In this paper, we briefly review the theory elaborated by Louis de Broglie who showed that in some circumstances, a particle tunneling through a dispersive refracting material may reverse its velocity with respect to that of its associated wave (phase velocity): this is a consequence of Rayleigh’s formula defining the group velocity. Within his “Double Solution Theory”, de Broglie re-interprets Dirac’s aether concept which was an early attempt to describe the matter-antimatter symmetry. In this new approach, de Broglie suggests that the (hidden) sub-quantum medium required by his theory be likened to the dispersive and refracting material with identical properties. A Riemannian generalization of this scheme restricted to a space-time section, and formulated within an holonomic frame is here considered. This procedure is shown to be founded and consistent if one refers to the extended formulation of General Relativity (EGR theory), wherein pre-exists a persistent field.

1 Introduction

The original wave function first predicted by Louis de Broglie [1] in his famous Wave Mechanics Theory, then was detected in 1927 by the American physicists Davisson and Germer in their famous experiment on electrons diffraction by a nickel crystal lattice.

In the late 1960’s, Louis de Broglie improved on his first theory which he called Double Solution Interpretation of Quantum Mechanics [2, 3].

His successive papers actually described the massive particle as being much closely related to its physical wave and constantly in phase with it.

The theory which grants the wave function a true physical reality as it should be, necessarily requires the existence of an underlying medium that permanently exchanges energy and momentum with the guided particle [4].

The hypothesis of such a concealed “thermostat” was brought forward by D. Bohm and J. P. Vigier [5] who referred to it as the sub-quantum medium.

They introduced a hydrodynamical model in which the (real) wave amplitude is represented by a fluid endowed with some specific irregular fluctuations so that the quantum theory receives a causal interpretation.

Francis Fer [6] successfully extended the double solution theory by building a non-linear and covariant equation wherein the “fluid” is taken as a physical entity. In the recent paper [7], the author proposed to generalize this model to an extended formulation of General Relativity [8], which allows to provide a physical solution to the fluid random perturbation requirement.

Based on his late conceptions, Louis de Broglie then completed a subsequent theory [9] on the guided particle: under specific circumstances the particle tunneling through a dispersive refracting material is shown to reverse velocity with respect to the associated wave phase velocity.

As a further assumption, Louis de Broglie identified the dispersive refracting material with the hidden medium [10] considered above.

In this case, the theoretical results obtained are describing the behavior of a pair particle-antiparticle which is close to the Stuckelberg-Feynmann picture [11], in which antiparticles are viewed as particles with negative energy that move backward in time.

Within this interpretation, the sub-quantum medium as derived from de Broglie’s theories, appears to provide a deeper understanding of Dirac’s aether theory [12], once popular before.

In this paper, we try to generalize this new concept by identifying the hidden medium with the persistent energy-momentum field tensor inherent to the EGR theory.

Such a generalization is here only restricted to a Riemannian space-time section \((t = \text{const})\), where the integration is further performed over a spatial volume. By doing so, we are able to find back the essential formulas set forth by Louis de Broglie in the Special Relativity formulation.

We assumed here a limited extension without loss of generality: a fully generalized theory is desirable, as for example the attempt suggested by E. B. Gliner [13], who has defined a “µ-medium” entirely derived from General Relativity considerations.

2 Short overview of the Double Solution Theory within wave mechanics (Louis de Broglie)

2.1 The reasons for implementing the theory

As an essential contribution to quantum physics, Louis de Broglie’s wave mechanics theory has successfully extended the wave-particle duality concepts to the whole physics.

Double solution theory which aimed at confirming the...
true physical nature of the wave function is based on two striking observations: within the Special Theory of Relativity, the frequency $v_0$ of a plane monochromatic wave is transformed as

$$ v = \frac{v_0}{\sqrt{1 - \beta^2}}, $$

whereas a clock’s frequency $v_0$ is transformed according to $v_c = v_0 \sqrt{1 - \beta^2}$ with the phase velocity

$$ \psi = \frac{c}{\beta} = \sqrt{\frac{\epsilon}{\mu}}. $$

The 4-vector defined by the gradient of the plane monochromatic wave is linked to the energy-momentum 4-vector of a particle by introducing Planck’s constant $\hbar$ as

$$ W = h\nu, \quad \lambda = \frac{\hbar}{p}, \quad (1) $$

where $p$ is the particle’s momentum and $\lambda$ is the wave length.

