Emergence of Particle Masses in Fractal Scaling Models of Matter

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Based on a fractal scaling model of matter, that reproduces systematic features in the distribution of elementary particle rest masses, the paper presents natural oscillations in chain systems of harmonic quantum oscillators as mechanism of particle mass generation.

1 Introduction

The origin of particle masses is one of the most important topics in modern physics. In this paper we won’t discuss the current situation in the standard theory and the Higgs mechanism. Based on a fractal scaling model [1] of natural oscillations in chain systems of harmonic oscillators we present an alternative mechanism of mass generation.

Possibly, natural oscillations of matter generate scaling distributions of physical properties in very different processes. Fractal scaling models [2] of oscillation processes are not based on any statements about the nature of the link or interaction between the elements of the oscillating chain system. Therefore the model statements are quite generally, what opens a wide field of possible applications.

Within the last 10 years many articles were published which show that scaling is a widely distributed natural phenomenon [3–7]. As well, scaling is a general property of inclusive distributions in high energy particle reactions [8] – the quantity of secondary particles increases in dependence on the logarithm of the collision energy.

Particularly, the observable mass distribution of celestial bodies is connected via scaling with the mass distribution of fundamental particles [9], that can be understood as contribution to the fundamental link between quantum – and astrophysics.

Based on observational data, Haramein, Hyson and Rauscher [10,11] discuss a scaling law for all organized matter utilizing the Schwarzschild condition, describing cosmological to subatomic structures. From their point of view the universality of scaling suggests an underlying polarizable structured vacuum of mini white and black holes. They discuss the manner in which this structured vacuum can be described in terms of resolution of scale analogous to a fractal scaling as a means of renormalization at the Planck distance.

In the framework of our model [1], particles are resonance states in chain systems of harmonic quantum oscillators and the masses of fundamental particles are connected by the scaling exponent \( \frac{1}{2} \). For example, the logarithm of the proton-to-electron mass ratio is 7\( \frac{1}{2} \), but the logarithm of the W-boson-to-proton mass ratio is 4\( \frac{1}{2} \). This means, they are connected by the equation:

\[
\ln \left( \frac{m_w}{m_{\text{proton}}} \right) = \ln \left( \frac{m_{\text{proton}}}{m_{\text{electron}}} \right) - 3
\]

The logarithm of the W-boson-to-electron mass ratio is 4\( \frac{1}{2} \) + 7\( \frac{1}{2} \) = 12:

\[
\ln \left( \frac{m_w}{m_{\text{electron}}} \right) = 12.
\]

Already within the eighties the scaling exponent \( \frac{1}{2} \) was found in the distribution of particle masses by V. A. Kolombet [12]. In addition, we have shown [9] that the masses of the most massive bodies in the Solar System are connected by the scaling exponent \( \frac{1}{2} \). The scaling exponent 3 \times \frac{1}{2} arises as consequence of natural oscillations in chain systems of similar harmonic oscillators [2]. If the natural frequency of one harmonic oscillator is known, one can calculate the complete fractal spectrum of natural frequencies of the chain system. Spectral nodes arise on the distance of \( \frac{1}{2} \) logarithmic units. Near spectral nodes the spectral density reaches local maximum and natural frequencies of the oscillating chain system are distributed maximum densely. We suspect, that stable particles correspond to main spectral nodes which represent rational number logarithms.

The colossal difference between the life times of stable and “normal” particles is amazing. The life-time of a proton is minimum 10\(^{34}\) times larger than the life of a neutron, although the mass difference between them is only 0.13% of the proton rest mass. From this point of view seems that the stability of a particle is not connected with its mass.

In the framework of the standard theory, the electron is stable because it’s the least massive particle with non-zero electric charge. Its decay would violate charge conservation. The proton is stable, because it’s the lightest baryon and the baryon number is conserved. Therefore the proton is the most important baryon, while the electron is the most important lepton and the proton-to-electron mass ratio can be understood as a fundamental physical constant. Within the standard theory, the W- and Z-bosons are elementary particles which mediate the weak force. The rest masses of all these particles are measured with high precision. The precise rest masses of other elementary or stable particles (quarks, neutrinos) are nearly unknown and not measured directly.

