Superfluidity Component of Solid ^{4}He and *Sound Particles* with Spin 1

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We present a new model for solid which is based on such a concept as the fluctuation motion of "solid particles" or "solid points". The fluctuation motion of "solid particles" in solid ^{4}He represents a longitudinal elastic wave which is in turn quantized by neutral longitudinal Bose *sound particles* with spin 1 with the rest mass m. Thus, first we remove a concept of "lattice" for solid by presentation of new model of one as a vibration of *sound particles* by natural frequency Ω_l. In this respect, we first postulate that the superfluid component of a solid ^{4}He is determined by means of *sound particles* with spin 1 in the condensate.

1 Introduction

The quantum solid is remarkable object which reveal macroscopic quantum phenomena, such as superfluidity and Bose-Einstein condensation (BEC) of solid ^{4}He [1] which were reported by many authors [2, 3].

The original theory proposed by Einstein in 1907 was of great historical relevance [4]. In the Einstein model, each atom oscillates relatively to its neighbors in the lattice which execute harmonic motions around fixed positions, the knots of the lattice. He treated the thermal property of the vibration of a lattice of N atoms as a $3N$ harmonic independent oscillator by identical own frequency Ω_0 which was quantized by application of the prescription developed by Plank in connection with the theory of Black Body radiation. The Einstein model could obtain the Dulong and Petit prediction at high temperature but could not reproduce an adequate representation of the the lattice at low temperatures. In 1912, Debye proposed to consider the model of the solid [5], by suggestion that the frequencies of the $3N$ harmonic independent oscillators are not equal as it was suggested by the Einstein model. In addition to his suggestion, the acoustic spectrum of solid may be treated as if the solid represented a homogeneous medium, except that the total number of independent elastic waves is cut off at $3N$, to agree with the number of degrees of freedom of N atoms. In this respect, Debye stated that one longitudinal and two transverse waves are excited in solid. These velocities of sound cannot be observed in a solid at frequencies above the cut-off frequency. Also, he suggested that phonon is a spinless. Thus, the Debye model correctly showed that the heat capacity is proportional to the T^3 law at low temperatures. At high temperatures, he obtained the Dulong-Petit prediction compatible to experimental results.

The other model of solid was presented by the authors of this letter in [6] where the solid was considered as continuum elastic medium consisting of neutral Fermi-atoms, fixed in the knots of lattice. In this case, we predicted that the lattice represents as the Bose-gas of Sound-Particles with finite masses m_l and m_t, corresponding to a longitudinal and a transverse elastic field. On the other hand, the lattice was considered as a new substance of matter consisting of sound particles, which excite the one longitudinal and one transverse elastic waves (this approach is differ from Debye one). These waves act on the Fermi-atoms which are stimulating a vibrations with the natural frequencies Ω_l and Ω_t. In this context, we introduced a new principle of elastic wave-particle duality, which allows us to build the lattice model. The given model leads to the same results as presented by Debye's theory.

However, we consider the model of solid by new way by introducing of such a concept as the fluctuation motion of "solid particles" or "solid points". In this respect, we remove a concept as a lattice of solid or an atoms, fixed in the knots of lattice because we deal with the "solid particle" which exist in any point of the solid. This "solid particle" is a similar to the "fluid particle" on the basis of hydrodynamics [7] (where "fluid particle" is determined as a very small volume V_0, in regard to the volume V of the liquid ($V_0 \ll V$), which consists of a macroscopic number of liquid atoms). The motion of "solid particle" describes the longitudinal elastic wave which in turn represents a Bose gas of neutral *sound particles* with spin 1 with finite mass m. In this letter, we present a new model of solid which describes a vibration of *sound particles* by natural frequency Ω_l. We postulate also that the superfluid component of a solid is determined by means of *sound particles* in the condensate.

