How Black Holes Violate the Conservation of Energy

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Black holes produce more energy than they consume thereby violating the conservation of energy and acting as perpetual motion machines.

1 Introduction

According to Stephen Hawking and Leonard Mlodinow [1]: "Because there is a law such as gravity, the Universe can and will create itself from nothing." Such views of gravity are usually attributed as being rooted in Einstein's general-relativistic space-time.

However, the field equations Einstein [2] used to describe the general-relativistic space-time are founded on the conservation of momentum and energy. How can a space-time derived based on the conservation of momentum and energy provide an ex nihilo source of energy sufficient to create a universe?

The answer is found in Karl Schwarzschild's solution [3] to the field equations, usually called the Schwarzschild metric. The Schwarzschild metric describes a gravitational field outside a spherical non-rotating mass. When the mass is compacted within its Schwarzschild radius it is commonly referred to as a black hole.

Herein the terms of the Schwarzschild metric are rearranged to display limits in the Schwarzschild metric that necessarily result from the conservation of momentum and energy. Then is shown how black holes violate the limits, acting as perpetual motion machines that produce more energy than they consume.

2 Expressing the Schwarzschild metric using velocities

In this section, the Schwarzschild metric is rearranged so as to be expressed using velocities measured with reference coordinates. This rearrangement, which appears as equation (8) at the end of this section, will make very clear the limits imposed within the Schwarzschild metric by the conservation of momentum and energy.

Einstein [4] originally expressed the principles of special relativity using velocities measured with reference coordinates. However, Einstein [2, Equations 47] expressed the field equations in more abstract terms, using tensors. Einstein was careful to show that the field equations, nevertheless, correspond to the conservation of momentum and energy [2, Equations 47a] and thus have a nexus to physical reality.

The Schwarzschild metric, as a solution to the field equations, also corresponds to the conservation of momentum and energy. Arrangement of the Schwarzschild metric as in (8) allows for an intuitive comprehension of exactly how momentum and energy is conserved.

For a compact mass M with a Schwarzschild radius R, the Schwarzschild metric is often expressed using reference space coordinates (r, θ, ϕ), coordinate time t and local time τ (often referred to as proper time τ), as

$$c^2 d\tau^2 = c^2\left(1-\frac{R}{r}\right)dt^2 - \frac{dr^2}{(1-R/r)} - r^2 d\theta^2 - (r sin\theta)^2 d\phi^2. \quad (1)$$

The Schwarzschild metric as shown in (1) can be rearranged to form (8), as shown below. To obtain (8) from (1), begin by multiplying both sides of (1) by $\left(\frac{1}{dt}\right)^2$ yielding

$$\begin{aligned} c^2\left(\frac{d\tau}{dt}\right)^2 &= c^2\left(1-\frac{R}{r}\right)\left(\frac{dt}{dt}\right)^2 - \frac{1}{1-R/r}\left(\frac{dr}{dt}\right)^2 \\ &- r^2\left(\frac{d\theta}{dt}\right)^2 - (r sin\theta)^2\left(\frac{d\phi}{dt}\right)^2, \end{aligned} \quad (2)$$

which allows motion in all dimensions to be measured with respect to the reference coordinates (r, θ, ϕ, t). The terms of (2) can be rearranged as

$$\begin{aligned} c^2 &= c^2\left(\frac{d\tau}{dt}\right)^2 + c^2\frac{R}{r} + \frac{1}{1-R/r}\left(\frac{dr}{dt}\right)^2 \\ &+ r^2\left(\frac{d\theta}{dt}\right)^2 + (r sin\theta)^2\left(\frac{d\phi}{dt}\right)^2. \end{aligned} \quad (3)$$

The terms in (3) can be grouped by defining three different velocities. A velocity through the three dimensions of curved space can be defined as

$$v_S = \sqrt{\frac{1}{1-R/r}\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 + (r sin\theta)^2\left(\frac{d\phi}{dt}\right)^2}. \quad (4)$$

A velocity of local time through a time dimension can be defined as

$$v_\tau = c\frac{d\tau}{dt}. \quad (5)$$

A gravitational velocity can be defined as

$$v_G = c\sqrt{\frac{R}{r}}. \quad (6)$$

Using the definitions in (4), (5) and (6), (3) reduces to

$$c^2 = v_\tau^2 + v_G^2 + v_S^2. \quad (7)$$

Equation (7) can be expressed using orthogonal vectors $\vec{v}_\tau$, $\vec{v}_G$ and $\vec{v}_S$ where $v_\tau = |\vec{v}_\tau|$, $v_G = |\vec{v}_G|$ and $v_S = |\vec{v}_S|$, and where

$$c = \left|\vec{v}_\tau + \vec{v}_G + \vec{v}_S\right|. \quad (8)$$

Equation (8) is mathematically equivalent to (1) and expresses the Schwarzschild metric as a relationship of vector velocities. The conservation of momentum and energy, as expressed in the Schwarzschild metric, requires that the magnitude of the sum of the velocities is always equal to the constant c. Before exploring the full implication of this relationship, the next section confirms that (8) conforms with what is predicted by special relativity.

