1 Introduction

Since the first observation of superdeformation in $^{152}$Dy [1] and in $^{194}$Hg [2] more than 350 settled SD bands in more than 100 nuclei have been will established in several mass regions of nuclear chart $A \approx 190$, 150, 130 [3–6]. With the aid of large γ-ray detectors arrays, new regions of SD nuclei have been discovered encircle mass $A \approx 80$, 60, 70, 90 regions. The $A \approx 190$ mass region is of special interest, more than 85 SD bands have now been observed in this mass region alone in Au, Hg, Tl, Pb, Bi and Po nuclei. The SD states in $A \approx 190$ mass region have been observed down to quite low spin also many SD bands have now been observed in this mass region alone in Au, Hg, Tl, Pb, Bi, and Po nuclei. The SD states in $A \approx 190$ mass region have been observed down to quite low spin also many SD bands have now been observed in this mass region alone in Au, Hg, Tl, Pb, Bi, and Po nuclei. The SD states in $A \approx 190$ mass region have been observed down to quite low spin also many SD bands have now been observed in this mass region alone in Au, Hg, Tl, Pb, Bi, and Po nuclei.

Spin assignment is one of the most difficult and still unsolved problems in the study of nuclear superdeformation, because spins have not been determined experimentally in SD nuclei. This is due to the difficulty of establishing the excitation of a SD band into known yrast states. Several related approaches to assign the spins of SD bands in terms of there excitation energy of a SD State $E(\nu)$, which is associated [7–9] with the successive gradual alignments of a pair of nucleons occupying specific high-N intruder orbitals in the presence of pairing correlations.

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2 Sketch of the Model

In the model used, the excitation energy of a SD State $E(I)$ and spin $I$ is expressed as:

$$I^2 = I(I+1) = \sum_n b_n E_n(I).$$

With $\hat{I} [I(I+1)]^{1/2}$. If we restrict to three terms only, then

$$I(I+1) = b_0 + b_1 E(I) + b_2 E^2(I).$$

Solving for $E(I)$ we get the two-parameters formula for $E(I)$

$$E(I) = E_0 + a \left(I + b I(I+1)\right)^{1/2}$$

with $a, b, E_0$ simply expressed by $b_0$, $b_1$ and $b_2$

$$a = \frac{1}{2b_2} \left(b_1^2 - 4b_0 b_2\right)^{1/2}$$
\[ b = \frac{4b_2}{b_1^2 - 4b_0b_2} \]  
\[ E_0 = a - \frac{b_1}{2b_2} \]

where \( b \) characterizes the nuclear softness.

The rigid rotor limit corresponds to \( b \rightarrow 0 \) and \( a, E_0 \) keeping finite. The value of the parameter \( a \) increases slowly with \( I \). It is expected that a better expression may be obtained if a weak \( I \) dependence of the parameter \( a \) is taken into account. So equation (3) is tentatively modified as follows:

\[ E(I) = a \left( [1 + b(I)(I + 1)]^{1/2} - 1 \right) + c(I + 1) \]

with an additional parameter \( c \). Leading to a form for the gamma transition energies

\[ E_γ(I) = a \left( [1 + b(I)(I + 1)]^{1/2} - [1 + b(I-2)(I-1)]^{1/2} \right) + 2c(2I+1). \]

The kinematic \( J^1 \) and dynamic \( J^2 \) moment of inertia associated with the \( a, b, c \) formula are:

\[ J^1 = ab[1 + b(I + 1)]^{1/2} + \frac{1}{2c} \]
\[ J^2 = ab[1 + b(I + 1)]^{1/2} - \frac{1}{2c}. \]

The bandhead moment of inertia is

\[ J_0 = \frac{\hbar^2}{ab + 2c}. \]

Each SD nucleus is described by three adjustable parameters \( a, b, c \) which are determined by fitting procedure of all known levels.

For the SD bands, one can extract the rotational frequency, dynamic and kinematic moment of inertia by using the experimental intra band \( E_γ \) transition energies as follows:

\[ \hbar \omega = \frac{1}{4} \left[ E_γ(I + 2) + E_γ(I) \right] \]
\[ J^2(I) = \frac{4\hbar^2}{\Delta E_γ} \]
\[ J^1(I - 1) = \frac{\hbar^2(2I - 1)}{E_γ} \]

where

\[ E_γ = E(I) - E(I - 2), \]
\[ \Delta E_γ = E_γ(I + 2) - E_γ(I). \]

It is seen that whereas the extracted \( J^1 \) depends on \( I \) proposition, \( J^2 \) does not.

