The Turning Point for the Recent Acceleration of the Universe with a Cosmological Constant

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The turning point and acceleration expansion of the universe are investigated according to the standard cosmological theory with a non-zero cosmological constant. Choosing the Hubble constant H_0, the radius of the present universe R_0, and the density parameter in matter $\Omega_{M,0}$ as three independent parameters, we have analytically examined the other properties of the universe such as the density parameter in dark energy, the cosmological constant, the mass of the universe, the turning point redshift, the age of the present universe, and the time-dependent radius, expansion rate, velocity, and acceleration parameter of the universe. It is shown that the turning point redshift is only dependent of the density parameter in matter, not explicitly on the Hubble constant and the radius of the present universe. The universe turned its expansion from past deceleration to recent acceleration at the moment when its size was about 3/5 of the present size if the density parameter in matter is about 0.3 (or the turning point redshift is 0.67). The expansion rate is very large in the early period and decreases with time to approach the Hubble constant at the present time. The expansion velocity exceeds the light speed in the early period. It decreases to the minimum at the turning point and then increases with time. The minimum and present expansion velocities are determined with the independent parameters. The solution of time-dependent radius shows the universe expands all the time. The universe with a larger present radius, smaller Hubble constant, and/or smaller density parameter in matter is elder. The universe with smaller density parameter in matter accelerates recently in a larger rate but less than unity.

1 Introduction

The measurements of type Ia supernovae to appear fainter and thus further away than expected have indicated that the universe turned its expansion from past deceleration to recent acceleration [1-4]. The dark energy, a hypothetical form of negative pressure, is generally suggested to be the cause for the universe to accelerate recently. The Einsteinian cosmological constant Λ, initially assumed for a static model of the universe, is the simplest candidate of the dark energy [5]. Quintessence such as the scalar field from the scalar-tensor theory or the five-dimensional Kaluza-Klein unification theory is usually considered as another candidate of the dark energy [6-9]. In the black hole universe model, proposed recently by the author, the dark energy is nothing but the accretion of mass in an increasing time rate from outside space, the mother universe [10-17]. In the black hole universe model, the cosmological constant can be represented as $\Lambda = 3(\dot{M}/M)^2$, where M is the universe mass and $\dot{M}$ is the time rate of the universe mass. However, when the universe turns or what the redshift of the turning point for the universe to turn its expansion from past deceleration to recent acceleration has not yet been consistently and precisely determined.

The turning point redshift $Z_{\rm TP}$ was determined to be ~ 0.5 by combining the redshift and luminosity observations of type Ia supernovae with the standard model of cosmology [2, 4]. The universe was considered to be flat (i.e., $k = 0$ with k the curvature of the universe) with a cold dark matter (CDM) and a constant dark energy density (i.e., the cosmological constant). To explain the measurements of type Ia supernovae with the flat universe model, the density parameters in matter and dark energy ($\Omega_{M,0}$ and $\Omega_{\Lambda,0}$) at the present time (t_0) were chosen to be

$$\Omega_{M,0} \equiv \frac{8\pi G\rho_M(t_0)}{3H_0^2} = 0.3, \tag{1}$$

$$\Omega_{\Lambda,0} \equiv \frac{\Lambda}{3H_0^2} = 0.7, \tag{2}$$

where G is the gravitational constant, $\rho_{M,0}$ is the mass density, and $H_0 \sim 50 - 70$ km/s/Mpc is the Hubble constant [18-21]. For a holographic dark energy, the turning point redshift depends on a free parameter [22]. The turning point redshift is $Z_{\rm TP} \sim 0.72$ if the free parameter is chosen to be unity. For the best fit to the type Ia supernova data, the free parameter is around 0.2, which leads to a smaller turning point redshift, $Z_{\rm TP} \sim 0.28$.

