Photon-assisted Spectroscopy of Dirac Electrons in Graphene

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The quantum Goos-Hanchen effect in graphene is investigated. The Goos-Hanchen phase shift is derived by solving the Dirac eigenvalue differential equation. This phase shift varies with the angle of incidence of the quasiparticle Dirac fermions on the barrier. Calculations show that the dependence of the phase shift on the angle of incidence is sensitive to the variation of the energy gap of graphene, the applied magnetic field and the frequency of the electromagnetic waves. The present results show that the conducting states in the sidebands is very effective in the phase shift for frequencies of the applied electromagnetic field. This investigation is very important for the application of graphene in nanoelectronics and nanophotonics.

1 Introduction

In recent years, the interest in novel device structures able to surmount the miniaturization limits imposed by silicon based transistors has led researchers to explore alternative technologies such as those originated in the field of semiconducting quantum dots, nanowire, graphene and carbon nanotubes [1, 2]. Graphene [3, 4] consists of a monolayer of carbon atoms forming a two-dimensional honeycomb lattice.

Graphene has been intensively studied due to its fascinating physical properties and potential applications in the field of nanoelectronics and another different field, for example, biosensor, hydrogen storage, and so on [5, 6]. In graphene, the energy bands touch the Fermi energy at six discrete points at the edges of the hexagonal Brillouin zone. Out of these six Fermi points, only two are inequivalent, they are commonly referred to as K and K´ points [7]. The quasiparticle excitation about K & K´ points obey linear Dirac like energy dispersion [8]. The presence of such Dirac like quasiparticle is expected to lead to a number of unusual electronic properties in graphene including relativistic quantum Hall effect [9], quasi-relativistic Klein tunneling [10, 11] and the lateral shift of these Dirac quasi-particles in graphene, which is known as Goos-Hanchen effect, Bragg reflector and wave guides [12–15]. The present paper is devoted to investigate the quantum Goos-Hanchen effect in graphene, taking into consideration the effect of electromagnetic waves of wide range of frequencies and magnetic field.

2 The Model

The transport of quasiparticle Dirac Fermions in monolayer graphene through a barrier of height, V_b, and width, d, is described by the following Dirac Hamiltonian, H_o, which is given as [4, 16]:

$$H_o = -i\hbar v_f \sigma \nabla + V_b, \tag{1}$$

where v_f is the Fermi velocity and $\sigma = (\sigma_x, \sigma_y)$ are the Pauli matrices. Since the graphene is connected to two leads and applying a top gate with gate voltage, V_g. Also, the transport of quasiparticle Dirac fermions are influenced by applying both magnetic field, B, and an electromagnetic field of amplitude, V_{ac}, and of wide range of frequencies, ω. So, accordingly Eq. (1) can be rewritten as follows:

$$H = -i\hbar v_f \sigma \nabla + V_b + eV_{sd} + eV_g + eV_{ac}\cos(\omega t) + \frac{\hbar eB}{2m^*}, \tag{2}$$

where V_{sd} is the bias voltage, $\hbar$ is reduced Planck's constant and m^* is the effective mass of quasiparticle Dirac fermions. Now, due to transmission of these quasi-particles Dirac fermions, a transition from central band to side-bands at energies [11, 17] $E \pm n\hbar\omega$, where n is an integer with values $0, \pm1, \pm2, \ldots$. The Dirac fermions Hamiltonian, H, (Eq. 2) operates in space of the two-component eigenfunction, Ψ, where Dirac eigenvalue differential equation is given by [11]:

$$H\Psi(r) = E\Psi(r), \tag{3}$$

where E is the scattered energy of quasi-particle Dirac fermions. The solution of Eq. (3) gives the following eigenfunctions [11,18]. The eigenfunction of incident quasi-particle Dirac fermions is

$$\Psi_{in}(r) = \sum_{n=1}^{\infty} J_n\left(\frac{eV_{ac}}{\hbar\omega}\right)[A + B], \tag{4}$$

where

$$A = \begin{pmatrix} 1 \\ se^{i\varphi} \end{pmatrix} \exp\left(i(k_x x + k_y y)\right),$$

$$B = r\begin{pmatrix} 1 \\ -se^{-i\varphi} \end{pmatrix} \exp\left(i(-k_x x + k_y y)\right),$$

