The Intensity of the Light Diffraction by Supersonic Longitudinal Waves in Solid

Vahan Minasyan and Valentin Samoilov
Scientific Center of Applied Research, JINR, Dubna, 141980, Russia
E-mails: mvahan@scar.jinr.ru; scar@off-serv.jinr.ru

First, we predict existence of transverse electromagnetic field created by supersonic longitudinal waves in solid. This electromagnetic wave with frequency of ultrasonic field is moved by velocity of supersonic field toward of direction propagation of one. The average Poynting vector of superposition field is calculated by presence of the transverse electromagnetic and the optical fields which in turn provides appearance the diffraction of light.

1 Introduction

In 1921 Brillouin have predicted that supersonic wave in ideal liquid acts as diffraction gratings for optical light [1]. His result justify were confirmed by Debay and Sears [2]. Further, Schaefer and Bergmann had shown that supersonic waves in crystal leads to light diffraction [3]. The description of latter experiment is that the diffraction pattern is formed by passing a monochromatic light beam through solid perpendicular to direction of ultrasonic wave propagation. Furthermore, the out-coming light is directed on diffraction pattern. As results of these experiment, a diffraction maximums of light intensity represent as a sources of light with own intensities. Each intensity of light source depends on the amplitude of acoustical power because at certain value of power ultrasound wave there is vanishing of certain diffraction maxima. Other important result is that the intensity of the first positive diffraction maximum is not equal to the intensity of the first negative minimum, due to distortion of the waveform in crystals by the departures from Hooke's law as suggested [4]. For theoretical explanation of experimental results, connected with interaction ultrasonic and optical waves in isotropy homogenous medium, were used of so called the Raman-Nath theory [5] and theory of photo-elastic linear effect [6] which were based on a concept that acoustic wave generates a periodical distribution of refractive index in the coordinate-time space. For improving of the theory photo-elastic effect, the theories were proposed by Fues and Ludloff [7], Mueller [8] as well as Melngailis, Maradudin and Seeger [9]. In this letter, we predict existence of transverse electromagnetic radiation due to strains created by supersonic longitudinal waves in solid. The presence of this electromagnetic field together with optical one provides appearance of superposition wave which forms diffracted light with it's maxima.

2 Creation of an electromagnetic field

A model of solid is considered as lattice of ions and gas of free electrons. Each ion coupled with a point of lattice knot by spring, creating of ion dipole. The knots of lattice define a position equilibrium of each ion which is vibrated by own frequency Ω_0.

The electron with negative charge $-e$ and ion with positive charge e are linked by a spring which in turn defines the frequency ω_0 of electron oscillation in the electron-ion dipole. Obviously, such dipoles are discussed within elementary dispersion theory [10]. Hence, we suggest that property of springs of ion dipole and ion-electron one is the same.

Now we attempt to investigate an acoustic property of solid. By under action of longtidunal acoustic wave which is excited into solid, there is an appearance of vector displacement $\vec{u}$ of each ions.

Consider the propagation of an ultrasonic plane traveling wave in cubic crystal. Due to laws of elastic field for solid [11], the vector vector displacement $\vec{u}$ satisfies to condition which defines a longitudinal supersonic field

$$\operatorname{curl} \vec{u} = 0 \tag{1}$$

and is defined by wave-equation

$$\nabla^2 \vec{u} - \frac{1}{c_l^2} \frac{d^2 \vec{u}}{dt^2} = 0\,, \tag{2}$$

where c_l is the velocity of a longitudinal ultrasonic wave which is determined by elastic coefficients.

The simple solution of (2) in respect to $\vec{u}$ has a following form

$$\vec{u} = \vec{u}_0 \vec{e}_x \sin(Kx + \Omega t)\,, \tag{3}$$

where u_0 is the amplitude of vector displacement; $\vec{e}_x$ is the unit vector determining the direction of axis OX in the coordinate system XYZ.

