The Dirac Electron in the Planck Vacuum Theory

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The nature of the Dirac electron (a massive point charge) and its negative-energy solutions are examined heuristically from the point of view of the Planck vacuum (PV) theory [1, 2]. At the end of the paper the concept of the vacuum state as previously viewed by the PV theory is expanded to include the massive-particle quantum vacuum [3, 4].

1 The Dirac equation

When a free, massless, bare charge travels in a straight line at a uniform velocity v, its bare Coulomb field e_*/r^2 perturbs (polarizes) the PV. If there were no PV, the bare field would propagate as a frozen pattern with the same velocity. However, the PV responds to the perturbation by producing magnetic and Faraday fields [1, 5] that interact with the bare charge in a iterative fashion that leads to the well-known relativistic electric and magnetic fields [6] that are ascribed to the charge as a single entity. The corresponding force perturbing the PV is e_*^2/r^2, where one of the charges e_* in the product e_*^2 belongs to the free charge and the other to the individual Planck particles making up the degenerate negative-energy PV. By contrast, the force between two free elementary charges observed in the laboratory is $e^2/r^2\,(=\alpha e_*^2/r^2)$, where e is the observed electronic charge and α is the fine structure constant.

In the Dirac electron, where the bare charge has a mass m, the response of the PV to the electron's uniform motion is much more complicated as now the massive charge perturbs the PV with *two* forces, the polarization force e_*^2/r^2 and the attractive curvature force mc^2/r [1]. The radius at which the magnitudes of these two forces are equal

$$\frac{mc^2}{r} = \frac{e_*^2}{r^2} \qquad \text{at } r = r_c \tag{1}$$

is the electron's Compton radius $r_c\,(= e_*^2/mc^2)$. The string of Compton relations [4]

$$r_c mc^2 = r_* m_* c^2 = e_*^2 = c\hbar \tag{2}$$

tie the electron ($r_c mc^2$) to the Planck particles ($r_* m_* c^2$) within the PV, where r_* and m_* are the Compton radius and mass of those particles. The charges in the product e_*^2 of (2) are assumed to be massless point charges.

The Dirac equation for the electron is [3, 7]

$$\left(c\,\widehat{\alpha}\,\widehat{p} + \beta mc^2\right)\psi = E\psi\,, \tag{3}$$

where the momentum operator and energy are given by

$$\widehat{p} = \hbar\,\nabla/i \qquad \text{and} \qquad E = \pm(m^2c^4 + c^2p^2)^{1/2} \tag{4}$$

and where $\widehat{\alpha}$ and β are defined in the references. The relativistic momentum is $p\,(= mv/\sqrt{1 - v^2/c^2})$.

As expressed in (3), the physics of the Dirac equation is difficult to understand. Using (2) to replace $\hbar$ in the momentum operator and inserting the result into (3), reduces (3) to

$$\left(\widehat{\alpha}\,\frac{e_*^2\nabla}{imc^2} + \beta\right)\psi = \frac{E}{mc^2}\,\psi\,, \tag{5}$$

where the charge product e_*^2 suggests the connection in (2) between the free electron and the PV. It is significant that neither the fine structure constant nor the observed electronic charge appear in the Dirac equation, for it further suggests that the bare charge of the electron interacts *directly* with the bare charges on the individual Planck particles within the PV, without the fine-structure-constant screening that leads to the Coulomb force e^2/r^2 in the first paragraph. Equation (5) leads immediately to the equation

$$\left(\widehat{\alpha}\,\frac{r_c\nabla}{i} + \beta\right)\psi = \frac{E}{mc^2}\,\psi \tag{6}$$

with its Del operator

$$r_c\,\nabla = \sum_{n=1}^{3} \widehat{x}_n\,\frac{\partial}{\partial x_n/r_c} \tag{7}$$

being scaled to the electron's Compton radius.

Through (2), (5), and (6), then, the connection of the Dirac equation to the PV is self evident — the Dirac equation represents the response of the PV to the two perturbations from the uniformly propagating electron. As an extension of this thinking, the quantum-field and Feynman-propagator formalisms of quantum electrodynamics are also associated with the PV response.