To combine the measurements of type Ia supernovae with the cosmological model, a redshift-luminosity distance relation is required. The often used relation is, however, a linearly approximate relation,

$$d_L(Z) \simeq c(1+Z)\int_0^Z \frac{du}{H(u)}, \tag{3}$$

which is only good for nearby objects (see the detail of the standard derivation given by [23]. Using this approximate redshift-luminosity distance relation to study the expansion of the universe constrained by the measurements of type Ia supernovae with redshift greater than unity, one cannot accurately determine the turning point redshift [24] (Zhang and tan 2007). In Eq. (3), c is the light speed, Z is the redshift of light from the object, and d_L is the luminosity, which is usually defined by

$$F = \frac{L}{4\pi d_L^2}, \tag{4}$$

where L is the luminosity of the object such as a supernova, F is the apparent brightness of the object (i.e., the object emission flux measured at the Earth).

In this study, we analytically derive the turning point redshift only from the cosmological model without combining the model with the type Ia supernova data of measurements and thus without using the approximate redshift-luminosity distance relation. The simplest cosmological model that describes the recent acceleration of the universe is governed by the Friedmann equation with a non-zero Einsteinian cosmological constant [1-2, 5]. The expansion characteristics of the universe described by this constant ΛCDM model depend on three independent parameters. There are many different ways or combinations to choose the three independent parameters. But no matter how to combine, the number of independent parameters is always three. We have chosen the Hubble constant H_0, the radius of the present universe R_0, and the density parameter in matter $\Omega_{M,0}$ as the three independent parameters and have further derived the turning point redshift. The derived turning point redshift is only dependent of the density parameter in matter $\Omega_{M,0}$, not dependent of the other two independent parameters R_0 and H_0 if the universe is flat.

Exact solutions of the Friedmann equation [25-26] with the cosmological constant were obtained by [27-28]. The physical solutions, however, have not yet been analyzed with the recent measurements of the universe, especially on the turning point redshift.

The objective of this study is to quantitatively study the turning point and expansion characteristics of the recent acceleration universe through analyzing and numerically solving the Friedmann equation with a non-zero cosmological constant. First, for each set of H_0, $\Omega_{M,0}$, and R_0, we analytically obtain the turning point redshift $Z_{\rm TP}$ and other cosmological parameters such as the density parameter in dark energy $\Omega_{\Lambda,0}$, the cosmological constant Λ, and the mass of the universe M. Then, we substitute the obtained M and Λ into the Friedmann equation to numerically solve the time-dependent expansion rate or Hubble parameter $H(t)$, velocity $\upsilon(t)$, radius $R(t)$, and acceleration parameter $q(t)$ of the universe. Third, from the solutions, we determine the age of the present universe. Finally, we discuss the significant results and summarize our concluding remarks.

2 Turning Point and Expansion Characteristics of the Universe

According to the standard cosmological theory, the expansion of the universe is governed by the Friedmann equation [25-26, 29]

$$H^2(t) \equiv \frac{\dot{R}^2(t)}{R^2(t)} = \frac{8\pi G\rho_M(t)}{3} - \frac{kc^2}{R^2(t)} + \frac{\Lambda}{3}, \tag{5}$$

(Friedmann 1922, 1924; Carroll et al. 1992) where the dot refers to the derivative with respect to time, G is the gravitational constant, $\rho_M(t)$ is the density of matter given by

$$\rho_M(t) = \frac{3M}{4\pi R^3(t)}, \tag{6}$$

and k is the curvature of the space given by -1, 0, 1 for the universe to be open, flat, and closed, respectively. For the flat universe (i.e., $k = 0$), Eq. (5) becomes

$$H^2(t) \equiv \frac{\dot{R}^2(t)}{R^2(t)} = \frac{2GM}{R^3(t)} + \frac{\Lambda}{3}. \tag{7}$$

The solution of Eq. (7) depends on three independent parameters: R_0, M, and Λ. There are many different combinations that can be considered as the three independent parameters such as (R_0, H_0, Λ), (R_0, H_0, $\Omega_{M,0}$), etc. In this study, we have chosen R_0, H_0, and $\Omega_{M,0}$ as the three independent parameters.

To describe the acceleration of the universe, we define the acceleration parameter as

$$q(t) \equiv \frac{R(t)\ddot{R}(t)}{\dot{R}^2(t)} = 1 + \frac{\dot{H}(t)}{H^2(t)}. \tag{8}$$

Traditionally, a negative sign is inserted in Eq. (8) for the deceleration parameter.