The life-times of electron and proton seem not measurable. In addition, there is no comparison between the life of a proton (\( \tau_{\text{proton}} > 10^{35} \) years) and the age of the visible universe (\( \tau_{\text{universe}} > 10^{10} \) years). Though, there is an interesting scale similarity between the product of the proton life \( \tau_{\text{proton}} \) > 10\(^{30}\) years and the proton mass generating frequency \( \omega_{\text{proton}} \), on
the one side, and the product of the age $\tau_{\text{universe}} > 10^{10}$ years of the visible universe and the Planck frequency $\omega_{\text{Planck}}$, on the other side:

$$\omega_{\text{proton}} = E_{\text{proton}}/h = 938 \text{ MeV}/h = 1.425 \cdot 10^{24} \text{ Hz}$$

$$\omega_{\text{proton}}\tau_{\text{proton}} > 10^{60}$$

(3)

$$\omega_{\text{Planck}} = \sqrt{(c^2/\hbar G)} = 1.855 \cdot 10^{43} \text{ Hz}$$

$$\omega_{\text{Planck}}\tau_{\text{universe}} > 10^{60}.$$  

(4)

If both products are of the same scale, we can write:

$$\omega_{\text{proton}}\tau_{\text{proton}} = \omega_{\text{Planck}}\tau_{\text{universe}}.$$  

(5)

Because the frequencies $\omega_{\text{proton}}$ and $\omega_{\text{Planck}}$ are fundamental constants, the equation (5) means that possibly exists a fundamental connection between the age of the visible universe and the proton life-time.

2 Methods

Based on the continued fraction method [13] we will search the natural frequencies of a chain system of many similar harmonic oscillators in this form:

$$\omega_{jk} = \omega_{00} \exp (S_{jk}).$$  

(6)

$\omega_{jk}$ is a set of natural frequencies of a chain system of similar harmonic oscillators, $\omega_{00}$ is the natural angular oscillation frequency of one oscillator, $S_{jk}$ is a set of finite continued fractions with integer elements:

$$S_{jk} = n_{j0} + \frac{1}{n_{j1} + \frac{1}{n_{j2} + \ldots + \frac{1}{n_j}}} = [n_{j0}; n_{j1}, n_{j2}, \ldots, n_j],$$

(7)

where $n_{j0}, n_{j1}, n_{j2}, \ldots, n_j \in Z$, $j = 0, \infty$. We investigate continued fractions (7) with a finite quantity of layers $k$, which generate discrete spectra, because in this case all $S_{jk}$ represent rational numbers. Possibly, the free links $n_{j0}$ and the partial denominators $n_{j1}, n_{j2}, \ldots, n_j$ could be interpreted as some kind of "quantum numbers". The present paper follows the Terskich [13] definition of a chain system, where the interaction between the elements proceeds only in their movement direction. Model spectra (7) are not only logarithmic invariant, but also fractal, because the discrete hyperbolic distribution of natural frequencies $\omega_{jk}$ repeats itself on each spectral layer.

The partial denominators run through positive and negative integer values. Ranges of relative low spectral density (spectral gaps) and ranges of relative high spectral density (spectral nodes) arise on each spectral layer. In addition to the first spectral layer, fig. 1 shows the second spectral layer $k = 2$ with $[n_{j1}] = 2$ (logarithmic representation). Maximum spectral density areas (spectral nodes) arise automatically on the distance of integer and half logarithmic units.

$$\frac{-2}{-3/2} \quad -1 \quad -1/2 \quad 0 \quad 1/2 \quad 1 \quad 3/2 \quad 2$$

Fig. 1: The spectrum (7) on the first layer $k = 1$, for $|n_{j0}| = 0, 1, 2, \ldots$ and $|n_{j1}| = 2, 3, 4, \ldots$ and, in addition, the second spectral layer $k = 2$, with $|n_{j0}| = 2$ and $|n_{j1}| = 2, 3, 4, \ldots$ (logarithmic representation).

Fractal scaling models of natural oscillations are not based on any statements about the nature of the link or interaction between the elements of the oscillating chain system. For this reason we assume that our model could be useful also for the analysis of natural oscillations in chain systems of harmonic quantum oscillators. We assume that in the case of natural oscillations the amplitudes are low, the oscillations are harmonic and the oscillation energy $E$ depends only on the frequency ($\hbar$ is the Planck constant):

$$E = \hbar \omega.$$  

(8)

In the framework of our model (6) all particles are resonance states of an oscillating chain system, in which to the oscillation energy (8) corresponds the particle mass $m$:

$$m = \omega \hbar / c^2.$$  

(9)

In this connection the equation (9) means that quantum oscillations generate mass. Under consideration of (6) now we can create a fractal scaling model of the mass spectrum of model particles. This mass spectrum is described by the same continued fraction 7, for $m_{jk} = \omega_{00} \hbar / c^2$:

$$\ln \left( m_{jk} / m_{00} \right) = [n_{j0}; n_{j1}, n_{j2}, \ldots, n_j].$$  

(10)

The frequency spectrum (7) and the mass spectrum (10) are isomorphic. The mass spectrum (10) is fractal and consequently it has a clear hierarchical structure, in which continued fractions (7) of the form $[n_{j0}; \infty]$ and $[n_{j0}; 2, \infty]$ define main spectral nodes, as fig. 1 shows.