If the particle is considered as that containing a rest energy $M_0c^2 = h\nu_0$, it is likened to a small clock of frequency $\nu_0$ so that when moving with velocity $v = \beta c$, its frequency different from that of the wave is then

$$ v = v_0 \sqrt{1 - \beta^2}. $$

In the spirit of the theory, the wave is a physical entity having a very small amplitude not arbitrarily normed and which is distinct from the $\psi$-wave reduced to a statistical quantity in the usual quantum mechanical formalism.

Let us call $\theta$ the physical wave which is connected to the $\psi$-wave by the relation $\psi = C\theta$, where $C$ is a normalizing factor.

The $\theta$-wave has then nature of a subjective probability representation formulated by means of the objective $\theta$-wave.

Therefore wave mechanics is complemented by the double solution theory, for $\psi$ and $\theta$ are two solutions of the same equation.

If the complete solution of the equation representing the $\theta$-wave (or, if preferred, the $\psi$-wave, since both waves are equivalent according to $\psi = C\theta$), as written’s as

$$ \theta = a(x, y, z, t) \exp \left[ \frac{i}{\hbar} \phi(x, y, z, t) \right], \quad \hbar = \frac{\hbar}{2\pi}, \quad (2) $$

where $a$ and $\phi$ are real functions, while the energy $W$ and the momentum $p$ of the particle localized at point $(x, y, z)$, at time $t$ are given by

$$ W = \partial_t \phi, \quad p = -\text{grad} \phi, \quad (3) $$

which in the case of a plane monochromatic wave, where one has

$$ \phi = \hbar \left[ v = \left( \frac{ax + \beta y + \gamma z}{A} \right) \right] $$

yields equation (1) for $W$ and $p$.

### 2.2 The guidance formula and the quantum potential

Taking Schrödinger’s equation for the scalar wave $\theta$, and $U$ being the external potential, we get

$$ \partial_t \theta = \frac{\hbar}{2m} \Delta \theta + \frac{i}{\hbar} U \theta. \quad (4) $$

This complex equation implies that $\theta$ be represented by two real functions linked by these two real equations which leads to

$$ \theta = a \exp \left( \frac{i \phi}{\hbar} \right), \quad (5) $$

where $a$ the wave’s amplitude, and $\phi$ its phase, both are real. Substituting this value into equation (4), it gives two important equations

$$ \begin{align*}
\partial_t \phi - U & = -\frac{1}{2m} \text{grad} (\phi)^2 = -\frac{\hbar^2}{2m} \frac{\Delta a}{a} \\
\partial_t (a^2) & = -\frac{1}{m} \text{div} (a^2 \text{grad} \phi) = 0
\end{align*} \quad (6) $$

If terms involving Planck’s constant $\hbar$ in equation (6) are neglected (which amounts to disregard quanta), and if we set $\phi = S$, this equation becomes

$$ \partial_t S - U = \frac{1}{2m} \text{grad} S^2. $$

As $S$ is the Jacobi function, this equation is the Jacobi equation of Classical Mechanics.

Only the term containing $\hbar^2$ is responsible for the particle’s motion being different from the classical motion.

The extra term in (6) can be interpreted as another potential $Q$ distinct from the classical $U$ potential

$$ Q = -\frac{\hbar^2}{2m} \frac{\Delta a}{a}. \quad (7) $$

One has thus a variable proper mass

$$ M_0 = m_0 + \frac{Q_0}{c^4}, \quad (8) $$

where, in the particle’s rest frame, $Q_0$ is a positive or negative variation of this rest mass and it represents the “quantum potential” which causes the wave function’s amplitude to vary.