A light that was emitted at time t is generally shifted towards the red when it is observed at the present time t_0 due to the expansion of the universe. The redshift of the light is given by

$$Z_H = \frac{R(t_0)}{R(t)} - 1. \tag{9}$$

The recent acceleration universe turned its expansion from past deceleration to recent acceleration at the moment when the acceleration parameter is equal to zero, i.e.,

$$q(t_{\rm TP}) = 0, \tag{10}$$

where $t_{\rm TP}$ is defined as the turning point - the time when the universe neither accelerates nor decelerates. It has been recognized for years but not yet theoretically determined.

Differentiating Eq. (7) with respect to time to get $\dot{H}(t)$ and using the turning point condition (10), we have the following relation

$$\Lambda = \frac{3GM}{R^3(t_{\rm TP})}. \tag{11}$$

Then, using Eq. (9), we have

$$\Lambda = \frac{3GM}{R^3(t_0)}\left(\frac{R^3(t_0)}{R^3(t_{TP})}\right) = \frac{3GM}{R_0^3}(Z_{TP}+1)^3, \qquad (12)$$

where we have replaced $R(t_0)$ by R_0 and denoted the redshift of observed light that was emitted at the turning point by Z_{TP} - the turning point redshift. From Eq. (12), the turning point redshift can be written as

$$Z_{TP} = \left(\frac{\Lambda R_0^3}{3GM}\right)^{1/3} - 1. \qquad (13)$$

At the present time t_0, Eq. (7) can be written as

$$1 = \Omega_{M,0} + \Omega_{\Lambda,0}, \qquad (14)$$

where the density parameters in matter and dark energy are defined respectively by

$$\Omega_{M,0} = \frac{8\pi G\rho_M(t_0)}{3H_0^2} = \frac{2GM}{H_0^2 R_0^3}, \qquad (15)$$

and

$$\Omega_{\Lambda,0} = \frac{\Lambda}{3H_0^2}. \qquad (16)$$

From Eqs. (15)-(16), we obtain

$$\frac{\Lambda R_0^3}{3GM} = 2\frac{1-\Omega_{M,0}}{\Omega_{M,0}}. \qquad (17)$$

Then, Eq. (13) reduces

$$Z_{TP} = \left(2\frac{1-\Omega_{M,0}}{\Omega_{M,0}}\right)^{1/3} - 1. \qquad (18)$$

Eq. (18) is a new result and has not been obtained before by any one. It is seen from Eq. (18) that the turning point redshift Z_{TP} is only dependent of the density parameter in matter Ω_M, not explicitly on another two independent parameter H_0 and R_0.

Figure 1 plots Z_{TP} as a function of Ω_M. The result indicates that, for the universe to be recently turned (i.e., $Z_{TP} > 0$), the density parameter in matter must be $\Omega_{M,0} < 2/3$ (or $\Omega_{\Lambda,0} > 1/3$). For the universe to be turned at $1 \gtrsim Z_{TP} \gtrsim 0.5$, the density parameter in matter must be $0.2 \lesssim \Omega_M \lesssim 0.4$. When $\Omega_{M,0} = 1$, we have $Z_{TP} = -1$, which implies that the flat universe will never be accelerated if the cosmological constant is zero. This is consistent with the gravitational physics because gravity always attracts.

Considering H_0, R_0, and $\Omega_{M,0}$ as three independent parameters in the flat universe model, we can determine Ω_Λ, M, Λ, and Z_{TP} by Eqs. (14)-(16) and (18). Substituting the determined M and Λ into Eq. (7), we can numerically solve the expansion parameters of the recent acceleration universe including the radius $R(t)$, expansion rate $H(t)$, expansion velocity $v(t)$, and acceleration parameter $q(t)$.