The appearance of the vector displacement for ions implies that each ion acquires the dipole moment $\vec{p} = e\vec{u}$ in of ion dipole. Consequently, we may argue that there is a presence of the electromagnetic field which may find by using of a moving equation for ion in the ion dipole

$$M \frac{d^2 \vec{u}}{dt^2} + q\vec{u} = e\vec{E}_l\,, \tag{4}$$

where $\vec{E}_l$ is the vector electric field which is induced by longitudinal ultrasonic wave; M is the mass of ion; the second term $q\vec{u}$ in left part represents as changing of quasi-elastic force which acts on ion in ion dipole, in this respect Ω_0 =

$\sqrt{\frac{q}{M}} = \omega_0 \sqrt{\frac{m}{M}}$ which is the resonance frequency or own frequency of ion determined via a resonance frequency ω_0 of electron into electron-ion dipole [10].

Using of the operation *rot* of the both part of (4) together with (1), we obtain a condition for longitudinal electromagnetic wave

$$\operatorname{curl} \vec{E}_l = 0 \,. \tag{5}$$

Now, substituting solution $\vec{u}$ from (3) in (4), we find the vector longitudinal electric field of longitudinal electromagnetic wave

$$\vec{E}_l = E_{0,l}\, \vec{e}_x \sin(Kx + \Omega t)\,, \tag{6}$$

where

$$E_{0,l} = \frac{M(\Omega_0^2 - \Omega^2)\, u_0}{e} \tag{7}$$

is the amplitude of longitudinal electric field.

On other hand, the ion dipole acquires a polarizability α, which is determined by following form

$$\vec{p} = \alpha \vec{E}_l = \frac{M(\Omega_0^2 - \Omega^2)\, \alpha \vec{u}}{e} \,. \tag{8}$$

The latter is compared with $\vec{p} = e\vec{u}$, and then, we find a polarizability α for ion dipole as it was made in the case of electron-ion one presented in [10]

$$\alpha = \frac{e^2}{M(\Omega_0^2 - \Omega^2)} \,. \tag{9}$$

Thus, the dielectric respond ε of ion medium takes a following form

$$\varepsilon = 1 + 4\pi N_0 \alpha = 1 + \frac{4\pi N_0 e^2}{M(\Omega_0^2 - \Omega^2)} \,, \tag{10}$$

where N_0 is the concentration of ions.

The dielectric respond ε of acoustic medium likes to optical one, therefore,

$$\sqrt{\varepsilon} = \frac{c}{c_l} \,, \tag{11}$$

where c is the velocity of electromagnetic wave in vacuum.

We note herein that a longitudinal electric wave with frequency Ω is propagated by velocity c_l of ultrasonic wave in the direction OX. In the presented theory, the vector electric induction $\vec{D}_l$ is determined as

$$\vec{D}_l = 4\pi \vec{P}_l + \vec{E}_l \,, \tag{12}$$

and

$$\vec{D}_l = \varepsilon \vec{E}_l \,, \tag{13}$$

where $\vec{P}_l = N_0 \vec{p}$ is the total polarization created by ion dipoles in acoustic medium.

Furthermore, the Maxwell equations for electromagnetic field in acoustic medium with a magnetic penetration $\mu = 1$ take following form

$$\operatorname{curl} \vec{E} + \frac{1}{c}\frac{d\vec{H}}{dt} = 0 \,, \tag{14}$$

$$\operatorname{curl} \vec{H} - \frac{1}{c}\frac{d\vec{D}}{dt} = 0 \,, \tag{15}$$

$$\operatorname{div} \vec{H} = 0 \,, \tag{16}$$

$$\operatorname{div} \vec{D} = 0 \tag{17}$$

with

$$\vec{D} = \varepsilon \vec{E} \,, \tag{18}$$

where $\vec{E} = \vec{E}(\vec{r}, t)$ and $\vec{H} = \vec{H}(\vec{r}, t)$ is the vectors of local electric and magnetic fields in acoustic medium; $\vec{D} = \vec{D}(\vec{r}, t)$ is the local electric induction in the coordinate-time space; $\vec{r}$ is the coordinate; t is the current time in space-time coordinate system.