By analogy with the classical formula $\partial_t S = E$, and $p = -\text{grad} S$, $E$ and $p$ being the classical energy and momentum, one may write

$$ \partial_t \phi = E, \quad -\text{grad} \phi = p. \quad (9) $$

As in non-relativistic mechanics, where $p$ is expressed as a function of velocity by the relation $p = mv$, one eventually finds the following results

$$ v = \frac{p}{m} = -\frac{1}{m} \text{grad} \phi, \quad (10) $$

which is the guidance formula.
It gives the particle’s velocity, at position \((x, y, z)\) and time \(t\) as a function of the local phase variation at this point.

Inspection shows that relativistic dynamics applied to the variable proper mass \(M_0\) eventually leads to the following result

\[
W = \frac{M_0 c^2}{\sqrt{1 - \beta^2}} = M_0 c^2 \sqrt{1 - \beta^2} + \frac{M_0 V^2}{\sqrt{1 - \beta^2}}
\]  

(11)

known as the Planck-Laue formula.

Here, the quantum force results from the variation of \(M_0 c^2\) as the particle moves.

2.3 Particles with internal vibration and the hidden thermodynamics

The idea of considering the particle as a small clock is of central importance here.

Let us look at the self energy \(M_0 c^2\) as the hidden heat content of a particle. One easily conceives that such a small clock has (in its proper system) an internal periodic energy of agitation which does not contribute to the whole momentum. This energy is similar to that of a heat containing body in the state of thermal equilibrium.

Let \(Q_0\) be the heat content of the particle in its rest frame, and viewed in a frame where the body has a velocity \(\beta c\), the contained heat will be

\[
Q = Q_0 \sqrt{1 - \beta^2} = M_0 c^2 \sqrt{1 - \beta^2} \equiv h \nu_0 \sqrt{1 - \beta^2}.
\]  

(12)

The particle thus appears as being at the same time a small clock of frequency

\[
\nu = \nu_0 \sqrt{1 - \beta^2}
\]

and a small reservoir of heat

\[
Q = Q_0 \sqrt{1 - \beta^2}
\]

moving with velocity \(\beta c\). If \(\phi\) is the wave phase \(a \exp(\frac{\phi}{\nu_0})\), where \(a\) and \(\phi\) are real, the guidance theory states that

\[
\partial_t \phi = \frac{M_0 c^2}{\sqrt{1 - \beta^2}} \nu \quad \text{and} \quad \text{grad} \phi = \frac{M_0 \nu}{\sqrt{1 - \beta^2}}.
\]  

(13)

The Planck-Laue equation may be written

\[
Q = M_0 c^2 \sqrt{1 - \beta^2} = \frac{M_0 c^2}{\sqrt{1 - \beta^2}} = \nu p. 
\]  

(14)

Combining (13) and (14) results in

\[
M_0 c^2 \sqrt{1 - \beta^2} = \partial_t \phi + \nu \text{ grad} \phi = \frac{d \phi}{dt}.
\]

Since the particle is regarded as a clock of proper frequency \(M_0 \nu_0^2\), the phase of its internal vibration expressed with \(a_i \exp(\frac{\phi_i}{\nu_0})\) and \(a_i\) and \(\phi_i\) real will be

\[
\phi_i = h \nu_0 \sqrt{1 - \beta^2} t = M_0 c^2 \sqrt{1 - \beta^2} t.
\]

thus we obtain

\[
d(\phi_i - \phi) = 0.
\]  

(15)

This fundamental result agrees with the assumption according to which the particle as it moves in its wave, remains constantly in phase with it.

3 Propagation in a dispersive refracting material

3.1 Group velocity

The classical wave is written as

\[
a \exp[2\pi i(\nu t - k r)];
\]  

(16)

it propagates along the direction given by the unit vector \(n\).

We next introduce the phase velocity \(\nu\) of the wave, which determines the velocity between two “phases” of the wave.