2 Analysis

For beginning let us analyze quantization of a quantum liquid (or quantum gas) which consists of N Bose or Fermi atoms with the mass M confined in the volume V. Considering a quantum liquid as a continuum medium, we investigate the fluctuation motion of "fluid particles" on the basis of hydrodynamics (where "fluid particle" is determined as a very small volume V_0, in regard to the volume V of the liquid ($V_0 \ll V$), which consists of a macroscopic number of liquid atoms).

In accordance with the hydrodynamics laws, the mass density ρ and pressure p for a liquid are presented as

$$\rho = \rho_0 + \rho^{'}$$

and

$$p = p_0 + p^{'},$$

where $\rho_0 = \frac{MN}{V}$ and p_0 are, respectively, the equilibrium mass density and pressure; $\rho^{'}$ and $p^{'}$ are the relative fluctuations of the mass density and pressure.

As is known, the continuity equation has the form:

$$\frac{\partial \rho^{'}}{\partial t} = -\rho_0 \operatorname{div} \vec{v}, \tag{1}$$

which may present as:

$$\rho^{'} = -\rho_0 \operatorname{div} \vec{u}, \tag{2}$$

where $\vec{v} = \frac{\partial \vec{u}}{\partial t}$ is the speed of a fluid particle; $\vec{u} = \vec{u}(\vec{r}, t)$ is the displacement vector of a fluid particle which describes a longitudinal sound wave.

On the other hand, Euler's equation in the first-order-of-smallness approximation takes the reduced form:

$$\frac{\partial \vec{v}}{\partial t} + \frac{\nabla p^{'}}{\rho_0} = 0. \tag{3}$$

Hence, we consider the fluctuation motion of fluid particles as adiabatic, deriving the following equation:

$$p^{'} = \left(\frac{\partial p}{\partial \rho_0}\right)_S \rho^{'} = c_l^2 \rho^{'}, \tag{4}$$

where S is the entropy of liquid; $c_l = \sqrt{\left(\frac{\partial p}{\partial \rho_0}\right)_S}$ is the speed of the longitudinal elastic wave.

As is known, the fluctuation motion of fluid particles represents as a potential one:

$$\operatorname{curl} \vec{v} = \operatorname{curl} \frac{\partial \vec{u}}{\partial t} = 0. \tag{5}$$

Thus, by using the above equation we may get to the wave equation for the vector of displacement $\vec{u} = \vec{u}(\vec{r}, t)$:

$$\nabla^2 \vec{u}(\vec{r}, t) - \frac{1}{c_l^2} \frac{\partial^2 \vec{u}(\vec{r}, t)}{\partial t^2} = 0, \tag{6}$$

which in turn gives a description of the longitudinal sound wave.

Now, we state that the longitudinal elastic wave consists of neutral spinless Bose *sound particles* with the non-zero rest mass m. Then, the displacement vector $u(\vec{r}, t)$ is expressed via a secondary quantization vector of the wave function of spinless Bose *sound particles* directed along the wave vector $\vec{k}$:

$$\vec{u}(\vec{r}, t) = u_l \left(\vec{\phi}(\vec{r}, t) + \vec{\phi}^{+}(\vec{r}, t)\right), \tag{7}$$

where u_l is the normalization constant which is the amplitude of oscillations; $\vec{\phi}(\vec{r}, t)$ is the second quantization vector wave functions for creation and annihilation of one longitudinal *sound particle* with the mass m whose direction $\vec{l}$ is directed towards the wave vector $\vec{k}$:

$$\vec{\phi}(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{a}_{\vec{k}}\, e^{i(\vec{k}\vec{r} - kc_l t)} \tag{8}$$

$$\vec{\phi}^{+}(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{a}^{+}_{\vec{k}}\, e^{-i(\vec{k}\vec{r} - kc_l t)} \tag{9}$$