J_n is the n^{th} order of Bessel function of first kind and the eigenfunction for the transmitted quasiparticle Dirac fermions through the barrier is given by:

$$\Psi_{tr}(r) = \sum_{n=1}^{\infty} J_n\left(\frac{eV_{ac}}{\hbar\omega}\right) t \begin{pmatrix} 1 \\ se^{i\varphi} \end{pmatrix} \exp\left(i(k_x x + k_y y)\right) \tag{5}$$

In Eqs. (4, 5), r and t are the reflection and transmission amplitude respectively and S = Sgn(E) is the signum function of E. The components of the wave vectors k_x and k_y outside the barrier are expressed in terms of the angle of incidence, φ, of the quasiparticles Dirac fermions as:

$$k_x = k_f \cos\varphi, \quad k_y = k_f \sin\varphi, \tag{6}$$

where k_f is the Fermi wave vector. The eigenfunction Ψ_b inside the region of the barrier is given by:

$$\Psi_b(r) = \sum_{n=1}^{\infty} J_n\left(\frac{eV_{ac}}{\hbar\omega}\right)[C + D], \tag{7}$$

where

$$C = \begin{pmatrix} \alpha \\ s'\beta e^{i\theta} \end{pmatrix} \exp\left(i(q_x x + k_y y)\right),$$

$$D = r\begin{pmatrix} \alpha \\ -s'\beta e^{-i\theta} \end{pmatrix} \exp\left(i(-q_x x + k_y y)\right),$$

$$q_x = (k_f'^2 - k_y^2)^{\frac{1}{2}}, \tag{8a}$$

and

$$\theta = \tan^{-1}\left(\frac{k_y}{q_x}\right) \tag{8b}$$

in which

$$k_f' = \frac{\sqrt{(V_b - \varepsilon)^2 - \frac{\varepsilon_g^2}{2}}}{\hbar v_f}, \tag{9}$$

where ε_g is the energy gap and ε is expressed as

$$\varepsilon = E - eV_g - n\hbar\omega - eV_{sd} - V_b + \frac{\hbar eB}{2m^*} \tag{10}$$

In Eq. (7), the parameters s', α, and β are given by:

$$s' = sgn\,(E - V_b) \tag{11}$$

$$\alpha = \sqrt{1 + \frac{\frac{s'\varepsilon_g}{2\hbar v_f}}{\sqrt{k_f'^2 + \frac{\varepsilon_g^2}{4(\hbar v_f)^2}}}} \tag{12}$$

This parameter, α, corresponds to K-point. Also, β is given by

$$\beta = \sqrt{1 - \frac{\frac{s'\varepsilon_g}{2\hbar v_f}}{\sqrt{k_f'^2 + \frac{\varepsilon_g^2}{4(\hbar v_f)^2}}}} \tag{13}$$

This parameter, β, corresponds to K'-point. Now, in order to find an expression for both the transmission coefficient, t, (Eq. 5) and the corresponding Goos-Hanchen phase shift, Φ, this is done by applying the boundary conditions at the boundaries of the barrier [11, 18]. This gives the transmission coefficient, t, as:

$$t = \sum_{n=1}^{\infty} J_n\left(\frac{eV_{ac}}{\hbar\omega}\right) \times \left[\frac{1}{\cos(q_x d) - F}\right], \tag{14}$$

where

$$F = i(s' s\gamma \sec(\varphi)\sec(\theta) + \tan(\varphi)\tan(\theta))\sin(q_x d)$$

and γ is expressed as:

$$\gamma = \frac{\sqrt{\frac{\varepsilon_g^2}{4(\hbar v_f)^2} + k_f'^2}}{k_f'} \tag{15}$$

The transmission coefficient, t, is related to the Goos-Hanchen phase shift, Φ, [12, 18] as:

$$t = \frac{e^{i\phi}}{f}, \tag{16}$$

where f is the Gaussian envelop of the shifted wave of quasiparticle Dirac fermions [12, 18, 19]. So, the expression for the phase shift is given by:

$$\Phi = \tan^{-1}\left[\frac{\sin(\theta)\sin(\varphi) + ss'\gamma}{\cos(\theta)\cos(\varphi)}\tan(q_x d)\right], \tag{17}$$

where d is the width of the barrier. We notice that the phase shift, Φ (Eq. 17) depends on the angle of incidence, ϕ of the quasiparticle Dirac fermion and on the barrier of height, V_b, and its width, d, and other parameters considered, for example, the energy gap, ε_g, the magnetic field, B, gate voltage, V_g, and the external pulsed photons of wide range of frequencies.