3 Equation (8) and special relativity

In the previous section, the Schwarzschild metric in (1) has been rearranged as (8) to provide a more concrete picture of the relationships necessary for conservation of momentum and energy.

Here is confirmed (8) is in accord with the case of special relativity for unaccelerated motion.

When there is no acceleration and therefore no gravity field, $R = 0$ and thus according to (6), $v_G = 0$ so that (8) reduces to

$$c = \left|\vec{v}_\tau + \vec{v}_S\right|. \tag{9}$$

When $R = 0$,

$$v_{S,R=0} = \sqrt{\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 + (r sin\theta)^2\left(\frac{d\phi}{dt}\right)^2}, \tag{10}$$

which expressed in Cartesian coordinates is the familiar form of velocity used in special relativity,

$$v_{S,R=0} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}. \tag{11}$$

Equation (9) accurately reproduces the relationship of velocity and time known from special relativity. As velocity v_S in the space dimensions increases, there is a corresponding decline in the velocity v_τ in the orthogonal time dimension. When velocity in the time dimension reaches its minimum value (i.e., $v_\tau = 0$) this indicates a maximum value (i.e., $v_S = c$) in the space dimensions has been reached.

Equation (9) can be rearranged to confirm it portrays exactly the relationship between coordinate time and local time that is known to occur in the case of special relativity. Specifically, from the relationship of the orthogonal vectors $\vec{v}_\tau$, and $\vec{v}_S$ in (9), it must be true that

$$c^2 = v_\tau^2 + v_S^2. \tag{12}$$

and thus from (5)

$$c^2 = c^2\left(\frac{d\tau}{dt}\right)^2 + v_S^2, \tag{13}$$

and therefore

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{v_S^2}{c^2}}, \tag{14}$$

which is a form of the well known Laplace factor indicating the relationship between local time and coordinate time for special relativity.

4 Equation (8) and limits imposed by the conservation of momentum and energy

The arrangement of the Schwarzschild metric in (8) allows for a more concrete explanation of the limitations inherent in the Schwarzschild metric that necessarily result from the conservation of momentum and energy.

The vector sum of $\vec{v}_\tau$, $\vec{v}_G$ and $\vec{v}_S$ establishes a maximum value of c for each individual vector velocity.

When $\vec{v}_\tau = 0$ and $\vec{v}_S = 0$, $\vec{v}_G$ reaches its maximum value of c. Gravitational velocity $\vec{v}_G$ cannot exceed its maximum value of c without violating (8).

According to the definition of v_G in (6), when $v_G = c$, then $r = R$. When $r < R$, then $v_G > c$; therefore, according to (8), $r < R$ never occurs. As shown by Weller [5], matter from space can never actually reach $r = R$, but if it could, it would go no farther. At $r = R$ and $v_G = c$, all motion through space stops ($\vec{v}_S = 0$) and local time stops ($\vec{v}_\tau = 0$, so $d\tau/dt = 0$). Without motion in time or space, matter cannot pass through radial location $r = R$.

This section has shown that because of the conservation of momentum and energy — as expressed by the Schwarzschild metric arranged as in (8) — matter from space cannot cross the Schwarzschild radius R to get to a location where $r < R$.

The following sections consider conservation of energy equivalence in the Schwarzschild metric and the result when energy conservation is not followed.

5 Apportionment of energy equivalence

Einstein [6] pioneered apportioning energy differently based on reference frames, using such an apportionment in his initial calculations deriving the value for the energy equivalence of a mass (i.e., $E = mc^2$).

This notion of apportionment of energy equivalence is a helpful tool in understanding the implications of violating the conservation of energy and momentum in the Schwarzschild metric. When considering apportionment of energy equivalence in the Schwarzschild metric, it is helpful to keep in mind how Einstein makes a distinction between "matter" and a "matterless" gravitational field defined by the field equations or by the Schwarzschild metric. According to the Einstein [2, p. 143], everything but the gravitation field is denoted as "matter". Therefore, matter when added to the matterless field includes not only matter in the ordinary sense, but the electromagnetic field as well.

How the Schwarzschild metric apportions energy equivalence can be understood from

$$c^2 = v_\tau^2 + c^2\frac{R}{r} + v_S^2. \tag{15}$$

which is (7) modified so as to replace υ_G with its equivalent given in (6). Equation (15) is mathematically equivalent to (1), just rearranged to aid in the explanation of the apportionment of energy equivalence.

Equation (15) can be put into perhaps more familiar terms by introducing a particle of mass m into the gravitation field. The energy equivalence mc^2 of the mass m is apportioned according to (15) as

$$mc^2 = m\upsilon_\tau^2 + mc^2\frac{R}{r} + m\upsilon_S^2. \qquad (16)$$

In order to provide insight into the nature of the gravitational energy component c^2R/r in (15) —which appears as mc^2R/r in (16) — the next section discusses briefly how this term came to reside in the Schwarzschild metric.