with the condition

$$\int \vec{\phi}^{+}(\vec{r}, t)\, \vec{\phi}(\vec{r}, t)\, dV = n_0 + \sum_{\vec{k} \neq 0} \hat{a}^{+}_{\vec{k}}\, \hat{a}_{\vec{k}} = \hat{n}, \tag{10}$$

where $\vec{a}^{+}_{\vec{k}}$ and $\vec{a}_{\vec{k}}$ are, respectively, the Bose vector-operators of creation and annihilation for a free *sound particle* with the energy $\frac{\hbar^2 k^2}{2m}$, described by the vector $\vec{k}$ whose direction coincides with the direction $\vec{l}$ of a traveling longitudinal elastic wave; $\hat{n}$ is the operator of the total number of *sound particles*; $\hat{n}_0$ is the total number of *sound particles* at the condensate level with the wave vector $\vec{k} = 0$.

Thus, as is seen, the displacement vector $\vec{u}(\vec{r}, t)$ satisfies wave-equation (6) and in turn takes the form:

$$\vec{u}(\vec{r}, t) = \vec{u_0} + \frac{u_l}{\sqrt{V}} \sum_{\vec{k} \neq 0} \left(\vec{a}_{\vec{k}}\, e^{i(\vec{k}\vec{r} - kc_l t)} + \vec{a}^{+}_{\vec{k}}\, e^{-i(\vec{k}\vec{r} - kc_l t)}\right). \tag{11}$$

While investigating a superfluid liquid, Bogoliubov [8] separated the atoms of liquid helium ^{4}He in the condensate from those atoms filling the states above the condensate. In an analogous manner, we may consider the vector operator $\vec{a}_0 = \vec{l}\sqrt{n_0}$ and $\vec{a}^{+}_0 = \vec{l}\sqrt{n_0}$ as c-numbers (where $\vec{l}$ is the unit vector in the direction of propagation of the sound wave) within the approximation of a macroscopic number of *sound particles* in the condensate $n_0 \gg 1$. These assumptions lead to a broken Bose-symmetry law for *sound particles* in the condensate. To extend the concept of a broken Bose-symmetry law for *sound particles* in the condensate, we apply the definition of BEC of *sound particles* in the condensate as was postulated by the Penrose-Onsager for the definition of BEC of helium atoms [9]:

$$\lim_{n_0, n \to \infty} \frac{n_0}{n} = const. \tag{12}$$

On the other hand, we may observe that presence of *sound particles* filling the condensate level with the wave vector $\vec{k} = 0$ leads to the appearance of the constant displacement $\vec{u}_0 = \frac{2u_l \vec{l}\sqrt{n_0}}{\sqrt{V}}$ of the *sound particles*.

To find the normalization constant u_l, we introduce the following condition which allows us to suggest that at absolute zero all *sound particles* fill the condensate level $\vec{k} = 0$.

This reasoning implies that at $n_0 = n$ the constant displacement takes a maximal value $2d = \sqrt{|\vec{u}_0|^2}$ which represents the maximal distance between two neighboring *sound particles*. On the other hand, this distance is determined by the formula $d = \left(\frac{3V}{4\pi n}\right)^{\frac{1}{3}}$, which is in turn substituted into the expression $2d = \sqrt{|\vec{u}_0|^2}$. Then, consequently, we get to the normalization constant $u_l = 0.65\left(\frac{n}{V}\right)^{-\frac{5}{6}}$.

The condition of conservation of density at each point of the solid stipulates that

$$\rho_0 = \frac{MN}{V} = \frac{mn}{V}, \tag{13}$$

which represents a connection of the mass m and density ρ_0 of *sound particles* with the mass M and density ρ_0 of the liquid helium atoms with mass M.