Consider now the superposition of two stationary waves having each a very close frequency: along the \(x\)-axis, they have distinct energies

\[
E_1 = A \sin 2\pi(\nu + d\nu) \left[ t - \frac{x}{\nu + d\nu} \right],
\]

\[
E_2 = A \sin 2\pi(\nu - d\nu) \left[ t - \frac{x}{\nu - d\nu} \right].
\]

thus next we have

\[
\frac{\nu + d\nu}{\nu + d\nu} = \frac{\nu}{\nu} + d\left(\frac{\nu}{\nu}\right), \quad \frac{(\nu - d\nu)}{\nu - d\nu} = \nu - d\left(\frac{\nu}{\nu}\right),
\]

and by adding both waves

\[
E = 2A \cos 2\pi d\nu \left[ t - x \left(\frac{d}{d\nu}\right)\left(\frac{\nu}{\nu}\right)\right] \sin 2\pi\nu \left[ t - \frac{x}{\nu}\right].
\]  

(17)

The term

\[
2A \cos 2\pi d\nu \left[ t - x \left(\frac{d}{d\nu}\right)\left(\frac{\nu}{\nu}\right)\right]
\]

may be regarded as the resulting amplitude that varies along with the so-called “group velocity” \(\nu_g\) such that

\[
\frac{1}{\nu_g} = \left(\frac{d}{d\nu}\right)\left(\frac{\nu}{\nu}\right).
\]  

(19)

Recalling the relation between the wave length \(\lambda\) and the material refracting index \(n\)

\[
\lambda = \frac{\nu}{\nu_g} = \frac{\nu_0}{n\nu}
\]  

(20)

where \(\nu_0\) is the wave velocity in a given reference material \((c\) in vacuum), we see that

\[
n = \frac{\nu_0}{\nu}, \quad \text{i.e. in vacuum} \quad n = \frac{c}{\nu}.
\]  

(21)

Now, we have the Rayleigh formulae

\[
\frac{1}{\nu_g} = \left(\frac{d}{d\nu}\right)\left(\frac{\nu}{\nu}\right) = \left(\frac{d}{d\nu}\right)\left(\frac{1}{\lambda}\right).
\]  

(22)
It is then easy to show that \([v]_0\) coincides with the velocity \(v\) of the particle, which is also expressed in terms of the wave energy \(W\) as
\[
[v]_0 = \frac{\partial W}{\partial k}.
\]
The velocity of the particle \(v\) may be directed either in the propagating orientation of the wave in which case
\[
p = k = \left(\frac{\hbar}{\lambda}\right)n,
\]
or in the opposite direction \(p = -k = -\left(\frac{\hbar}{\lambda}\right)n\).
When the particle’s velocity \(v > 0\), and \(p = k\), we have the Hamiltonian form
\[
v = \frac{\partial W}{\partial p}.
\]

### 3.2 Influence of the refracting material

Let us recall the relativistic form of the Doppler’s formulae:
\[
v_0 = \frac{\nu \left(1 - \frac{c^2}{v^2}\right)}{\sqrt{1 - \beta^2}},
\]
where as usual \(v_0\) is the wave’s frequency in the frame attached to the particle.

Considering the classical relation \(W = h\nu\) connecting the particle energy and its wave frequency, and taking into account (23), we have
\[
W = W_0 \sqrt{1 - \beta^2} \left(1 - \frac{\nu}{\tilde{v}}\right).
\]
However, inspection shows that the usual formula
\[
W = \frac{W_0}{\sqrt{1 - \beta^2}}
\]
holds only if
\[
1 - \frac{\nu}{\tilde{v}} = 1 - \beta^2,
\]
which implies
\[
\tilde{v} = c^2
\]
and this latter relation is satisfied provided we set
\[
W = \frac{M_o c^2}{\sqrt{1 - \beta^2}}, \quad p = \frac{M_o \nu}{\sqrt{1 - \beta^2}},
\]
where \(M_o\) is the particle’s proper mass which includes an extra term \(\delta M_o\) resulting from the quantum potential \(Q\) contribution.
When the particle whose internal frequency is \(v_0 = \frac{M_o c^2}{n}\) has travelled a distance \(dn\) during \(dt\), its internal phase \(\phi_i\) has changed by
\[
d\phi_i = M_o c^2 \sqrt{1 - \beta^2} dt = d\phi,
\]
where \(n\) is the unit vector normal to the phase surface.