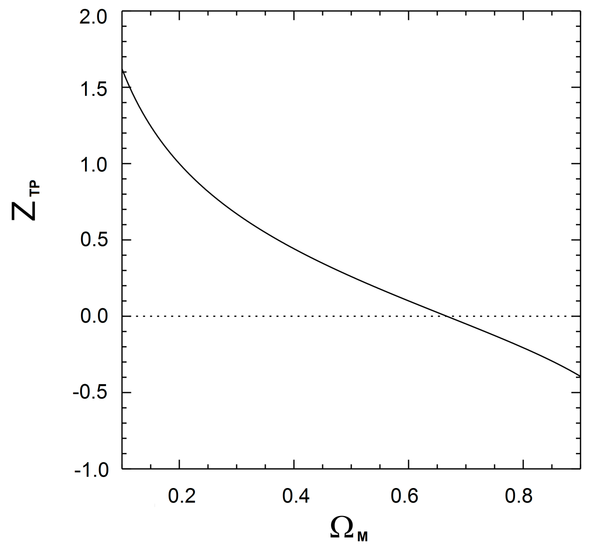

Fig. 1: Turning point redshift Z_{TP} versus density parameter in matter $\Omega_{M,0}$.

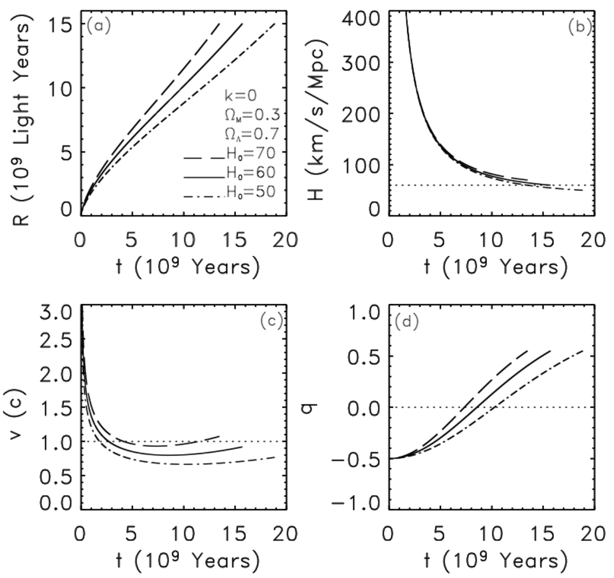

Fig. 2: Expansion characteristics of the universe when $\Omega_{M,0} = 0.3$, $R_0 = 15$ billion light years, and $H_0 = 50, 60, 70$ km/s/Mpc. (a) Radius of the universe $R(t)$, (b) expansion rate $H(t)$, (c) expansion velocity $v(t)$, (d) acceleration parameter $q(t)$.

Figure 2 plots these expansion parameters – $R(t)$, $H(t)$, $v(t)$, and $q(t)$ – as functions of time. We have chosen $H_0 = 50, 60, 70$ km/s/Mpc, $\Omega_{M,0} = 0.3$, and $R_0 = 15$ billion light years, which are displayed in Figure 2a. Three types of lines (dotted-dashed, solid, and dashed) correspond to the results with three different Hubble constants. With these three sets

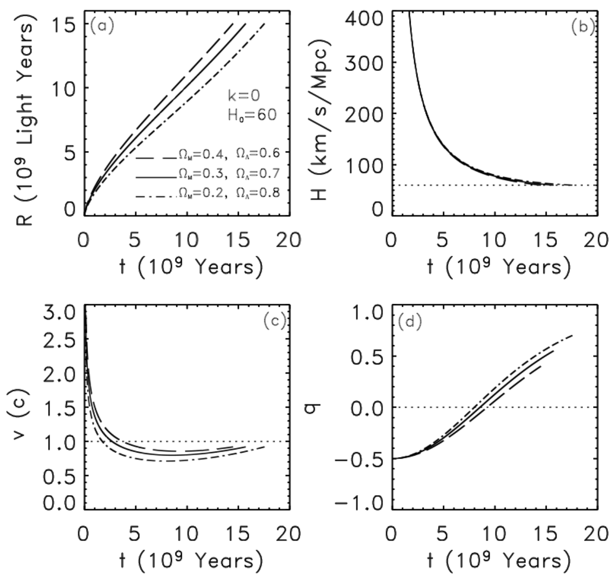

Fig. 3: Expansion characteristics of the universe when $\Omega_{M,0} = 0.2, 0.3, 0.4$ and $H_0 = 60$ km/s/Mpc with the same R_0. (a) Radius of the universe $R(t)$, (b) expansion rate $H(t)$, (c) expansion velocity $\upsilon(t)$, (d) acceleration parameter $q(t)$.

of parameters, we have $M = 1.7, 2.4, 3.3 \times 10^{53}$ kg, $\Lambda = 5.5, 8.0, 10.8 \times 10^{-36}$ s^{-2}, $\Omega_{\Lambda,0} = 0.7$, and $Z_{TP} = 0.67$.

