Is Space-Time Curved?

Benjamin Prather
E-mail: benjamin.prather@gmail.com

This paper considers the possibility of a teleparallel approximation of general relativity where the underlying space-time of a compact massive source is related to the isotropic coordinate chart rather than the geometric chart. This results in a 20 percent reduction of the expected shadow radius of compact objects. The observation of the shadow radius of Sagittarius A* should be possible in the near future using VLBI. The theoretical reduction is within the uncertainty of the expected shadow radius, however any observation less than a critical radius would indicate that gravity is not the result of space-time curvature alone. If space-time curvature does not act alone it is simpler to adopt the teleparallel view, with the tetrad field representing the index of refraction of the required material field in a flat space-time.

Introduction

General relativity is highly successful in explaining the first order corrective terms to Newtonian gravity observed in the classical solar system test known at the time of its proposal. Further, it has predicted higher order effects not originally anticipated such as the orbital decay of binary pulsars. Any competing theory of gravity must agree with general relativity in these predictions. The bounds on these measurements have significantly improved since the introduction of general relativity [1].

The central tenant of general relativity is that gravity is a pseudo-force due to the curvature of space-time. This produces a theory lacking an absolute sense of parallelism. General relativity has been expressed as a teleparallel theory, thus restoring absolute parallelism [2].

The teleparallel equivalent of general relativity allows the curvature of a metric to be rephrased as contorsion in a flat space-time due to a tetrad field [2]. The geodesic equation becomes non-inertial forces as a result of the variation in the local index of refraction and motion the tetrad field represents.

This paper considers the implications of a teleparallel theory of gravity where the underlying space-time corresponds to a flattened version of the isotropic solutions rather than the usual geometric coordinates. This non-inertial flattening process produces pseudo-forces, which are taken to be actual forces due to the presence of a material field.

Globally, space-time is likely to be closer to a DeSitter space-time than the Minkowski space-time used in the limiting behaviour here. In this sense, space-time is demonstrably curved. The issue here is the local nature of space-time in the presence of strong gravitational fields.

1 Is space-time curved?

Despite its broad empirical success, and lack of any viable alternatives, general relativity continues to generate detractors who raise philosophical objections to its core propositions. These detractors, near or beyond the fringe of science, often lack the mathematical knowledge needed to properly discuss general relativity in a rigorous setting. Indeed, many of these objections stem from a rejection of the abstract mathematics required for general relativity or perceived errors in general relativity arising from subtle misunderstandings of these advanced notions.

The descendants of neo-Kantianism assert that space-time curvature caused by matter and energy is impossible, since matter and energy already require the concepts of space and time. A Galilean space-time is also claimed by these critics to be necessary to form an understanding of the world [3].

As Lie groups, however, the Poincaré group is equally descriptive as the Galilean group. These differences in symmetries can be empirically measured, strongly favouring a Minkowskian space-time over a Galilean space-time. In both geometries it is almost always helpful to select a convenient fixed frame to work within. General relativity, in its usual presentation, breaks this Lie symmetry globally.

General relativity can also be expressed as a teleparallel theory, restoring absolute parallelism by replacing the curvature of space with an embedded tetrad field. Tetrad fields can be viewed as representing the flow and refractive properties of a Lorentzian aether.

In classical fluid mechanical one can use a Lagrangian reference frame co-moving with a fluid or an inertial Eulerian reference frame. In a relativistic aether, using the Levi-Civita connection produces the Lagrangian description while using the Weitzenböck connection produces the Eulerian description.

The geodesic equation then becomes changing speed due to an index of refraction, bending due to Huygens' principle and frame dragging due to advection. In the teleparallel equivalent of general relativity this tetrad field exists as an independent structure. This can be viewed as a flowing index of refraction emerging in the absence of a refractive medium.

This theory can be bashed into a flat model using a non-inertial transformation. The use of a non-inertial reference frame introduces pseudo-forces to the equations of motion. Interpreting these forces as originating from a material field creates a flat theory of gravity while simultaneously providing a material medium responsible for the tetrad field.

It is in this sense that the question is raised, is space-time curved?