As we see in above, due to action of ultrasonic wave on the solid there is changed a polarization of ion dipole by creation electric field $\vec{E}_l$ and electric induction $\vec{D}_l$. Therefore, we search a solution of Maxwell equations by introducing the vector electric field by following form

$$\vec{E} = \vec{E}_t + \vec{E}_l - \operatorname{grad} \phi \tag{19}$$

and

$$\vec{H} = \operatorname{curl} \vec{A} \,, \tag{20}$$

where

$$\vec{E}_t = -\frac{d\vec{A}}{cdt} \,, \tag{21}$$

where ϕ and $\vec{A}$ are, respectively, the scalar and vector potential of electromagnetic wave.

As result, the solution of Maxwell equations leads to following expression

$$\operatorname{grad} \phi = \vec{E}_l \,. \tag{22}$$

In turn, using of (6) we find a scalar potential

$$\phi = \phi_0 \cos(Kx + \Omega t)\,, \tag{23}$$

where $\phi_0 = -\frac{E_{0,l}}{K}$.

As we see the gradient of scalar potential $\operatorname{grad} \phi$ of electromagnetic wave neutralizes the longitudinal electric field $\vec{E}_l$.

After simple calculation, we obtain a following equations for vector potential $\vec{A}$ of transverse electromagnetic field

$$\nabla^2 \vec{A} - \frac{\varepsilon}{c^2}\frac{d^2\vec{A}}{dt^2} = 0 \tag{24}$$

with condition of plane transverse wave

$$\operatorname{div} \vec{A} = 0 \,. \tag{25}$$

The solution of (24) and (25) may present by plane transverse wave with frequency Ω which is moved by velocity c_l along of direction of unit vector $\vec{s}$

$$\vec{A} = \vec{A}_0 \sin\,(K\vec{s}\vec{r} + \Omega t) \tag{26}$$

and

$$\vec{A} \cdot \vec{s} = 0 \,, \tag{27}$$

where $K = \frac{\Omega\sqrt{\varepsilon}}{c}$ is the wave number of transverse electromagnetic wave; $\vec{s}$ is the unit vector in direction of wave normal; $\vec{A}_0$ is the vector amplitude of vector potential. In turn, the vector electric transverse wave $\vec{E}_t$ takes a following form

$$\vec{E}_t = \vec{E}_0 \cos(K\vec{s}\vec{r} + \Omega t)\,, \tag{28}$$

where the vector amplitude $\vec{E}_0$ of vector electric wave equals to

$$\vec{E}_0 = -\frac{\Omega\vec{A}_0}{c}\,.$$

Consequently, we found a transverse electromagnetic radiation which is induced by longitudinal ultrasonic wave. To find the vector amplitude $\vec{E}_0$, we using of the law conservation energy. In turn, the energy W_a of ultrasonic wave is transformed by energy W_t of transverse electromagnetic radiation, namely, there is a condition $W_a = W_t$ because there is absence the longitudinal electric field $\vec{E}_l$ which was neutralized by the gradient of scalar potential grad ϕ of electromagnetic wave as it was shown in above

$$W_a = \frac{M}{2}\left[\left(\frac{d\vec{u}}{dt}\right)^2 + \frac{1}{c_l^2}\left(\frac{d\vec{u}}{dx}\right)^2\right] = M\,\Omega^2 u_0^2 \cos^2(Kx + \Omega t)\,, \tag{29}$$

$$W_t = \frac{\varepsilon}{4\pi}\,E_0^2 \cos^2(K\vec{s}\vec{r} + \Omega t)\,. \tag{30}$$

At comparing of (29) and (30), we may argue that vector of wave normal $\vec{s}$ is directed along of axis OX or $\vec{s} = \vec{e}_x$, and then, we arrive to finally form of

$$\vec{E}_t = \vec{E}_0 \cos(Kx + \Omega t) \tag{31}$$

with condition

$$\frac{\varepsilon}{4\pi}\,E_0^2 = M\Omega^2 u_0^2\,. \tag{32}$$

Obviously, the law conservation energy plays an important role for determination of the transverse traveling plane wave.

