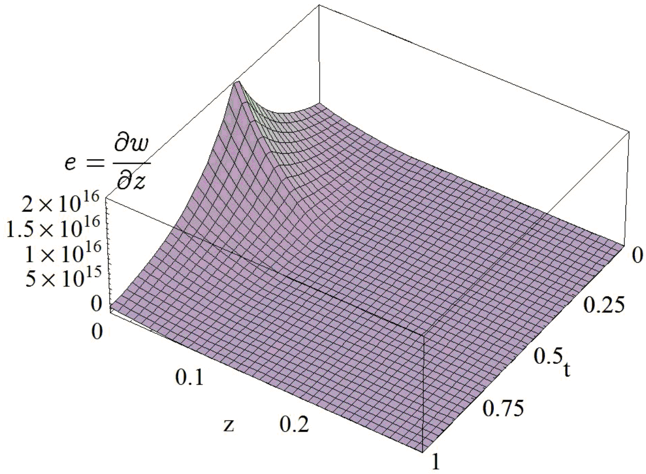

Fig. 5: The dimensionless volume dilation e.

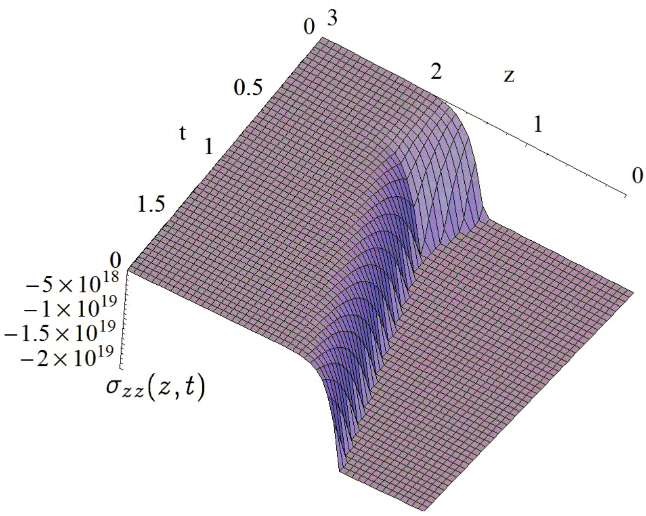

Fig. 6: The dimensionless stresses σ_{zz} distributions.

the equations (11) and (12) will be reduced to a separable form and can be solved directly and therefore using the dimensionless boundary conditions we obtain:

• The solution of the dimensionless temperature equation

$$\theta(z,t) = \left[\vartheta_1 e^{-Az} + \vartheta_2 e^{-\frac{z}{\varphi}}\right] e^{-\left|\frac{t-2t_p}{t_p}\right|}; \qquad (16)$$

where $A^2 = \frac{(1-t_p)}{t_p(t_p-\tau)}$, $H = \frac{(t_p-1)}{t_p\varphi}$,
$\vartheta_1 = \left[\frac{1}{At_p\varphi\sqrt{\alpha\tau_p}} - \frac{H}{A\varphi(\frac{1}{\varphi^2}-A^2)}\right]$, $\vartheta_2 = \frac{H}{(\frac{1}{\varphi^2}-A^2)}$;

• The solution of the dimensionless displacement equation

$$w(z,t) = \left[W_1 e^{-Bz} + W_2 e^{-Az} + W_3 e^{-\frac{z}{\varphi}}\right] e^{-\left|\frac{t-2t_p}{t_p}\right|}, \quad (17)$$

where $W_1 = \left[\frac{GA\vartheta_1}{(A^2-\frac{B^2}{t_p^2})} + \frac{G\vartheta_2}{\varphi(A^2-\frac{B^2}{t_p^2})}\right]$, $W_2 = -\frac{GA\vartheta_1}{(A^2-\frac{B^2}{t_p^2})}$,
$W_3 = -\frac{G\vartheta_2}{\varphi(\frac{1}{\varphi^2}-\frac{B^2}{t_p^2})}$;

• The solution of the dimensionless stresses equation

$$\sigma_{xx} = \sigma_{yy} = -e^{-\left|\frac{t-2t_p}{t_p}\right|}\left[\gamma_0\left(\vartheta_1 e^{-Az} + \vartheta_2 e^{-\frac{z}{\varphi}}\right) + \right.$$
$$\left. +\lambda_1\left(W_1 B e^{-Bz} + W_2 A e^{-Az} + W_3\,\frac{1}{\varphi}\,e^{-\frac{z}{\varphi}}\right)\right], \quad (18)$$

$$\sigma_{zz} = -e^{-\left|\frac{t-2t_p}{t_p}\right|}\left[\gamma_0(\vartheta_1 e^{-Az} + \vartheta_2 e^{-\frac{z}{\varphi}}) + \right.$$
$$\left. +\lambda_2\left(W_1 B e^{-Bz} + W_2 A e^{-Az} + W_3\,\frac{1}{\varphi}\,e^{-\frac{z}{\varphi}}\right)\right], \quad (19)$$

where $\lambda = 7.76{\times}10^{10}$ kg/m sec^2,
$\rho = 8954$ Kg/m^3, $\mu = 3.86{\times}10^{10}$ kg/m sec^2,
$\alpha_t = 1.78{\times}10^{-5}$, $c_E = 383.1$ J/kgK, $t_p = 0.1$ sec,
$k = 386$ W/mK, $\lambda + 2\mu = 1.548{\times}10^{11}$ kg/m sec^2,
$\tau_q = 0.7{\times}10^{-12}$ sec, $\tau_\theta = 89{\times}10^{-12}$ sec,
$\varphi = 0.2$ m, $\gamma = (3\lambda + 2\mu)\alpha_t = 5.518{\times}10^6$ kg/m sec^2,
$\beta = 2{\times}10^{13}$, $\delta = 1.7{\times}10^{-6}$, $A = \beta\tau_q = 14$,
$I_1 = I_0(1-R) = 1{\times}10^{13}$ W/m^2.

4 Discussion and conclusion

In this paper the thermoelastic waves in a homogeneous isotropic semi-infinite medium caused by an ultrashort pulsed laser heating having exponentially decay, based on the dual phase lag modification of Fourier's law have been investigated. The problem formulated in the dimensionless form and then solved analytically for the temperature, the stress, and the displacement by inventing a new sort of the hybridized separation of variables by the source structure function. The obtained analytical solutions are tested numerically using for Cu as a target medium.

The results are presented graphically. The obtained results indicated that due to the very high power of the laser pulse at the surface in a very short duration the temperature distribution possessing a wave nature with finite speed as in Fig. 2. The medium responses to the laser heating by increasing change in the displacement distribution with increasing time duration as in Fig. 3. The thermoelastic characteristics (stresses components $\sigma_{xx} = \sigma_{yy}$ and volume dilation $e = \frac{\partial w}{\partial z}$) of the medium possess wave nature as in Fig. 4 and Fig. 5. Fig. 6. depicts that the stress component σ_{zz} have wave nature with wave front has its maximum at the average of the laser pulse duration. By these results it is expected that the dual phase lag heat conduction model will serve to be more realistic to handle practically the laser problems with very high heat flux and/or ultrashort time heating duration.

Submitted on May 06, 2009 / Accepted on May 15, 2009

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