A New Theoretical Derivation of the Fine Structure Constant

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The present paper is devoted to a new derivation of the expression given already earlier for the fine structure constant α. This expression is exactly the same as that what we published several times since 1986. The equation $1/\alpha = \pi^4 \sqrt{2}\ m_{qm}/m_0$ (m_0 being the rest mass of the electron and m_{qm} the quantum-mechanical fraction of it) is precisely confirmed. The new derivation is based on relations for the energy density in the interior of a macroscopically resting electron within the framework of our standing wave model. This model is strongly supported by the present investigation. Two equations for the energy density inside of an electron were set equal, one of them is taken from classical electrodynamics, the other uses relations from quantum mechanics, special relativity theory and four-dimensional space. As the final theoretical equation for the fine structure constant is unchanged, the numerical value as published in 2008 is still maintained: $1/\alpha = 137.035\,999\,252$.

1 Introduction

In the fine structure constant $\alpha = e^2/\hbar c$ the constants of the electron charge e, Planck's constant h and the light velocity c are flowing together. These fundamental constants play a leading role in electrodynamics (ED), quantum mechanics (QM) and special relativity theory (SRT). Pauli [1] has called the explanation of the fine structure constant one of the most important problems of modern atomic physics. Mac Gregor 1971 [2] discussed α as an universal scaling factor. Here we present a new derivation for the fine structure constant obtained by equalizing two expressions for the energy density of the electromagnetic field inside the electron. One of these relations is based on ED, the other one is based on QM and SRT. In our opinion the new derivation is extraordinarily beautiful, simple and elegant.

We have developed a model of a macroscopically resting extended electron, called standing wave model. This model is based on the assumption that there is an internal energy flux along a closed curve of everywhere the same curvature. The energy flux takes place with velocity of light and is located on the surface of a sphere with radius r_m. The curve is denoted as spherical loop. It has an arc length $4\pi\rho_m$, where $\rho_m = r_m/\sqrt{2}$ is its radius of curvature and it consists of four semi circles. The internal motion produces the spin, magnetic moment and the electromagnetic field of the electron. In a set of publications [3–5] the authors have reported about these subjects.

Moreover, a study of the internal energy transport allowed us to derive a relation for the fine structure constant by investigating longitudinal and transversal standing waves inside of the electron. Here a new explanation of the fine structure constant is presented, also based on the standing wave model of the macroscopically resting electron but following a way which is essentially new.

We are convinced the new way of deriving α is of peculiar interest in understanding the structure of elementary particles. Therefore, we would like to open a discussion about our ideas and procedures.

2 Energy density based on electrodynamics

From classical electrodynamics applied to our standing wave model we were able to calculate the energy contributions of the electromagnetic field to the self-energy of an electron in the whole space. The energy flux is located on the surface of a sphere with the radius [5]

$$r_m = \frac{\hbar}{\sqrt{2} m_{qm} c}, \tag{1}$$

where m_{qm} denotes the quantum-mechanical fraction of the rest mass m_0 of the electron. Quantities which have a subscript m are related to the surface or the interior of a sphere with radius r_m and a subscript qm shall indicate that the corresponding quantity is related to quantum mechanics. Inside the sphere there are a transversal electric field with a field strength $\mathbf{E}^{\mathbf{t}}_{\mathbf{m}}$, and a magnetic field with a field strength $\mathbf{H_m}$. The absolute values of both field strengths are equal inside the sphere of radius r_m [5]:

$$\frac{e}{r_m^2} = |\mathbf{E}^{\mathbf{t}}_{\mathbf{m}}| = |\mathbf{H_m}|. \tag{2}$$

These fields are supposed to be homogeneous inside, i.e. the magnitudes of the field strenghts do not depend on the position. The volume of the sphere is given by

$$V_m = \frac{4}{3}\pi r_m^3. \tag{3}$$

The energy densities of the electric and magnetic fields can be taken from the field strength squares [6]:

$$u_E = \frac{1}{8\pi}|\mathbf{E}^{\mathbf{t}}_{\mathbf{m}}|^2 \tag{4}$$

$$u_H = \frac{1}{8\pi}|\mathbf{H_m}|^2. \quad (5)$$

The total energy density u_s of the electromagnetic field inside the electron is

$$u_s = u_E + u_H = \frac{2}{8\pi}\frac{e^2}{r_m^4}. \quad (6)$$

By integration over the sphere and using eq. (1) as well as the definition of the fine structure constant, the corresponding field energy is obtained

$$\begin{aligned} W_s &= \frac{2}{8\pi}\int_0^{r_m}\int_0^{\pi}\int_0^{2\pi}\frac{e^2}{r_m^4}r^2\sin\varphi\, d\theta\, d\varphi\, dr \\ &= \frac{2}{3}\frac{e^2}{2r_m} = \frac{2}{3}\frac{\alpha}{\sqrt{2}}m_{qm}c^2. \end{aligned} \quad (7)$$

The subscript s shall indicate that the corresponding quantities are related to the standing wave model.