Now, we consider the Hamiltonian operator $\hat{H}_l$ of a liquid [8]:

$$\hat{H}_l = \frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV + \frac{1}{2}\int\left(\frac{c_l\rho'}{\sqrt{\rho_0}}\right)^2 dV. \tag{14}$$

Substituting ρ' from (2) into (14), we obtain

$$\hat{H}_l = \frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV + \frac{\rho_0}{2}\int\left(c_l \operatorname{div}\vec{u}\right)^2 dV. \tag{15}$$

Using Dirac's approach in [10] for quantization of the electromagnetic field, we have:

$$\frac{\partial\vec{u}(\vec{r},t)}{\partial t} = -\frac{ic_l\vec{u}_l}{\sqrt{V}}\sum_{\vec{k}} k\left(\vec{a}_{\vec{k}}\,e^{-ikc_lt} - \vec{a}^+_{-\vec{k}}\,e^{ikc_lt}\right)e^{i\vec{k}\vec{r}} \tag{16}$$

as well as

$$\operatorname{div}\vec{u}(\vec{r},t) = \frac{i\vec{u}_l}{\sqrt{V}}\sum_{\vec{k}}\vec{k}\left(\vec{a}_{\vec{k}}\,e^{-ikc_lt} + \vec{a}^+_{-\vec{k}}\,e^{ikc_lt}\right)e^{i\vec{k}\vec{r}}. \tag{17}$$

Now, introducing (16) and (17) into (15) and using

$$\frac{1}{V}\int e^{i(\vec{k}_1+\vec{k}_2)\vec{r}} = \delta^3_{\vec{k}_1+\vec{k}_2},$$

we obtain the terms in the right side of the Hamiltonian of the system presented in (15):

$$\frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV = -\frac{\rho_0 c_l^2 u_l^2}{2}\sum_{\vec{k}} k^2\left(\vec{a}_{\vec{k}} - \vec{a}^+_{-\vec{k}}\right)\left(\vec{a}_{-\vec{k}} - \vec{a}^+_{\vec{k}}\right)$$

and

$$\frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV = \frac{\rho_0 c_l^2 u_l^2}{2}\sum_{\vec{k}} k^2\left(\vec{a}_{\vec{k}} + \vec{a}^+_{-\vec{k}}\right)\left(\vec{a}_{-\vec{k}} + \vec{a}^+_{\vec{k}}\right).$$

These expressions determine the reduced form of the Hamiltonian operator $\hat{H}_l$ by the form:

$$\hat{H}_l = 2\sum_{\vec{k}}\rho_0 u_l^2 c_l^2 k^2 \vec{a}^+_{\vec{k}} a_{\vec{k}}, \tag{18}$$

where u_l^2 is defined by the first term in the right side of (18) which represents the kinetic energy of a *sound particle* $\frac{\hbar^2k^2}{2m}$, if we suggest:

$$2\rho_0 u_l^2 c_l^2 k^2 = \frac{\hbar^2k^2}{2m}. \tag{19}$$

Then,

$$u_l^2 = \frac{\hbar^2}{4c_l^2 m\rho_0},$$

which allows one to determine the mass m of a *sound particle* using the value of the normalization constant $u_l = 0.65\left(\frac{n}{V}\right)^{-\frac{5}{6}}$ and (13):

$$m = \frac{\hbar}{c_l}\left(\frac{n}{V}\right)^{\frac{1}{3}}. \tag{20}$$

Thus, the Hamiltonian operator $\hat{H}_l$ describes an ideal Bose gas of a spinless *sound particles*:

$$\hat{H}_l = \sum_{\vec{k}}\frac{\hbar^2k^2}{2m}\vec{a}^+_{\vec{k}} a_{\vec{k}}. \tag{21}$$

3 Bose quasiparticles in solid

Now let us analyze quantization of a solid ^{4}He which consists of N atoms with the mass M confined in the volume V. Considering a solid ^{4}He as a continuum medium, we investigate the fluctuation motion of "solid particles" on the basis of hydrodynamics (where "solid particle" is determined as a very small volume V_0, in regard to the volume V of the solid ($V_0 \ll V$), which consists of a macroscopic number of ^{4}He atoms in solid).