Figure 2a shows that $R(t)$ increases with time to approach R_0 at the present time t_0. In comparison with a linear relation, the radius-time curves bend down at $R \lesssim 3R_0/5$ and then slightly go up at $R \gtrsim 3R_0/5$. The flat universe turned its expansion from past deceleration to recent acceleration at the time when the size of the universe was about three-fifth of the present universe (i.e., at $Z_{TP} \simeq 2/3$) due to the dark energy or non-zero cosmological constant. Figure 2b indicates that the expansion rate or Hubble parameter $H(t)$ decreases with time (or $\dot{H}(t) < 0$) to approach the Hubble constant H_0 at the present time. The dotted line refers to $H_0 = 60$ km/s/Mpc. Figure 2c shows that the expansion velocity decreases with time to the minimum at the turning point and then increases with time to approach $\upsilon_0 = H_0 R_0$, which exceeds the light speed in the case of $H_0 = 70$ km/s/Mpc and $R_0 = 15$ billion light years. In the early period, the expansion velocity can be much greater than the light speed. The minimum expansion velocity is determined by $\upsilon_{\min} = (2GM)^{1/3}\Lambda^{1/6}$. From Figure 2d, that the universe turned its expansion from past deceleration to recent acceleration can be seen in more obviously. The dotted line refers to $q = 0$. Each curve of $q(t)$ intersects with the dotted line at the turning point. For a different Hubble constant, the turning point t_{TP} is different. The acceleration parameter is negative (i.e., deceleration) before the turning point and positive (i.e., acceleration) after the turning point. At the present time, the acceleration parameter is slightly over 0.5.

Figure 3 also plots the four expansion parameters $R(t)$, $H(t)$, $\upsilon(t)$, and $q(t)$ as functions of time. In this plot, we have chosen a single $H_0 = 60$ km/s/Mpc but three $\Omega_M = 0.2, 0.3, 0.4$ with the same R_0. The three types of lines correspond to the results with three different density parameters. With these three sets of parameters, we have $M = 2.4 \times 10^{53}$ kg, $\Lambda = 8.0 \times 10^{-36}$ s^{-2}, $\Omega_{\Lambda,0} = 0.8, 0.7, 0.6$, and $Z_{TP} = 1, 0.67, 0.5$. The results are basically similar to Figure 2. The turning point redshift is single in the case of Figure 2 but multiple in the case of Figure 3. The radius-time curves (Figure 3a) also bend down relative to the linear relation in the past and go up recently, which implies that the flat universe was decelerated in the past and accelerated recently. The decreasing profiles of expansion rate $H(t)$ with time only slightly different among different density parameters (Figure 3b). The expansion velocity reaches the minimum $\upsilon_{\min}$ at the turning point and approaches υ_0 at t_0 (Figure 3c). The acceleration parameter at t_0 is greater if the universe contains more dark energy relative to matter (Figure 3d). For a different density parameter, the turning point t_{TP} is different. The acceleration parameter is negative (i.e., deceleration) before the turning point and positive (i.e., acceleration) after the turning point.

From Figures 2 and 3, we can find the present time or the age of the present universe with $R_0 = 15$ billion light year. For a different H_0 or $\Omega_{M,0}$, the age of the present universe should be different. The age of the present universe determined based on Figures 2 and 3 is plotted as a function of H_0 in Figure 4a and as a function of $\Omega_{M,0}$ in Figure 4b. It is seen that the age of the present universe decreases with H_0 and $\Omega_{M,0}$ when R_0 is fixed. For $R_0 = 15$ billion light year, $H_0 = 50 - 70$ km/s/Mpc, and $\Omega_{M,0} = 0.3$, the age of the universe is in the range of $\sim 13 - 19$ billion years, slightly less than H_0^{-1}. The universe is elder if it turned earlier (i.e., smaller $\Omega_{M,0}$) or has a smaller expansion rate.