3 Energy density based on QM, SRT and four dimensional space

We start from the three dimensional surface $S_{qm} = 2\pi^2 R^3$ of a four dimensional sphere (cf Schmutzer 1958 [7]). Choosing for the radius $R = \pi r_m$ there follows

$$S_{qm} = 2\pi^5 r_m^3. \quad (8)$$

The zero point energy inside this sphere is given by

$$W_{qm} = \frac{1}{2}\hbar\omega_0, \quad (9)$$

where ω_0 is the lowest possible, positive eigen frequency of the corresponding basic harmonic oscillator. According to the standing wave model this harmonic oscillator describes the electron. From the de Broglie relation

$$E = \hbar\omega_0 = m_0 c^2 \quad (10)$$

there follows

$$W_{qm} = \frac{1}{2}m_0 c^2, \quad (11)$$

and the energy density can be obtained from (8) and (11)

$$u_{qm} = \frac{W_{qm}}{S_{qm}} = \frac{m_0 c^2}{4\pi^5 r_m^3}. \quad (12)$$

4 Fine structure constant

A calculation of the values of u_s and u_{qm} show that they are very close to each other. This stimulated us to set

$$u_s = u_{qm}. \quad (13)$$

Indeed, using (1), (6) and (12), we obtain

$$u_s = u_{qm} \Leftrightarrow \frac{e^2}{\hbar c} = \frac{1}{\sqrt{2}\pi^4}\frac{m_0}{m_{qm}}. \quad (14)$$

Now, using the definition of the fine structure constant, for the inverse of it there follows immediately

$$\frac{1}{\alpha} = \pi^4\sqrt{2}\frac{m_{qm}}{m_0}, \quad (15)$$

where m_0 denotes the rest mass of the electron and m_{qm} its quantum-mechanical fraction. Just the same relation has been found earlier in an other way [3–5]. There, we have shown that both, m_0 and m_{qm}, are depending on α. Solving equation (15) the latest theoretical value of the inverse fine structure constant is [5]

$$\frac{1}{\alpha} = 137.035\,999\,252. \quad (16)$$

This value has to be compared with the semi experimental value 137.035 999 084(51) obtained by combining theory and experiment of the anomalous magnetic moment of the electron [8], as well as with the value 137.035 999 074(44), which is the latest CODATA value [9] from 2010. Furthermore, the ratio m_0/m_{qm} is obtained to be

$$\frac{m_0}{m_{qm}} = 1.005\,263\,277. \quad (17)$$

If we replace (as an alternative) m_0 in (11) by m_{qm} and simultaneously e^2 in (6) by e_i^2 (e_i is the intrinsic or bare charge of the electron) then we have exactly the wonderful relation

$$\frac{\hbar c}{e_i^2} = \pi^4\sqrt{2} = 137.757\,257... \quad (18)$$

The equations (15) and (18) are identical if

$$\frac{m_0}{m_{qm}} = \frac{e^2}{e_i^2}. \quad (19)$$

5 Discussion and conclusions

The numerical value of the fine-structure constant α was often denoted to be a mystery, a magic number and an enigma. A lot of more or less obscure relations have been published with the aim to understand the origin, theoretical background and the numerical value of the fine structure constant, see for example the comprehensive compilation of Kragh 2003 [10]. Why a derivation like the present one has not been carried out earlier? Probably it was the lack of an accurate model of an extended electron. No such model was available, see for example Mac Gregor 1992 [11]. We are convinced that without an understanding of the geometry and inner dynamics of the electron, a consistent understanding of the fine structure constant will not be possible. The simplicity of the present explanation of the fine structure constant is really surprising. Nevertheless, a more detailed discussion and interpretation of the roots of the fine structure constant would be very desirable. So far it concerns the history it should be remarked that already König 1951 [12] found as a byproduct in a rather complicated argumentation the same expression for α as we found here but without the factor m_{qm}/m_0. A difference between the theoretical and the experimental value of 0.53 % might be the reason that his paper, entitled "An electromagnetic wave picture of micro processes", have found very little attention.

We do not intend to give here a comprehensive discussion of the many aspects which are coupled with the fine structure constant. Several essays have been published devoted to to different aspects (Bahcall and Schmidt 1967 [13] (variation of α with time), Jehle 1972 [14] and 1977 [15] (flux quantization, loops, general discussion), Wilczek 2007 [16] (fundamental constants), Jordan 1939 [17] (cosmological constancy), Peik et al 2004 [18] (temporal limit), Dehnen et al. 1961 [19] (independence on gravitation field), Srianand et al. 2004 [20] (limits on time variation), Schönfeld 1996 [21] (self-energy analysis, see also [3–5])). We would like to remark and underline only two aspects of the present results: one is the exponent four at π which is obviously connected to the four dimensions of our world, the other is that the present result supports strongly the independence of the fine structure constant on time and space, i.e. expresses the cosmological constancy of alpha which was studied by theory and experiment in the last time. Naturally an experiment can give only an upper limit of time or position variation, compare [17–20].

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