To do the transition from quantum liquid to the solid ^{4}He, we introduce a concept as the fluctuation motion of "solid particles" or "solid points". In this respect, we remove such concept as a "lattice" of solid ^{4}He or such concept as an atoms, fixed in the knots of lattice because "solid particles" exist in any point of the solid. The motion of "solid particles" describe an elastic wave consisting of the *sound particles* with spin 1 which in turn are vibrated by the natural frequency Ω_l.

In this respect, we may express the vector displacement of a longitudinal ultrasonic wave $u_l(\vec{r},t)$ via the second quantization vector wave functions of one *sound particle* with spin 1. Then, Eqs. (8) and (9) take the forms:

$$\vec{\phi}(\vec{r},t) = \frac{1}{\sqrt{V}}\sum_{\vec{k},\sigma}\vec{a}_{\vec{k},\sigma}e^{i(\vec{k}\vec{r}-kc_lt)} \tag{22}$$

$$\vec{\phi}^+(\vec{r},t) = \frac{1}{\sqrt{V}}\sum_{\vec{k},\sigma}\vec{a}^+_{\vec{k},\sigma}e^{-i(\vec{k}\vec{r}-kc_lt)} \tag{23}$$

with condition

$$\int\phi^+(\vec{r},\sigma)\,\phi(\vec{r},\sigma)\,dV = n_0 + \sum_{\vec{k}\neq0,\sigma}\hat{a}^+_{\vec{k},\sigma}\hat{a}_{\vec{k},\sigma} = \hat{n}, \tag{24}$$

where a free sound particles have the mass m and the value of its spin z-component $\sigma = 0; \pm 1$. In this respect, the vector-operators $\vec{a}^+_{\vec{k},\sigma}$, $\vec{a}_{\vec{k},\sigma}$ satisfy the Bose commutation relations as:

$$\left[\hat{a}_{\vec{k},\sigma}, \hat{a}^+_{\vec{k}',\sigma'}\right] = \delta_{\vec{k},\vec{k}'} \cdot \delta_{\sigma,\sigma'}$$

$$[\hat{a}_{\vec{k},\sigma}, \hat{a}_{\vec{k}',\sigma'}] = 0$$

$$[\hat{a}^+_{\vec{k},\sigma}, \hat{a}^+_{\vec{k}',\sigma'}] = 0.$$

In this case, the Hamiltonian operator $\hat{H}$ of the solid ^{4}He is represented by the form:

$$\hat{H} = \frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV + \frac{1}{2}\int\left(\frac{c_l\rho'}{\sqrt{\rho_0}}\right)^2 dV + \frac{\rho_0}{2}\int(\Omega_l\vec{u}_l)^2\, dV, \tag{25}$$

where

$$\frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV = -\frac{\rho_0 c_l^2 u_l^2}{2}\sum_{\vec{k},\sigma} k^2\left(\vec{a}_{\vec{k},\sigma} - \vec{a}^+_{-\vec{k},\sigma}\right)\left(\vec{a}_{-\vec{k},\sigma} - \vec{a}^+_{\vec{k},\sigma}\right),$$

$$\frac{\rho_0}{2}\int(\operatorname{div}\vec{u})^2\, dV = \frac{\rho_0 c_l^2 u_l^2}{2}\sum_{\vec{k}} k^2\left(\vec{a}_{\vec{k},\sigma} + \vec{a}^+_{-\vec{k},\sigma}\right)\left(\vec{a}_{-\vec{k},\sigma} + \vec{a}^+_{\vec{k},\sigma}\right)$$

and

$$\frac{\rho_0}{2}\int(\Omega_l\vec{u}_l)^2\, dV = \frac{\rho_0\Omega^2 u_l^2}{2}\sum_{\vec{k}}\left(\vec{a}_{\vec{k},\sigma} + \vec{a}^+_{-\vec{k},\sigma}\right)\left(\vec{a}_{-\vec{k},\sigma} + \vec{a}^+_{\vec{k},\sigma}\right).$$