2 The Klein paradox

The "hole" theory of Dirac [7] that leads to the Dirac vacuum will be presented here along with the Klein paradox as the two are intimately related. Consider an electrostatic potential of the form

$$e\phi = \begin{cases} 0 & \text{for } z < 0 \text{ (Region I)} \\ V_0 & \text{for } z > 0 \text{ (Region II)} \end{cases} \tag{8}$$

acting on the negative-energy vacuum state (corresponding to the negative E in (4)) with a free electron from $z < 0$ being scattered off the potential step at $z = 0$, beyond which $V_0 > E + mc^2 > 2mc^2$. This scattering problem leads to the Klein paradox that is reviewed below.

The scattering problem is readily solved [8, pp.127–131]. For the free electron in Region I, $E^2 = m^2c^4 + c^2p^2$; and for Region II, $(E - V_0)^2 = m^2c^4 + (cp')^2$, where E is the total electron energy in Region I, and p and p' are the z-directed electron momenta in Regions I and II respectively.

The Dirac equation (with motion in the z-direction) for $z < 0$ is

$$\left(c\,\alpha_z\,\widehat{p}_z + \beta mc^2\right)\psi = E\psi \tag{9}$$

and for $z > 0$ is

$$\left(c\,\alpha_z\,\widehat{p}_z + \beta mc^2\right)\psi = (E - V_0)\psi\,. \tag{10}$$

The resulting incident and reflected electron wavefunctions are

$$\psi_{\mathrm{I}} = A \begin{pmatrix} 1 \\ 0 \\ \frac{cp}{E+mc^2} \\ 0 \end{pmatrix} e^{ipz/\hbar} \tag{11}$$

and

$$\psi_{\mathrm{R}} = B \begin{pmatrix} 1 \\ 0 \\ \frac{-cp}{E+mc^2} \\ 0 \end{pmatrix} e^{-ipz/\hbar} \tag{12}$$

respectively, where $cp = \sqrt{E^2 - m^2c^4}$. The transmitted wave turns out to be

$$\psi_{\mathrm{T}} = D \begin{pmatrix} 1 \\ 0 \\ \frac{cp'}{V_0-E-mc^2} \\ 0 \end{pmatrix} e^{ip'z/\hbar}\,, \tag{13}$$

where $cp' = \sqrt{(V_0 - E)^2 - m^2c^4}$. It should be noted that the imaginary exponent in (13) represents a propagating wave which results from $V_0 > E + mc^2$; in particular, the particle motion in Region II is not damped as expected classically and quantum-mechanically when $V_0 < E + mc^2$.

The constants A, B, and D are determined from the continuity condition

$$\psi_{\mathrm{I}} + \psi_{\mathrm{R}} = \psi_{\mathrm{T}} \tag{14}$$

at $z = 0$ and lead to the parameter

$$\Gamma \equiv \left(\frac{V_0 - E + mc^2}{V_0 - E - mc^2}\,\frac{E + mc^2}{E - mc^2}\right)^{1/2} > 1\,. \tag{15}$$

The particle currents are calculated from the expectation values of

$$j_z(x) = c\,\psi^{\dagger}(x)\,\alpha_z\psi(x) \tag{16}$$

and yield j_I, j_R, and j_T for the incident, reflected, and transmitted currents respectively. The resulting normalized reflection and transmission currents become

$$\frac{j_R}{j_I} = -\left(\frac{1+\Gamma}{1-\Gamma}\right)^2, \tag{17}$$

$$\frac{j_T}{j_I} = -\frac{4\Gamma}{(1-\Gamma)^2}\,. \tag{18}$$

Since Γ is positive, (17) gives

$$\left|\frac{j_R}{j_I}\right| - 1 > 0 \tag{19}$$

for the excess reflected current; i.e., the reflected current is *greater* than the incident current! This seemingly irrational result is known as the Klein paradox.