3 Analysis of \( \Delta I=1 \) signature splitting in SD signature partner

To investigate the \( \Delta I=1 \) staggering in signature partner pairs of odd SD band, one must extract the differences between the average transition \( I+1 \rightarrow I \) and \( I \rightarrow I-1 \) energies in one band the transition \( I+1 \rightarrow I \) and \( I \rightarrow I-1 \) energies in the signature partner

\[ \Delta E_γ(I) = \frac{1}{2} \left[ E_γ(I+1) - E_γ(I) \right] \]

where \( E_γ(I) \) is proposed in equation (8).

4 Numerical Calculation and Discussions

Our selected data set includes fourteen signature partner pairs in ten odd SD nuclei in the A ~ 190 mass region, namely:

- \(^{191}\text{Hg} \) (SD2, SD3)
- \(^{193}\text{Hg} \) (SD1, SD2)
- \(^{193}\text{Hg} \) (SD3, SD4)
- \(^{194}\text{Tl} \) (SD1, SD2)
- \(^{195}\text{Tl} \) (SD1, SD2)
- \(^{193}\text{Pb} \) (SD3, SD4)
- \(^{193}\text{Pb} \) (SD1, SD2)
- \(^{197}\text{Pb} \) (SD1, SD2)
- \(^{197}\text{Bi} \) (SD2, SD3)

The experimental transition energies are taken from reference [3]. To parameterize the spins of the SD bands, we assumed various values for the bandhead spin \( I_0 \) for each SD band and the model parameters \( a, b, c \) are adjusted by using a computer simulated search program in order to obtain a minimum root mean square deviation

\[ \chi = \left[ \frac{1}{N} \sum_{i=1}^{N} \frac{E_γ^{\text{exp}}(I) - E_γ^{\text{Theor}}(I)}{\Delta E_γ^{\text{exp}}(I)} \right]^{1/2}. \]

Of the calculated energies \( E_γ^{\text{cal}} \) from the observed energies \( E_γ^{\text{exp}} \), where \( N \) is the number of data points considered and \( \Delta E_γ^{\text{exp}} \) is the uncertainty of the \( \gamma \)-transition energies. The fitting procedure was repeated with spin \( I_0 \) fixed at the nearest half integer.

Table (1) gives the optimized model parameters \( a, b, c \), the bandhead spin proposition \( I_0 \) and the lowest transition energies \( E_γ(I_0+2 \rightarrow I_0) \) for each SD band.

The systematic behavior of kinematic \( J^1 \) and dynamic \( J^2 \) moments of inertia are guideline for the spin prediction and to understand the properties of the SD bands. We studied the variation of \( J^1 \) and \( J^2 \) as a function of rotational frequency \( \hbar \omega \). The value of \( J^1 \) and \( J^2 \) approaches each other at the bandhead spin \( I_0 \). The \( J^1 \) moment of inertia is found to be smaller than that of \( J^2 \) for all values of \( \hbar \omega \). Both \( J^1 \) and \( J^2 \) plots are concave upwards. In general the bandhead moments of inertia in our selected signature partners odd-A SD nuclei \( J_0 \approx (94 \pm 4)\hbar \text{MeV}^{-1} \) are longer than that of the yrast SD bands in neighboring even-even nuclei. The best fitted parameters were used to calculate the theoretical transition energies extracted from our proposed model.
Table 1: The calculated best model parameters $a, b, c$ and suggested bandhead spins $I_0$ for our selected signature partners in the odd SD nuclei in $A \approx 190$ region.