3 Diffraction of light

First step, we consider an incident optical light into solid which is directed along of axis OZ in the coordinate space XYZ with electric vector $\vec{E}_e$

$$\vec{E}_e = \vec{E}_{0,e} \cos(kz + \omega t)\,, \tag{33}$$

where $k = \frac{\omega\sqrt{\varepsilon_0}}{c}$ is the wave number; ω is the frequency of light; ε_0 is the dielectric respond of optical medium created by electron dipoles [10]

$$\frac{\varepsilon_0 - 1}{\varepsilon_0 + 2} = \frac{4\pi N_0 e^2}{3m(\omega_0^2 - \omega^2)}\,, \tag{34}$$

where ω_0 is the own frequency of electron in electron-ion dipole; m is the mass of electron.

The interaction of ultrasonic waves with incident optical light in a crystal involves the relation between intensity of out coming light from solid and the strain created by ultrasonic wave.

Consequently, the superposition vector electric $\vec{E}_s$ field in acoustic-optical medium is determined by sum of vectors of electric transverse $\vec{E}_t$ and optical $\vec{E}_e$ waves

$$\vec{E}_s = \vec{E}_0 \cos(Kx + \Omega t) + \vec{E}_{0,e} \cos(kz + \omega t)\,. \tag{35}$$

The average Poynting vector of superposition field $\langle\vec{S}\rangle$ in acoustic-optical medium is expressed via the average Poynting vectors of $\langle\vec{S}_e\rangle$ and $\langle\vec{S}_t\rangle$ corresponding to the optical and the transverse electromagnetic waves

$$\langle\vec{S}\rangle = \frac{c}{\sqrt{\varepsilon_0}}\,w_e\vec{e}_z + \frac{c}{\sqrt{\varepsilon}}\,w_t\vec{e}_x\,, \tag{36}$$

where w_e and w_t are, respectively, the average density energies of the optical and the transverse electromagnetic waves

$$w_e = \frac{\varepsilon_0 E_{0,e}^2}{4\pi} \cdot \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \cos^2(kz + \omega t)\,dt = \frac{\varepsilon_0 E_{0,e}^2}{8\pi} \tag{37}$$

and

$$w_t = \frac{\varepsilon E_0^2}{4\pi} \cdot \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \cos^2(Kx + \Omega t)\,dt = \frac{M\Omega^2 u_0^2}{2} \tag{38}$$

by using of condition (32).

Thus, the average Poynting vector of superposition field $\langle\vec{S}\rangle$ is presented via intensities of the optical I_e and the transverse electromagnetic wave I_t

$$\langle\vec{S}\rangle = I_e\vec{e}_z + I_t\vec{e}_x\,, \tag{39}$$

where

$$I_e = \frac{E_{0,e}^2\,c\,\sqrt{\varepsilon_0}}{8\pi} \tag{40}$$

and

$$I_t = \frac{M\,\Omega^2 u_0^2 c_l}{2}\,. \tag{41}$$

This result shows that the intensity of transverse electromagnetic wave I_t represents as amplitude of acoustic field.

Obviously, we may rewrite down (39) by complex form within theory function of the complex variables

$$\langle\vec{S}\rangle = I_e + iI_t = \sqrt{I_e^2 + I_t^2}\,\exp(i\theta)\,, \tag{42}$$

where θ is the angle propagation of observation light in the coordinate system XYZ in regard to OZ

$$\theta = \operatorname{arcctg}\left(\frac{I_e}{I_t}\right), \tag{43}$$

which is chosen by the condition $0 \leqslant \operatorname{arcctg}\left(\frac{I_e}{I_t}\right) \leqslant \pi$.

Using of identity

$$\exp(iz\cos\psi) = \sum_{m=-\infty}^{m=\infty} J_m(z)\, i^m \exp(im\psi)\,, \qquad (44)$$

where $\psi = \arccos\theta$.