These expressions determine the reduced form of the Hamiltonian operator $\hat{H}$:

$$\hat{H}_l = \sum_{\vec{k}\neq 0,\sigma}\left(\frac{\hbar^2k^2}{2m} + mv^2\right)\vec{a}^+_{\vec{k},\sigma}a_{\vec{k},\sigma} + \frac{mv^2}{2}\sum_{\vec{k}\neq 0,\sigma}\left(\vec{a}^+_{-\vec{k},\sigma}\vec{a}^+_{\vec{k},\sigma} + \vec{a}_{\vec{k},\sigma}\vec{a}_{-\vec{k},\sigma}\right), \tag{26}$$

where we denote $v = \frac{\hbar\Omega_l}{\sqrt{2}mc_l}$, which in turn is the speed of sound particle in a solid.

For the evolution of the energy level, it is necessary to diagonalize the Hamiltonian $\hat{H}_l$, which can be accomplished by introducing the vector Bose-operators $\vec{b}^+_{\vec{k}}$ and $\vec{b}_{\vec{k}}$ [11]:

$$\vec{a}_{\vec{k},\sigma} = \frac{\vec{b}_{\vec{k},\sigma} + L_{\vec{k}}\vec{b}^+_{-\vec{k},\sigma}}{\sqrt{1 - L^2_{\vec{k}}}}, \tag{27}$$

where $L_{\vec{k}}$ is the unknown real symmetrical function of the wave vector $\vec{k}$.

By substituting (27) into (26), we obtain

$$\hat{H} = \sum_{\vec{k}\neq 0}\varepsilon_{\vec{k}}\vec{b}^+_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}, \tag{28}$$

where $\vec{b}^+_{\vec{k},\sigma}$ and $\vec{b}_{\vec{k},\sigma}$ are the creation and annihilation operators of Bose quasiparticles with spin 1 with the energy:

$$\varepsilon_{\vec{k}} = \left[\left(\frac{\hbar^2k^2}{2m}\right)^2 + \hbar^2k^2v^2\right]^{1/2}. \tag{29}$$

In this context, the real symmetrical function $L_{\vec{k}}$ of the wave vector $\vec{k}$ is found to be

$$L^2_{\vec{k}} = \frac{\frac{\hbar^2k^2}{2m} + mv^2 - \varepsilon_{\vec{k}}}{\frac{\hbar^2k^2}{2m} + mv^2 + \varepsilon_{\vec{k}}}. \tag{30}$$

Thus, the average energy of the system takes the form:

$$\overline{\hat{H}} = \sum_{\vec{k}\neq 0}\varepsilon_{\vec{k}}\overline{\vec{b}^+_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}}, \tag{31}$$

where $\overline{\vec{b}^+_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}}$ is the average number of Bose quasiparticles with spin 1 with the wave vector $\vec{k}$ at the temperature T:

$$\overline{\vec{b}^+_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}} = \frac{1}{e^{\frac{\varepsilon_{\vec{k}}}{kT}} - 1}. \tag{32}$$

Thus, we have found the spectrum of free quasiparticles with spin 1 which is similar to Bogoliubov's one [8]. In fact, the Hamiltonian of system (31) describes an ideal Bose gas consisting of phonons with spin 1 at a small wave number $k \ll \frac{2mv}{\hbar}$ but at $k \gg \frac{2mv}{\hbar}$ the Hamiltonian operator describes an ideal gas of *sound particles*.

4 BEC of *sound particles*

As opposed to London's postulation concerning BEC of atoms [12], we state that *sound particles* in the condensate define the superfluid component of solid ^{4}He. Consequently, statistical equilibrium equation (10) takes the following form:

$$n_{0,T} + \sum_{\vec{k}\neq 0}\overline{\vec{a}^+_{\vec{k},\sigma}\vec{a}_{\vec{k},\sigma}} = n, \tag{33}$$

where $\overline{\vec{a}^+_{\vec{k},\sigma}\vec{a}_{\vec{k},\sigma}}$ is the average number of *sound particles* with the wave vector $\vec{k}$ at the temperature T.