<table>
<thead>
<tr>
<th>SD Bands</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$I_0$</th>
<th>$E_\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{191}$Hg(SD2)</td>
<td>19074.6639</td>
<td>3.0809</td>
<td>2.3765</td>
<td>10.5</td>
<td>252.4</td>
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<tr>
<td>$^{191}$Hg(SD3)</td>
<td>15810.8517</td>
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<td>233.2</td>
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<tr>
<td>$^{193}$Hg(SD2)</td>
<td>12654.6097</td>
<td>4.3858</td>
<td>2.6051</td>
<td>10.5</td>
<td>254</td>
</tr>
<tr>
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<td>12243.4329</td>
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<td>2.6289</td>
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<tr>
<td>$^{193}$Hg(SD4)</td>
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<td>4.3858</td>
<td>2.6051</td>
<td>10.5</td>
<td>254</td>
</tr>
<tr>
<td>$^{193}$Hg(SD5)</td>
<td>7279.9405</td>
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<td>-0.9723</td>
<td>10.5</td>
<td>284.5</td>
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<tr>
<td>$^{193}$Hg(SD4)</td>
<td>22034.6647</td>
<td>2.4110</td>
<td>2.4673</td>
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</tr>
<tr>
<td>$^{191}$Tl(SD1)</td>
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<td>276.77</td>
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<tr>
<td>$^{191}$Tl(SD2)</td>
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<td>0.8532</td>
<td>-5.2350</td>
<td>12.5</td>
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<tr>
<td>$^{193}$Tl(SD1)</td>
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<td>227.3</td>
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<tr>
<td>$^{193}$Tl(SD2)</td>
<td>6380.8736</td>
<td>5.3776</td>
<td>3.5196</td>
<td>8.5</td>
<td>206.6</td>
</tr>
<tr>
<td>$^{195}$Tl(SD1)</td>
<td>6380.8738</td>
<td>5.3776</td>
<td>3.5196</td>
<td>8.5</td>
<td>206.6</td>
</tr>
<tr>
<td>$^{195}$Tl(SD2)</td>
<td>33124.3911</td>
<td>2.4266</td>
<td>1.2551</td>
<td>6.5</td>
<td>167.5</td>
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<tr>
<td>$^{193}$Pb(SD3)</td>
<td>16892.1756</td>
<td>3.5957</td>
<td>2.2986</td>
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<tr>
<td>$^{193}$Pb(SD4)</td>
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<td>3.6196</td>
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<td>213.2</td>
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<tr>
<td>$^{193}$Pb(SD5)</td>
<td>3574.7877</td>
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<tr>
<td>$^{195}$Pb(SD1)</td>
<td>600.9413</td>
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<td>$^{195}$Pb(SD2)</td>
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<td>$^{195}$Pb(SD3)</td>
<td>2362.3559</td>
<td>13.4225</td>
<td>3.9167</td>
<td>7.5</td>
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<tr>
<td>$^{195}$Pb(SD4)</td>
<td>18884.3711</td>
<td>3.5500</td>
<td>2.0732</td>
<td>8.5</td>
<td>213.6</td>
</tr>
<tr>
<td>$^{195}$Pb(SD1)</td>
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<td>2.5870</td>
<td>3.8497</td>
<td>7.5</td>
<td>183.7</td>
</tr>
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<td>$^{195}$Pb(SD2)</td>
<td>724986.6813</td>
<td>0.1798</td>
<td>-1.3692</td>
<td>6.5</td>
<td>204.6</td>
</tr>
<tr>
<td>$^{197}$Bi (SD)</td>
<td>6.09E+08</td>
<td>8.24E-07</td>
<td>-245.7806</td>
<td>8.5</td>
<td>166.2</td>
</tr>
<tr>
<td>$^{197}$Bi (SD3)</td>
<td>6.09E+08</td>
<td>8.24E-07</td>
<td>-245.7806</td>
<td>9.5</td>
<td>186.7</td>
</tr>
</tbody>
</table>

To investigate the $\Delta I=1$ signature splitting, the difference between the averaged transitions $I+2\rightarrow I$ and $I\rightarrow I-2$ energies in one band and the transition $I+1\rightarrow I-1$ energies in its signature partner $\Delta^2E(I)$ are determined and its value as a function of spin I for each signature partner pair are plotted in figure (1). Most of these signature partners show large amplitude staggering with the exception of $^{193}$Hg (SD1, SD2), $^{195}$Pb (SD5, SD6) and $^{195}$Pb (SD3, SD4).

A clear out amplification of $\Delta^2E(I)$ is seen in $^{193}$Pb (SD3, SD4). For most cases one finds that $\Delta^2E(I)$ is very small at lower spins, increasing faster and faster as the spin I increase. The of $\Delta^2E(I)$ in $^{197}$Tl (SD1, SD2) and $^{195}$Tl (SD1, SD2) are remarkable similar.

5 Conclusion

The nuclear superdeformed rotational bands of signature partners of odd-mass number in the $A\sim 190$ region have been studied in the framework of a simple formula based on collective rotational model containing three parameters. The formula connected directly the unknown spin and the energy of the level the spins of the observed levels were extracted by assuming various values to the lowest spin of the bandhead at the nearest half integer. The optimized three parameters have been deduced by using a computer simulated search program in order to obtain a minimum root mean square deviation of the calculated transition energies from the measured energies.

The calculated transition energies, level, spins, rotational frequencies, kinematic and dynamic moments of inertia are examined for fourteen signature partner pairs. To investigate the $\Delta I=1$ signature splitting for each signature partner pair, we calculated the difference between the average transitions $I+2\rightarrow I$ and $I\rightarrow I-2$ energies in one band and the transition $I+1\rightarrow I-1$ energies in its signature partner. Most of the signature partners in this region show large amplitude $\Delta I=1$ staggering.

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References

Fig. 1: The $\Delta I=1$ signature splitting in some signature partners of odd-A superdeformed nuclei.