The most natural and Occam's-razor-consistent conclusion to be drawn from (19), however, is that the excess electron (or electrons) in the reflected current is (are) coming from the right ($z > 0$) of the step at $z = 0$ and proceeding in the negative z direction away from the step. Furthermore, the minus sign on the normalized transmission current in (18) implies that no electrons are entering Region II — the total electron current (reflected plus "transmitted") travels in the negative z direction away from the step. Then, given the experimental fact of electron-positron pair creation, it is reasonable to conclude that the incident free electron creates such pairs when it "collides" with the stressed portion of the vacuum ($z > 0$), the positrons (Dirac "holes") proceeding to the right *into the vacuum* after the collision [8, fig. 5.6]. That is, positrons (like neutrinos [9]) travel within the vacuum, not free space!

The evidence of the created positrons is felt in free space as the positron fields, analogous to the zero-point fields whose source is the zero-point agitation of the Planck particles within the PV. The curving of the positrons in a laboratory magnetic field is due to that field permeating the PV and acting on the "holes" within. (In the PV-theory view of things, the free electron is not seen as propagating *within* the vacuum state — only the electron force-fields (e_*^2/r^2 and mc^2/r) permeate that vacuum; consequently, the electron is not colliding with the negative-energy Planck particles making up the vacuum.)

3 Summary and comments

The total r-directed perturbing force the electron exerts on the PV is

$$F_e = \frac{e_*^2}{r^2} - \frac{mc^2}{r} = \frac{e_*^2}{r^2}\left(1 - \frac{r}{r_c}\right), \tag{20}$$

where the force vanishes at the electron's Compton radius r_c. For $r > r_c$ the force compresses the vacuum and for $r < r_c$ the vacuum is forced to expand. Ignoring the second term in (20) for convenience and concentrating on the region $r < r_c$, the lessons from the preceding section can be applied to the internal electron dynamics.

Recalling that the bare charge of the free electron interacts directly with the individual Planck particles in the PV, the electron-Planck-particle potential (e_*^2/r) in the inequality $e_*^2/r > E + mc^2$ leads to

$$r < \frac{e_*^2}{E + mc^2} = \frac{r_c}{1 + E/mc^2} < \frac{r_c}{2}\,, \tag{21}$$

where the positive and negative energy levels in (4) now *overlap*, and where any small perturbation to the PV can result in an electron-positron pair being created (the electron traveling in free space and the positron in the PV). The smaller the radius r, the more sensitive the PV is to such disruption.

The electron mass results from the massless bare charge being driven by ultra-high-frequency photons of the zero-point electromagnetic vacuum [4, 10]; so the bear charge of the electron exhibits a small random motion about its center-of-motion. The resulting massive-charge collisions with the sensitized PV produce a cloud of electron-positron pairs around that charge. The massive free charge then exhibits an exchange type of scattering [3, p.323] with some of the electrons in the pairs, increasing the free electron's apparent size in the process.

In the current PV theory it is assumed that the total quantum vacuum, which consists of the electromagnetic vacuum and the massive-particle vacuum [3, 4], exists in free space as virtual particles. However, the simple picture presented in the previous paragraphs and in Section 2 concerning pair creation modifies that view significantly. It is the massive-particle quantum vacuum that overlaps the positive energy levels of the free-space electron in the previous discussion. Thus, as the appearance of this latter vacuum in free space requires a sufficiently stressed vacuum state (in the above region $r < r_c/2$ e.g.), it is more reasonable to assume that the massive-particle component of the quantum vacuum does not exist in free space except under stressful conditions.

Consequently, it seems reasonable to conclude that the PV is a composite state patterned, perhaps, after the hierarchy of Compton relations

$$r_e m_e c^2 = r_p m_p c^2 = \cdots = r_* m_* c^2 = e_*^2 , \qquad (22)$$

where the products $r_e m_e$, $r_p m_p$, and $r_* m_*$ refer to the electron, proton, and Planck particle respectively. The dots between the proton and Planck-particle products represent any number of heaver intermediate-particle states. The components of this expanded vacuum state correspond to the sub-vacua associated with these particles; e.g., the electron-positron Dirac vacuum ($r_e m_e c^2$) in the electron case. If these assumptions are correct, then the negative-energy states in (4) no longer end in a negative-energy infinity — as the energy decreases it passes through the succession of sub-vacuum states, finally ending its increasingly negative-energy descent at the Planck-particle stage $r_* m_* c^2$. In summary, the PV model now includes the massive-particle quantum vacuum which corresponds to the collection of sub-vacuum states in (22).

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