To find the form $\overline{\vec{a}^+_{\vec{k},\sigma}\vec{a}_{\vec{k},\sigma}}$, we use the linear transformation presented in (22):

$$\overline{\vec{a}^+_{\vec{k},\sigma}\vec{a}_{\vec{k},\sigma}} = \frac{1 + L^2_{\vec{p}}}{1 - L^2_{\vec{p}}}\overline{\vec{b}^+_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}} + \frac{L_{\vec{k}}}{1 - L^2_{\vec{k}}}\left(\overline{\vec{b}^+_{\vec{k},\sigma}\vec{b}^+_{-\vec{k},\sigma}} + \overline{\vec{b}_{\vec{k},\sigma}\vec{b}_{-\vec{k},\sigma}}\right) + \frac{L^2_{\vec{k}}}{1 - L^2_{\vec{k}}}.$$

According to the Bloch-De-Dominicis theorem, we have

$$\overline{\vec{b}^{+}_{\vec{k},\sigma}\vec{b}^{+}_{-\vec{k},\sigma}} = \overline{\vec{b}_{\vec{k},\sigma}\vec{b}_{-\vec{k},\sigma}} = 0.$$

In this respect, the equation for the density of *sound particles* in the condensate takes the following form:

$$\frac{n_{0,T}}{V} = \frac{n}{V} - \frac{1}{V}\sum_{\vec{k}\neq 0,\sigma}\frac{L^2_{\vec{k}}}{1-L^2_{\vec{k}}} - \frac{1}{V}\sum_{\vec{k}\neq 0,\sigma}\frac{1+L^2_{\vec{k}}}{1-L^2_{\vec{k}}}\overline{\vec{b}^{+}_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}}. \quad (34)$$

Obviously, at the lambda transition $T = T_\lambda$ the density of *sound particles* $\frac{n_{0,T_\lambda}}{V} = 0$. Hence, we note that the mass m and density $\frac{n}{V}$ of *sound particles* are expressed via the mass of ions M and density of ions $\frac{N}{V}$ when solving a system of two equations presented in (13) and (20):

$$\frac{n}{V} = \left(\frac{Mc_l}{\hbar}\frac{N}{V}\right)^{\frac{3}{4}} \quad (35)$$

and

$$m = \left(\frac{\hbar}{c_l}\right)^{\frac{3}{4}}\left(\frac{MN}{V}\right)^{\frac{1}{4}}. \quad (36)$$

At $T \to 0$ it follows $\overline{\vec{b}^{+}_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}} = 0$. Then taking into account the coefficient with number 3 before integral on the right side of equation (34) because it reflects the value of spin z-component $\sigma = 0; \pm 1$, we obtain

$$\frac{n_{0,T}}{n} = 1 - \frac{m^3 v^3}{\hbar^3\pi^2\frac{n}{V}}. \quad (37)$$

5 Conclusions

Thus, in this letter, we propose new model for solids which is different from the well-known models of Einstein and Debye because: 1) we suggest that the atoms are the Fermi particles which are absent in the Einstein and Debye models; 2) we remove such concept as lattice of solid by introducing a concept as the fluctuation motion of "solid particles" or "solid points". Thus, we deal with the "solid particle" which exist in any point of the solid; 3) In our model, we argue that the phonons in solid have spin 1 which is different from one presented by Einstein and Debye models; 4) in fact, in this letter, we first postulate that the superfluid component of a solid ^{4}He is determined by means of *sound particles* in the condensate as opposed to London's postulation concerning BEC of atoms [12]. Consequently, such reasoning allows us to consider the model of solid in a new light.

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