The average Poynting vector of superposition field $\langle\vec{S}\rangle$ is explicated on the spectrum of number m light sources with intensity I_m

$$\langle\vec{S}\rangle = \sum_{m=-\infty}^{m=\infty} I_m\,, \qquad (45)$$

where

$$I_m = \sqrt{I_t^2 + I_e^2}\, J_m\left(\mathrm{arcctg}\left(\frac{I_e}{I_t}\right)\right) i^m \exp(im\psi)\,, \qquad (46)$$

but $J_m(z)$ is the Bessel function of m order.

Thus, there is a diffraction of light by action of ultrasonic wave. In this respect, the central diffraction maximum point corresponds to $m = 0$ with intensity $I_{m=0}$

$$I_{m=0} = \sqrt{I_t^2 + I_e^2}\, J_0\left(\mathrm{arcctg}\left(\frac{I_e}{I_t}\right)\right). \qquad (47)$$

In the case, when arcctg $\frac{I_e}{I_t} = 2.4$ (at $z = 2.4$, the Bessel function equals zero $J_0(z) = 0$, that implies $I_{m=0} = 0$. In this respect, there is observed a vanishing of central diffraction maximum at certainly value of amplitude I_t acoustic field.

The main result of above-mentioned experiment [4,9] is that the intensity of the first positive diffraction maximum $I_{m=1}$ is not equal to the intensity of the first negative minimum $I_{m=-1}$. Due to presented herein theory, the intensity of the first positive diffraction maximum is

$$I_{m=1} = i\sqrt{I_t^2 + I_e^2}\, J_1\left(\mathrm{arcctg}\left(\frac{I_e}{I_t}\right)\right) \exp(\psi)\,, \qquad (48)$$

but the intensity of the first negative diffraction maximum is

$$I_{m=-1} = -i\sqrt{I_t^2 + I_e^2}\, J_{-1}\left(\mathrm{arcctg}\left(\frac{I_e}{I_t}\right)\right) \exp(-\psi)\,. \qquad (49)$$

It is easy to show that $I_{m=1} \neq I_{m=-1}$. Indeed, at comparing $I_{m=1}$ and $I_{m=-1}$, we have

$$J_{-1} = -J_1$$

and

$$\exp(\psi) \neq \exp(-\psi)\,,$$

which is fulfilled always because the there is a condition for observation angle $\theta \neq \frac{\pi}{2}$. Consequently, we proved that evidence $I_{m=1} \neq I_{m=-1}$ confirms the experimental data.

Thus, as we have been seen the longitudinal ultrasonic wave induces the traveling transverse electromagnetic field which together with optical light provides an appearance diffraction of light.

Submitted on February 04, 2010 / Accepted on February 09, 2010

References

1. Brillouin L. *Ann. de Physique*, 1921, v. 17, 102.
2. Debay P. and Sears F.W. Scattering of light by supersonic waves. *Proc. Nat. Acad. Sci. Wash.*, 1932, v. 18, 409.
3. Schaefer C. and Bergmann L. *Naturuwiss*, 1935, v. 23, 799.
4. Seeger A. and Buck O. *Z. Natureforsch*, 1960, v. 15a, 1057.
5. Raman C.V., Nath N.S.N. *Proc. Indian. Acad. Sci.*, 1935, v. 2, 406.
6. Estermann R. and Wannier G. *Helv. Phys. Acta*, 1936, v. 9, 520; Pockels F. Lehrduch der Kristalloptic. Leipzig, 1906.
7. Fues E. and Ludloff H. *Sitzungsber. d. press. Akad. Math. phys. Kl.*, 1935, v. 14, 222.
8. Mueller M. *Physical Review*, 1937, v. 52, 223.
9. Melngailis J., Maradudin A.A., and Seeger A. *Physical Review*, 1963, v. 131, 1972.
10. Born M. and Wolf E. Principles of optics. Pergamon press, Oxford, 1964.
11. Landau L.D. and Lifshiz E.M. Theory of elasticity. (Course of Theoretical Physics, v. 11.) Moscow, 2003.