6 Schwarzschild's description of gravity

One of the issues Schwarzschild [3, see §4] faced when deriving the Schwarzschild metric was how to describe the effects of gravity. He chose to do so using a positive integration constant that depends on the value of the mass at the origin. As a result the Newtonian gravitational constant G appears in the Schwarzschild metric. In (1) the gravitational constant G appears as part of the definition of the Schwarzschild radius R. In both Newtonian physics and the Schwarzschild metric, the Schwarzschild radius (R) — the location where Newtonian escape velocity (i.e., υ_G) is equal to c — is defined as

$$R = \frac{2GM}{c^2}. \qquad (17)$$

When the Schwarzschild metric is arranged as in (15), gravitational energy component c^2R/r increases toward infinity as radial location r decreases toward zero. This suggests the location of an unlimited energy source within the Schwarzschild metric; however, total gravitational energy is limited by the requirement that energy be conserved, as illustrated by the hypothetical described in the next section.

7 A hypothetical illustrating the conservation of energy equivalence in the Schwarzschild metric

The total energy-equivalence of a system comprised of a mass M can be defined as

$$E_M = Mc^2, \qquad (18)$$

where the energy of magnetic fields is included in M, or neglected. If a mass m is added to the system, the additional energy E added to the system as a result of the presence of mass m is also well known to be

$$E = mc^2. \qquad (19)$$

Thus if the system consisting of mass M and mass m were dissolved into radiation, the total resulting energy would be equal to

$$E_M + E = Mc^2 + mc^2. \qquad (20)$$

In order for the conservation of energy to be maintained in the system as a whole, any gravitational energy E_G or any energy from motion E_K that is added to the system as a result of the presence of mass m must be included as part of the additional energy E described in (19). Therefore, the additional energy E present in the system as a result of adding mass m can be expressed as

$$E = mc^2 = E_K + E_G + E_\tau, \qquad (21)$$

where E_τ is the portion of energy E that is not represented by gravitational energy component E_G or motion energy component E_K.

Equation (21) is the apportionment of energy equivalence shown in (16). To confirm this, in (21) set $E_G = mc^2R/r$, $E_K = m\upsilon_S^2$ and $E_\tau = m\upsilon_\tau^2$ to obtain (16).

The apportionment of energy equivalence in (16) and (21) indicates why crossing the Schwarzschild radius R violates the conservation of energy. When the particle reaches the Schwarzschild radius R — i.e., $r = R$ — the entire energy equivalence of mass m, is consumed by the gravitation component, i.e., $E_G = mc^2R/R = mc^2$. There is no energy left for mass m to travel in time (i.e., $E_\tau = 0$) or in space (i.e., $E_K = 0$). Therefore at locations $r = R$, all motion in time and space must stop, preventing mass m from ever crossing the critical radius.

If mass m were from space to cross the Schwarzschild radius R, the gravitational energy component $E_G = mc^2R/r$ would exceed the total energy equivalence $E = mc^2$ violating the conservation of energy.

If the particle were allowed to reach $r = 0$, gravitational energy component $E_G = mc^2R/r$ would approach infinity before becoming undefined.

8 How black holes act as perpetual motion machines

A perpetual motion machine is a hypothetical machine that violates the conservation of energy by producing more energy than it consumes.

According to the conservation of momentum and energy described by the Schwarzschild metric, see (8) and (15), a particle can never from space cross the Schwarzschild radius R of a compact mass M.

When a black hole is formed from a compacting mass M, the last particle on the surface of the mass that reaches and crosses the Schwarzschild radius R violates (8). Every particle thereafter that from space crosses R violates (8).

Further, from (16), each particle of mass m that reaches a radial location $r < R$, produces an amount of gravitational energy ($E_G = mc^2R/r$) that is greater than its total energy equivalence mc^2, as can only happen in a perpetual motion machine. When a particle is allowed to approach and reach $r = 0$, the ultimate perpetual motion machine is created which from the finite energy equivalence mc^2 of the particle produces an unlimited amount of gravitational energy as the particle approaches $r = 0$.

9 Concluding Remarks

Describing the effects of gravity using a gravitational constant and violating the conservation of momentum and energy described by the Schwarzschild metric can hypothetically result in black holes that act as perpetual motion machines able to produce an unlimited amount of energy. However, the existence of such perpetual motion machines is not in accordance with the conservation of momentum and energy as expressed in Einstein's general-relativistic space-time.

Special mathematical calculations, including use of specially selected coordinates, have been used to explain how a particle can cross the Schwarzschild radius allowing black holes to form. Critiquing these mathematical calculations is beyond the scope of this short paper. The author has directly addressed some of this subject matter in a companion paper [5].

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