The identity of the corresponding wave phase variation
\[
d\phi = \partial_\phi dt + \partial_\psi \phi dr = (\partial_\psi \phi + v \text{ grad } \phi) dt
\]
is also expressed by
\[
\partial_\psi \phi + \partial_\psi \phi dr = d\phi_i,
\]
and it leads to
\[
\frac{M_o c^2}{\sqrt{1 - \beta^2}} - \frac{M_o \nu^2}{\sqrt{1 - \beta^2}} = M_o c^2 \sqrt{1 - \beta^2} - \nu^2.
\]
The situation is different in a refracting material which is likened to a “potential” \(P\) acting on the particle so that we write
\[
W = \frac{M_o c^2}{\sqrt{1 - \beta^2}} + P,
\]
\[
p = \frac{M_o \nu}{\sqrt{1 - \beta^2}} = \nu \left(1 - \frac{c^2}{v^2}\right) = \frac{M_0 c^2}{\sqrt{1 - \beta^2}}.
\]
Now taking into account equation (23), the equation (24) reads (re-instating \(h\))
\[
\frac{1}{h} d_\phi = v_0 \sqrt{1 - \beta^2} = \nu \left(1 - \frac{\nu}{\tilde{v}}\right)
\]
yielding
\[
W - \nu^2 \frac{W - P}{c^2} = W \left(1 - \frac{\nu}{\tilde{v}}\right)
\]
from which we infer the expression of the potential \(P\)
\[
P = W \left(1 - \frac{c^2}{\tilde{v}}\right) = h\nu \left(1 - \frac{c^2}{\tilde{v}}\right)
\]
and with the Rayleigh formulae (22)
\[
P = W \left[1 - n \frac{\partial (\nu v)}{\partial \nu}\right]
\]
(we assume \(v_0 = c\), for the phase \(\phi\) of the wave along the \(x\)-axis we find \(d\phi = W dt - k dx\) with
\[
k = v \frac{W - P}{c^2} = \frac{\hbar}{\lambda}.
\]
The phase concordance \(h d\phi = h d\phi\) readily implies
\[
(W - k\nu) dt = \left(W - \nu^2 \frac{W - P}{c^2}\right) dt
\]
and taking into account (28),
\[
d\phi_i = \frac{W}{\hbar} \left(1 - \frac{\nu}{\tilde{v}}\right) dt = 2\pi \nu \left(1 - \frac{\nu}{\tilde{v}}\right) dt.
\]
Now applying the Doppler formulae (23), and bearing in mind the transformation \(dt_0 = \sqrt{1 - \beta^2}\), we can write
\[
d\phi = 2\pi v_0 dt_0 = 2\pi \nu \left(1 - \frac{\nu}{\tilde{v}}\right) dt.
\]
One easily sees that the equivalence of (32) and (33) fully justifies the form of the “potential” \(P\).
4 The particle-antiparticle state

4.1 Reduction of the EGR tensor to the Riemannian scheme

4.1.1 Massive tensor in the EGR formulation

Setting the 4-unit velocity \( u^\alpha = \frac{dx^\alpha}{dt} \) which obeys here

\[ g_{\alpha\beta} u^\alpha u^\beta = g^{\alpha\beta} u_\alpha u_\beta = 1. \]

Expressed in mixed indices, the usual Riemannian massive tensor is well known

\[ (T^a_b)_\text{Riem} = \rho_0 c^2 u^a u_b, \quad (34) \]

where \( \rho_0 \) is the proper density of the mass.

In the EGR formulation, the massive tensor is given by

\[ (T^a_b)_\text{EGR} = (\rho_0)_{\text{EGR}} c^2 (u^b)_\text{EGR} + (T^a_b)_\text{field}. \quad (35) \]

The EGR world velocity is not explicitly written but it carries a small correction w.r.t. to the regular Riemannian velocity \( u^\alpha \).

The EGR density \( \rho_0 \) is also modified, as was shown in our paper [8] which explains the random perturbation of the fluid.