3 Results and Discussion

Numerical calculations are performed for phase shift, Φ, (Eq. 17) as shown below. For monolayer graphene, the values of both barrier height, V_b, and its width are respectively V_b = 120 meV and d = 80 nm [16, 18, 19]. Also, the value of the Fermi-velocity, v_f is approximately 10^6 m/s, and the effective mass of quasiparticle Dirac fermions is approximately m^* = 0.054 me [16, 18, 19]. The engineering of band gap of graphene generates a pathway for possible graphene-based nanoelectronics and nanophotonics devices. It is possible to open and tune the band gap of graphene by applying electric field [20] or by doping [21]. So, in our calculations we take the value of the energy gap of graphene to be ε_g = 0 eV, 0.03 eV, 0.05 eV [22].

The features of our results are the following:

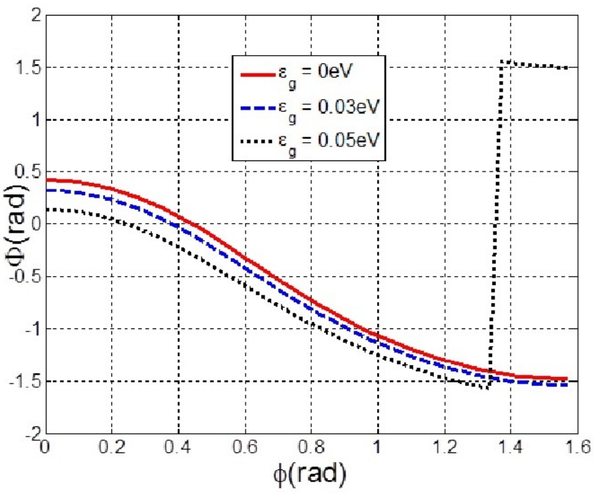

Fig. 1: The variation of Goos-Hanchen phase shift, Φ, with angle of incidence, ϕ, at different values of energy gap.

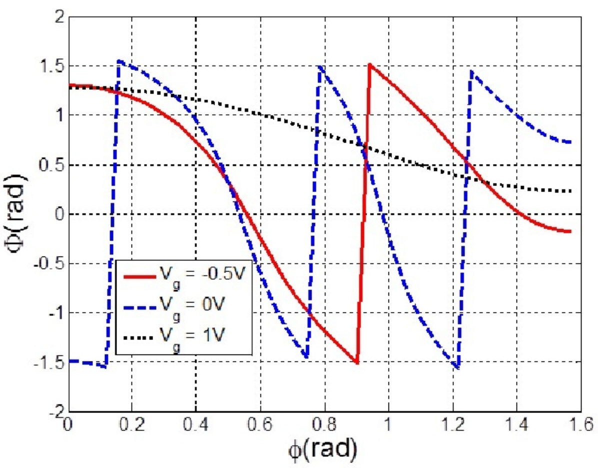

Fig. 2: The variation of Goos- Hanchen phase shift, Φ, with angle of incidence, ϕ, at different values of gate voltage.