3 Discussions and Conclusions

The open or closed universe can also be recently accelerated by the dark energy. Since k is not zero, the density parameters will be quite different in order for the universe to be turned from deceleration to acceleration at a similar turning point. The details on the turning point and expansion characteristics of the open and closed universes will be studied in future.

Consequently, the turning point and accelerating expansion of the flat universe has been investigated according to the cosmological theory with a non-zero cosmological constant. Choosing six sets of H_0, R_0, and $\Omega_{M,0}$, we have quantitatively determined $\Omega_{M,0}$, Λ, M, Z_{TP}, t_0, $R(t)$, $H(t)$, $\upsilon(t)$, and $q(t)$. Analyzing these results, we can conclude the following remarks.

To turn the expansion from deceleration to acceleration, the flat universe must contain enough amount of dark energy $\Omega_{\Lambda,0} > 1/3$. The turning point redshift depends only on the density parameter in matter $Z_{TP} = [2(1 - \Omega_{M,0})/\Omega_{M,0}]^{1/3} - 1$. The flat universe will never be accelerated if the cosmologi-

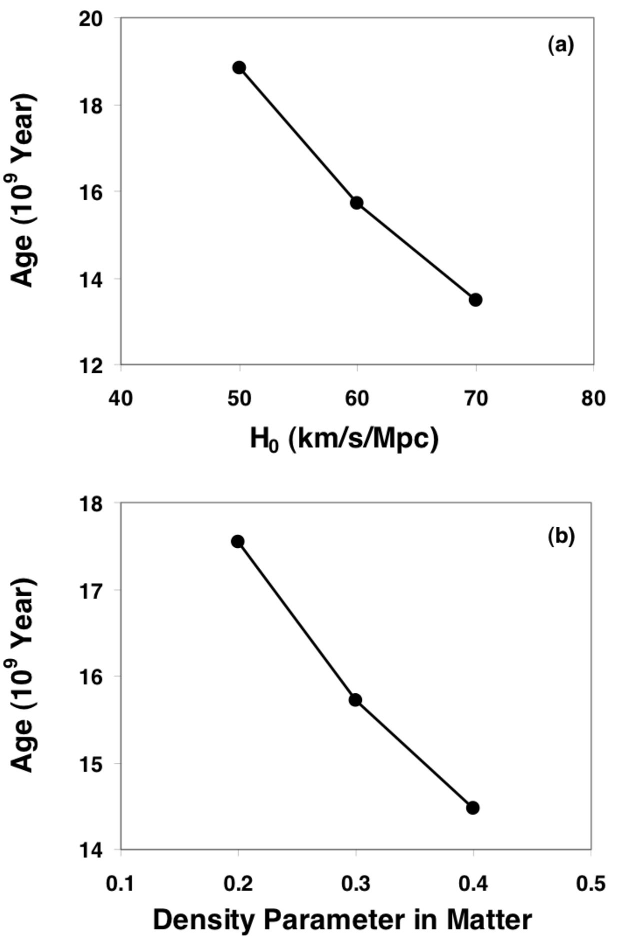

Fig. 4: Age of the universe as a function of H_0 (a) and $\Omega_{M,0}$ (b).

cal constant is zero. For the flat universe to be turned from deceleration to acceleration at $0.5 \lesssim Z_{TP} \lesssim 1$, the density parameter in matter must be $0.4 \gtrsim \Omega_{M,0} \gtrsim 0.2$. The radius of the universe generally increases with time. The expanding profiles are belong to the M_1 type of exact solutions given by [27-28]. The expansion rate of the universe rapidly decreases with time to approach the Hubble constant. The expansion velocity decreases with time to the minimum $\upsilon_{\min} = (2GM)^{1/3}\Lambda^{1/6}$ at the turning point and then increases with time to approach $\upsilon_0 = H_0 R_0$. The acceleration parameter also increases with time and changes from negative to positive at the turning point. The acceleration of the present universe is larger if it contains more dark energy. The age of the universe depends on all of R_0, H_0, and $\Omega_{M,0}$. The flat universe with a fixed R_0 should be elder for smaller H_0 or $\Omega_{M,0}$ due to the expansion velocity smaller.

Overall, this study has shown the constraints and characteristics of the recent acceleration universe, which deepens our understanding of the turning and accelerating of the universe from past deceleration to recent acceleration.

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