A More Elegant Argument that \( P \neq NP \)

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In April 2011, Craig Alan Feinstein published a paper in *Progress in Physics* entitled “An elegant argument that \( P \neq NP \)” [1]. Since then, Craig Alan Feinstein has discovered how to make that argument much simpler. In this letter, we present this argument.

Consider the following problem: Let \( \{s_1, \ldots, s_n\} \) be a set of \( n \) integers and \( t \) be another integer. We want to determine whether there exists a subset of \( \{s_1, \ldots, s_n\} \) for which the sum of its elements equals \( t \). We shall consider the sum of the elements of the empty set to be zero. This problem is called the SUBSET-SUM problem [2].

Let \( k \in \{1, \ldots, n\} \). Then the SUBSET-SUM problem is equivalent to determining whether there exist sets \( I^+ \subseteq \{1, \ldots, k\} \) and \( I^- \subseteq \{k+1, \ldots, n\} \) such that

\[
\sum_{i \in I^+} s_i = t - \sum_{i \in I^-} s_i.
\]

There is nothing that can be done to make this equation simpler. Then since there are \( 2^k \) possible expressions on the left-hand side of this equation and \( 2^{n-k} \) possible expressions on the right-hand side of this equation, we can find a lower-bound for the worst-case running-time of an algorithm that solves the SUBSET-SUM problem by minimizing \( 2^k + 2^{n-k} \) subject to \( k \in \{1, \ldots, n\} \).

When we do this, we find that

\[
2^k + 2^{n-k} = 2^{\lfloor n/2 \rfloor} + 2^{n-\lfloor n/2 \rfloor} = \Theta(\sqrt{2^n})
\]

is the solution, so it is impossible to solve the SUBSET-SUM problem in \( o(\sqrt{2^n}) \) time with a deterministic and exact algorithm. This lower-bound is tight [1]. And this conclusion implies that \( P \neq NP \) [2].

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References