1. Fig. 1, shows the dependence of the Goos-Hanchen phase shift, Φ, on the angle of incidence ϕ at different values of energy gap, ε_g. As shown from the figure that the phase shift, Φ, decreases as the angle of incidence, ϕ, increase for the considered values of the energy gap, ε_g. It must be noticed that for ε_g = 0.05eV, for angle of incidence $\phi \approx$1.335 rad, the phase shift, Φ , increases from -1.571 rad to 1.549 rad and then slightly decreases. This result shows the strong dependence of Goos-Hanchen phase shift on the engineered band gap of graphene [18, 23]. This result shows that the phase shift, Φ , can be enhanced by certain energy gap at the Dirac points.
2. Fig. 2 shows the dependence of the phase shift, Φ, on the angle of incidence, ϕ , at different values of the gate voltage, V_g. As shown from the figure that for large values of gate voltage, V_g, for example, V_g = 1V, the phase shift, Φ , decreases as the angle of incidence, ϕ, increase and phase shift takes only positive values. While for values of V_g = 0V or V_g = -0.5V, the value of phase shift oscillates between negative and positive values. It is well known that the tunneling of quasiparticle Dirac fermions could be controlled by changing the barrier height, V_b, this could be easily implemented by applying a gate voltage, V_g, to graphene [11,24–26].
3. Fig. 3 shows the dependence of the phase shift, Φ, on the angle of incidence, ϕ ,at different values of magnetic field, B. As shown from the figure that for B = 0.5 T, the phase shift decreases gradually as the angle of incidence, ϕ , increases to value Φ = 1.335 rad and then increases to Φ = 1.549 rad at ϕ = 1.374 rad and very slightly decreases. While for values B = 5 T and 10 T the value of the phase shift, Φ, is negative and decreases up to *Phi* = -1.561 rad when ϕ = 0.8635rad (when B = 5T) and then increases to Φ = 1.529 rad when ϕ = 0.902 rad and then decreases as the angle of incidence increases. For B = 10 T, the value of phase shift is negative and decreases as the value of ϕ increases up to ϕ = 0.432 rad and increases up to Φ = 1.547 rad and ϕ = 0.471 rad and decreases as the angle of incidence increases. This result shows that how a magnetic field modifies the transport of quasiparticle Dirac fermions in graphene with certain barrier height and certain energy gap [26].
4. Fig. 4 shows the variation of the phase shift, Φ, at different values of frequencies, ν , of the pulsed electromagnetic field. As shown from the figure, for higher frequencies 400 THz, 800 THz and 1000 THz, the value of the phase shift, Φ, decreases as the angle of incidence increases. We notice that in this range of frequencies, the value of phase shift is negative. While for microwave frequencies, MW = 300 GHz the value of the phase shift, Φ, decreases as the angle of incidence increases up to ϕ = 1.021 rad and then the phase shift increases up to Φ = 1.55 rad and ϕ = 1.06 rad and then decreases gradually.

This result shows that the conducting states in the side bands can be effective in the Goos-Hanchen phase shift for a certain frequency of the applied electromagnetic signal [27]. This result is very important for tailoring graphene for photonic nano-devices.

The present results show that the Goos-Hanchen phase shift can be modulated by both intrinsic parameters, for example, the barrier height, the energy gap and the extrinsic parameters, for example, magnetic field and the induced photons of electromagnetic field. The present research is very important for the applications of graphene in different nanoelectronics and nanophotonic devices.

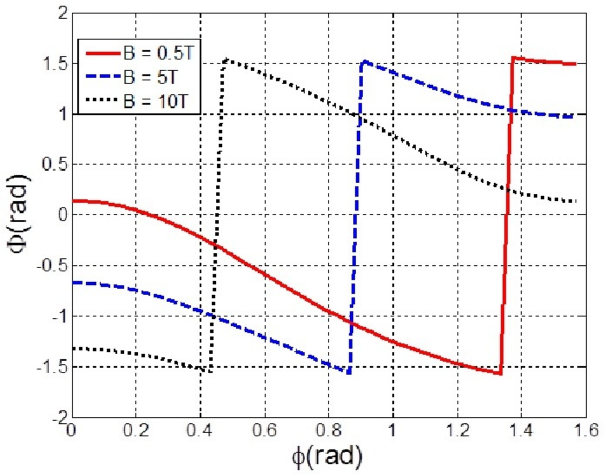

Fig. 3: The variation of Goos-Hanchen phase shift, Φ, with angle of incidence, ϕ, at different values of magnetic field.

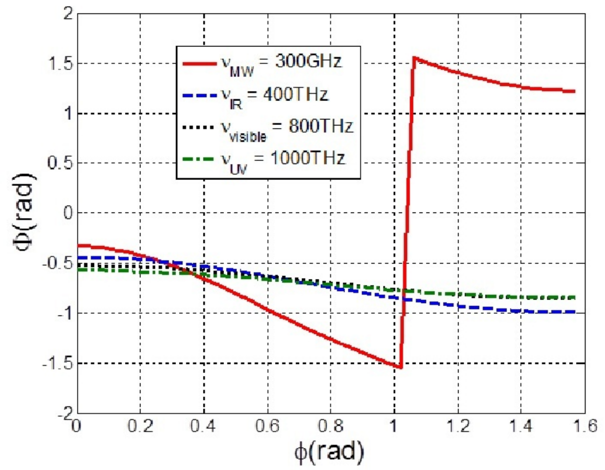

Fig. 4: The variation of Goos-Hanchen phase shift, Φ, with angle of incidence, ϕ, at different values of electromagnetic wave frequencies.

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