Let us now express \((T^a_b)_\text{EGR}\) in terms of the Riemannian representation

\[ (T^a_b)_\text{EGR} = (T^a_b)^* \quad \text{Riem}. \quad (36) \]

With respect to \((T^a_b)^* \text{Riem}\), the tensor \((T^a_b)^* \text{Riem}\) is obviously only modified through the Riemannian proper density \( \rho \) we denote \( \rho^* \) since now.

Having said that, we come across a difficulty since the quantity \((T^a_b)_\text{EGR}\) is antisymmetric whereas \((T^a_b)^* \text{Riem}\) is symmetric.

In order to avoid this ambiguity, we restrict ourselves to a space-time section \( x^4 = \text{const} \). In this case, we consider the tensor \((T^a_b)_\text{EGR}\) which we split up into

\[ (T^a_b)_\text{EGR} = (T^a_b)^* \quad \text{Riem}, \quad (37) \]

\[ (T^a_b)_\text{EGR} = (T^a_b)^* \quad \text{Riem}. \quad (38) \]

Inspection shows that each of the EGR tensors components when considered separately in (37) and (38) is now symmetric.

4.1.2 The modified proper mass

We write down the above components

\[ (T^a_b)^* \text{Riem} = \rho_0 c^2 u^a u_b, \quad (39) \]

\[ (T^a_b)^* \text{Riem} = \rho_0 c^2 u^a u_b. \quad (40) \]

This amounts to state that the proper density \( \rho_0 \) is modified by absorbing the EGR free field component \((T^a_b)_\text{field}\) tensor.

By the modification, we do not necessarily mean an “increase”, as will be seen in the next sections.

4.2 Refracting material

4.2.1 Energy-momentum tensor

We now consider a dispersive refracting material which is characterized by a given (variable) index denoted by \( n \).

Unlike a propagation in vacuum, a particle progressing through this material will be subject to a specific “influence” which is acting upon the tensor \((T^a_b)_\text{Riem}\). Thus, the energy-momentum tensor of the system will thus be chosen to be

\[ (T^a_b)_\text{Riem} = \rho_0 c^2 u^b u_4 - \delta^a_4 b(n), \quad (41) \]

where \( b(n) \) is a scalar term representing the magnitude of the influence and which is depending on the refracting index \( n \). The tensor \( \delta^a_4 b(n) \) is reminiscent of a "pressure term" which appears in the perfect fluid solution except that no equation of state exists.

Equation (41) yields

\[ (T^a_4)^* \text{Riem} = \rho_0 c^2 u^a u_4, \quad (42) \]

\[ (T^a_4)_\text{Riem} = \rho_0 c^2 + b(n), \quad (43) \]

Applying the relation \( u^\alpha c = v^\alpha u^\alpha \), equation (42) becomes

\[ (T^a_4)^* \text{Riem} = \rho_0 c v^a. \quad (44) \]

4.2.2 Integration over the hypersurface \( x^4 = \text{const} \)

Integration of (43) over the spatial volume \( V \) yields

\[ \frac{1}{c} \int \rho_0 c^2 \sqrt{-g} dV + \frac{1}{c} \int b(n) \sqrt{-g} dV, \quad (45) \]

while integrating (44), we get a 3-momentum vector

\[ \frac{1}{c} \int \rho_0 c v^\alpha \sqrt{-g} dV, \quad (47) \]

\[ \frac{1}{c} \int \rho_0 c v^\alpha \sqrt{-g} dV, \quad (48) \]

4.2.3 Matching the formulas of de Broglie

Let us multiply, respectively, (46) and (48) by \( u^4 \)

\[ u^4 c \left( P^a \right)_\text{Riem} = u^4 m_0 c^2 + u^4 B(n); \quad (49) \]

if we set \( P = u^4 B(n) \), we retrieve de Broglie’s first formula (25)

\[ u^4 c \left( P^a \right)_\text{Riem} = W = \frac{m_0 c^2}{\sqrt{1 - \beta^2}} + P(n) \quad (50) \]

as well as the second formula (26)

\[ u^4 \left( P^a \right)_\text{Riem} = p = \frac{m_0 v^\alpha}{\sqrt{1 - \beta^2}}. \quad (51) \]  

Patrick Marquet. The Matter-Antimatter Concept Revisited
5 A new aspect of the antiparticle concept

5.1 Proper mass

In §4.1.2 we have considered the modified proper density \( \rho_0 \), resulted from the EGR persistent free field “absorbed” by the tensor in the Riemannian scheme.