3 Results

Based on (10) in the present paper we will calculate a list of model particle masses which correspond to the main spectral nodes and compare this list with rest masses of well measured stable and fundamental particles – hadrons, leptons, gauge bosons and Higgs bosons.

The model mass spectrum (10) is logarithmically symmetric and the main spectral nodes arise on the distance of 1 and $1/2$ logarithmic units, as fig. 1 shows. The mass $m_{00}$ in (10) corresponds to the main spectral node $S_{00} = [0; \infty]$, because $\ln(m_{00}/m_{00}) = 0$. Let's assume that $m_{00}$ is the electron rest mass $0.510998910(13) \text{ MeV}/c^2$ [14]. In this case (10) describes the mass spectrum that corresponds to the natural frequency spectrum (7) of a chain system of vibrating electrons. Further stable or fundamental model particles correspond to further main spectral nodes of the form $[n_{j0}; \infty]$ and $[n_{j0}; 2]$. Actually, near the node $[12; \infty]$ we find the W- and Z-bosons,
Table 1: The calculated $S$-values (7) of $\frac{1}{2}$ logarithmic units width and the corresponding calculated model mass-intervals of main spectral nodes for the electron calibrated model mass spectrum. The deviation $d = \ln (m/m_0) - S$ is indicated.

<table>
<thead>
<tr>
<th>$S$</th>
<th>calculated (10) mass-interval $m_0 c^2$ (MeV)</th>
<th>corresponding particle</th>
<th>particle mass $m c^2$ (MeV) [14, 15]</th>
<th>$\ln (m/m_0)$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[0; \infty]$</td>
<td>0.451 – 0.579</td>
<td>electron ($m_e$)</td>
<td>0.510998910 ± 0.000000013</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$[7; 2, \infty]$</td>
<td>815 – 1047</td>
<td>proton</td>
<td>938.27203 ± 0.00008</td>
<td>7.515</td>
<td>0.015</td>
</tr>
<tr>
<td>$[7; 2, \infty]$</td>
<td>815 – 1047</td>
<td>neutron</td>
<td>939.565346 ± 0.000023</td>
<td>7.517</td>
<td>0.017</td>
</tr>
<tr>
<td>$[12; \infty]$</td>
<td>73395 – 94241</td>
<td>W-boson</td>
<td>80398 ± 25</td>
<td>11.966</td>
<td>–0.034</td>
</tr>
<tr>
<td>$[12; \infty]$</td>
<td>73395 – 94241</td>
<td>Z-boson</td>
<td>91187.6 ± 2.1</td>
<td>12.092</td>
<td>0.092</td>
</tr>
<tr>
<td>$[12; 2, \infty]$</td>
<td>121008 – 155377</td>
<td>Higgs-boson?</td>
<td>125500 ± 540</td>
<td>12.411</td>
<td>–0.089</td>
</tr>
<tr>
<td>$[13; \infty]$</td>
<td>199509 – 256174</td>
<td>EWSB?</td>
<td>51.528</td>
<td>0.028</td>
<td></td>
</tr>
<tr>
<td>$[51; 2, \infty]$</td>
<td>(1.048 – 1.345) × 10^{22}</td>
<td>Planck mass</td>
<td>1.220896(6) × 10^{22}</td>
<td>51.528</td>
<td>0.028</td>
</tr>
</tbody>
</table>

but near the node $[7; 2, \infty]$ the proton and neutron masses, as table 1 shows.

Theoretically, a chain system of vibrating protons generates the same spectrum (10). Also in this case, stable or fundamental model particles correspond to main spectral nodes of the form $[n_0; \infty]$ and $[n_0; 2, \infty]$, but relative to the electron calibrated spectrum, they are moved by $-\frac{7}{2}$ logarithmic units. Actually, if $m_0$ is the proton rest mass $M_0 = 538.27203(8)$ MeV/$c^2$ [14], then the electron corresponds to the node $[-7; -2, \infty]$, but the W- and Z-bosons correspond to node $[4; 2, \infty]$.

Consequently, the core claims of our model don’t depend on the selection of the calibration mass $m_0$, if it is the rest mass of a fundamental resonance state that corresponds to a main spectral node. As mentioned already, this is why the model spectrum (10) is logarithmically symmetric.

Because a chain system of any similar harmonic oscillators generates the spectrum (10), $m_0$ can be much less than the electron mass. Only one condition has to be fulfilled: $m_0$ has to correspond to a main spectral node of the model spectrum (10). On this background all particles can be interpreted as resonance states in a chain system of harmonic quantum oscillators, in which the rest mass of each single oscillator goes to zero. In the framework of our oscillation model this way can be understood the transition of massless to massive states.

Within our model particles arise as resonance states in chain systems of harmonic quantum oscillators and their mass distribution is logarithmically symmetric. In [11] we have investigated the distribution of hadrons (baryons and mesons) in dependence on their rest masses. We have shown that all known baryons are distributed over an interval of 2 logarithmic units, of $[7; 2, \infty]$ to $[9; 2, \infty]$. Maximum of baryons occupy the logarithmic center $[8; 2, \infty]$ of this interval. Maximum of mesons occupy the spectral node $[8; \infty]$ that split up the interval of $[0; \infty]$ to $[12; \infty]$ between the electron and the W- and Z-bosons proportionally of $\frac{2}{7}$. In addition, we have shown that the mass distribution of leptons isn’t different of the baryon and meson mass distributions, but follows them.

The rest mass of the most massive lepton (tauon) is near the maximum of the baryon and meson mass distributions.

In the framework of our model [1], the Planck frequency $\omega_{\text{Planck}}$ corresponds to a main spectral node of the model mass spectrum (10). Actually, relative to the proton mass generating frequency $\omega_{\text{proton}}$, the Planck frequency $\omega_{\text{Planck}}$ corresponds to the main node $[44; \infty]$ of the frequency spectrum (6):

$$\ln \frac{\omega_{\text{Planck}}}{\omega_{\text{proton}}} = \ln \frac{1.855 \times 10^{43}}{1.425 \times 10^{24}} \approx 44.\quad (11)$$

Relative to the electron mass generating frequency $\omega_e$ the Planck frequency $\omega_{\text{Planck}}$ corresponds to the spectral node $[51; 2, \infty]$:

$$\ln \frac{\omega_{\text{Planck}}}{\omega_e} = \ln \frac{1.855 \times 10^{43}}{7.884 \times 10^{35}} \approx 51.5 = 44 + 7.5.\quad (12)$$

The Planck frequency $\omega_{\text{Planck}}$ is $e^{44}$ times larger than the proton mass generating frequency $\omega_{\text{proton}}$ and the same relationship is between the Planck mass $m_{\text{Planck}}$ and the proton rest mass $m_{\text{proton}}$:

$$\ln \frac{m_{\text{Planck}}}{m_{\text{proton}}} = \ln \frac{2.177 \times 10^{-8}}{1.673 \times 10^{-27}} \approx 44\quad (13)$$

$$m_{\text{Planck}} = \sqrt{h c / G} = 2.177 \times 10^{-8} \text{ kg.}\quad (14)$$

The Planck mass $m_{\text{Planck}} \approx 21.77 \mu g$ corresponds to the main node $[44; \infty]$ of the proton calibrated mass spectrum (10) and therefore, probably, $m_{\text{Planck}}$ is the rest mass of a fundamental particle. In the framework of our model [1] the gravitational constant $G$ is connected directly with the fundamental particles masses. Now we can calculate $G$ based on the proton rest mass $m_{\text{proton}}$:

$$G = \frac{h c}{(e^{44} m_{\text{proton}})^2} \quad (14)$$

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Resume

In the framework of the present model discrete scaling mass distributions arise as result of natural oscillations in chain systems of harmonic quantum oscillators. With high precision, the masses of known fundamental and stable particles are connected by the model scaling factor $\frac{1}{2}$. Presumably, the complete mass distribution of particles is logarithmically symmetric and, possibly, massive particles arise as resonance states in chain systems of quantum oscillators.

Within our model any chain system of harmonic quantum oscillators generates the same mass spectrum (10) and the corresponding to the spectral node $[12; 2, \infty]$ observed particle mass of 125 GeV [15] can be interpreted as resonance state in a chain system of oscillating protons, for example.

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