Having established the required generalization, we now revert to the classical formulation as suggested by de Broglie.

The corresponding modified proper mass \( m_0^* \) should always be positive, therefore we are bound to set

\[
p = k \quad \text{if} \quad v > 0, \quad p = -k \quad \text{if} \quad v < 0.
\]

With these, we infer

\[
\frac{m_0^*}{\sqrt{1 - \beta^2}} = \pm \frac{W - P}{c^2} \quad \text{(53)}
\]

that is

\[
m_0^* = \pm \frac{W}{\sqrt{v}} \sqrt{1 - \beta^2} \quad \text{(54)}
\]

For propagation in vacuum we have \( P = 0 \), \( v = v_0 = c^2/\sqrt{v} \), and \( W = m_0 c^2/\sqrt{1 - \beta^2} \) which implies, a expected,

\[
m_0^* = m_0.
\]

5.2 Antiparticles state

The early theory of antiparticles is due to P. A. M. Dirac after he derived his famous relativistic equation revealing the electron-positron symmetric state. In order to explain the production of a pair “electron-positron”, Dirac postulated the presence of an underlying medium filled with electrons \( e \) bearing a negative energy \(-m_0 c^2\).

An external energy input \( 2m_0 c^2 \) would cause an negative energy electron to emerge from the medium as a positive energy one, thus become observable. The resulting “hole” would constitute, in this picture, an “observable” particle, positon, bearing a positive charge.

With Louis de Broglie, we follow this postulate: we consider that the hidden medium should also be filled with particles bearing a negative proper energy. Therefore the proper mass “modification” discussed above is expressed by

\[
m_0^* = -m_0 \quad \text{(55)}
\]

and is true in the medium.

At this point, two fundamental situations are to be considered as follows:

1) The “normal” situation where \( P = 0, m_0^* \), and \( v = v_0 \);
2) The “singular” situation where \( P = 2W \), in which case, according to (28) and (29), the following relations are obtained

\[
\frac{\partial(\nu \nu)}{\partial v} = -1.
\]

Hence, in the “singular” situation b),

\[
\frac{1}{\sqrt{v}} \frac{\partial}{\partial v} \frac{\nu}{\nu^2} = \frac{v}{c^2} = \frac{1}{v_0},
\]

from which is inferred

\[
W = \frac{m_0^* c^2}{\sqrt{1 - \beta^2}} \quad \text{and} \quad p = -\frac{m_0^* c^2}{\sqrt{1 - \beta^2}}.
\]

On the other hand

\[
k = v_0 \frac{(W - P)}{c^2} = \frac{m_0^* v_0}{\sqrt{1 - \beta^2}}, \quad k = \frac{m_0 v_0}{\sqrt{1 - \beta^2}}
\]

From which is inferred

\[
p = -k = -\frac{m_0 v_0}{\sqrt{1 - \beta^2}} \quad \text{(57)}
\]

Within this interpretation, the observed antiparticle has an opposite charge, a positive rest mass \( m_0 \) and a reversed velocity \( v_0 \) with respect to the phase wave propagation.

The state of electron-positon requires negative energies bounded to the sub-quantum medium which can be now further explicited.

The external energy input \( 2m_0 c^2 \) causes a positive (observable) energy of the electron to emerge from the medium according to

\[
-m_0 c^2 + 2m_0 c^2 = m_0 c^2.
\]

However, the charge conservation law requires the simultaneous emergence of an electron with positive rest energy \( m_0 c^2 \) implying for the hidden medium to supply a total energy of \( 2m_0 c^2 \). In other words, we should have

\[
Q = 2m_0 c^2.
\]

5.3 Introducing the quantum potential

Following the same pattern as above, the quantum potential \( Q \) is now assumed to act as a dispersive refracting material.

This means that \( Q = P \) where the definition (8) holds now, for \( m_0^* \).

\[
Q = M_0 c^2 - m_0^* c^2.
\]

Since \( m_0^* c^2 = -m_0 c^2 \), we have with (59)

\[
M_0 = m_0.
\]

The energy and the momentum of the antiparticle are now given by

\[
W = \frac{M_0 c^2}{\sqrt{1 - \beta^2}} = -\frac{m_0 c^2}{\sqrt{1 - \beta^2}},
\]

\[
p = \frac{M_0 v}{\sqrt{1 - \beta^2}} = -\frac{m_0 v_0}{\sqrt{1 - \beta^2}} = -k.
\]
6 Concluding remarks

According to the double solution theory, there exists a close relationship between the guidance formula, and the relativistic thermodynamics.

Following this argument, it is interesting to try to connect the entropy with the particle/antiparticle production process as it is derived above.

We first recall the classical action integral for the free particle:

$$a = \int L dt = -\int M_0 c^2 \sqrt{1 - \beta^2} \, dt.$$  \hspace{1cm} (63)

If we choose a period $T_i$ of the particle’s internal vibration (its proper mass is $M_0$) as the integration interval, from (12) we have

$$\frac{1}{T_i} = \frac{m_0 c^2}{\hbar} \sqrt{1 - \beta^2}.$$  \hspace{1cm} (64)

so that a “cyclic” action integral be defined as

$$\frac{a}{\hbar} = -\int_0^{T_i} M_0 c^2 \sqrt{1 - \beta^2} \, dt \approx -\frac{M_0 c^2}{m_0 c^2}$$  \hspace{1cm} (65)

($T_i$ is assumed to be always short so that $M_0$ and $\beta^2 = c^2$ can be considered as constants over the integration interval).

Denoting the hidden thermostat’s entropy by $s$, we set

$$\delta s = \frac{W}{R} = \frac{a}{\hbar},$$  \hspace{1cm} (66)

where $W$ is Boltzmann’s constant.

Since

$$\delta Q_0 = \delta m_0 c^2;$$

we obtain

$$\delta s = -W \frac{\delta Q_0}{m_0 c^2}.$$  \hspace{1cm} (67)

An entropy has thus been determined for the single particle surrounded by its guiding wave. According to Boltzmann’s relation

$$s = W \ln P,$$

where $P = \exp \left( \frac{W}{R} \right)$ is the probability characterizing the system.

In this view, the prevailing plane monochromatic wave representing the quantized (stable) stationary states corresponds to an entropy maxima, whereas the other states also exist but with a much reduced probability.

Now, we revert to the hidden sub-quantum medium which thus supplies the equivalent heat quantity

$$Q_0 = \mathcal{Q}.$$  \hspace{1cm} (68)

The definition (8) can be re-written as

$$Q_0 = M_0 c^2 - m_0 c^2.$$  \hspace{1cm} (69)

Therefore, according to the formula (67), the medium is needed to supply an energy of $2m_0 c^2$ that is characterized by an entropy decrease of $2W$.

Its probability being reduced, this explains why an antiparticle is unstable.

So, the thermodynamics approach, which could at first glance seem strange in quantum theory, eventually finds here a consistent ground. It is linked to "probability" situations which fit in the physical processes involving wave “packet” propagations within the guidance of the single particle.

We have tried here to provide a physical interpretation of the sub-quantum medium from which the particle-antiparticle symmetry originates within the double solution theory elaborated by Louis de Broglie. In the Riemannian approximation which we have presented above, the introduction of a term generalizing the quantum potential would appear as that having a somewhat degree of arbitrariness. However, if one refers to our extended general relativity theory (EGR theory), the introduction of this term is no longer arbitrary as it naturally arises from its main feature.

References


Patrick Marquet. The Matter-Antimatter Concept Revisited