2 Flat teleparallel approximation

In general relativity, the gravity of a compact, spherically symmetric, uncharged, acceleration-free and isolated mass generates can be described by the well known Schwarzschild metric in spherical coordinates.

$$r_s = \frac{2GM}{c^2}\,, \tag{1}$$

$$\mathbf{g}_{ij} = \mathrm{diag}\left(\left(1-\frac{r_s}{r}\right)c^2,\ \left(1-\frac{r_s}{r}\right)^{-1},\ r^2,\ r^2\sin^2\theta\right). \tag{2}$$

This solution implies that the speed of light depends on the angle of inclination of the trajectory relative to the coordinate chart. It is possible to transform the radial component to a new chart where the speed of light is isotropic [4].

$$r = r'\left(1+\frac{r_s}{4r'}\right), \qquad r' = r\left(\frac{1}{2}-\frac{r_s}{4r}+\sqrt{\frac{1}{4}\left(1-\frac{r_s}{r}\right)}\right), \tag{3}$$

$$\mathbf{g}_{ij} = \left(1+\frac{r_s}{4r'}\right)^4 \mathrm{diag}\left(\frac{(4r'-r_s)^2}{(4r'+r_s)^6}\,c^2,\ 1,\ r^2,\ r^2\sin^2\theta\right). \tag{4}$$

A flat teleparallel approximation of general relativity can be made by eliminating the $\left(1+\frac{r_s}{4r'}\right)^4$ coefficient. This results in a flat space-time with an index of refraction.

$$\mathbf{g}_{ij} = \mathrm{diag}\left(\frac{(4r'-r_s)^2}{(4r'+r_s)^6}\,c^2,\ 1,\ r^2,\ r^2\sin^2\theta\right). \tag{5}$$

Determining the pseudo-forces caused by this non-inertial transformation, much less the fields needed to generate them, is beyond the scope of this paper. All that is of interest here is that a model should exist with this geometry in the limit of the Schwarzschild metric, and has an event horizon a quarter the size of general relativity.

Bashing the Schwarzschild metric into a flat teleparallel theory may be a convenient way to get a model that agrees with observation, but is a very ad hoc way to approach the problem. A far better approach would be to build a teleparallel theory from the ground up based on first principles. Once the numerous obstacles are overcome, any resulting theory will agree with general relativity in the weak field limit.

This would require differences in the strong field limit to distinguish between theories. Given the significant change in event horizon radius, the optical shadow radius of a compact object should provide a useful parameter to compare potential theories in the strong field limit.

3 Optical shadow, General Relativity

An image showing the neighbourhood of the singularity, including the event horizon, photon sphere and optical shadow is given in Figure 1.

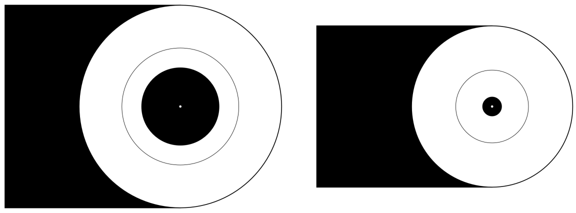

Fig. 1: Singularity Neighbourhood. The neighbourhood of a compact gravitational source is shown, with the Schwarzschild solution to the left and the flattened version to the right. The central black circle represents the event horizon, with a white circle showing the location of the singularity. This is surrounded by a thin circle representing the photon sphere. The outermost circle represents the optical shadow of the black hole, which is shown extending to the left using a tangent line approximation.

The depicted shadow region is a right circular cylinder with the singularity on its axis. The optical radius can be defined as the largest radius such that no unbound trajectory can have both an infinite length within the depicted shadow region and avoid the event horizon. In general relativity, the optical radius is $r_{crit} = \frac{3}{2}\sqrt{3}\,r_s$ [5].

4 Optical shadow, flat teleparallel approximation

In the flat teleparallel approximation, the dynamics are expected to be identical except for a rescaling of the radius near the singularity.

Almost all of the shadowing effect occurs near the singularity. An estimate of the asymptotic trajectory can be made using a tangent line to a circle about the singularity with a radius of r_{crit}. The radius of this circle is transformed by (3).

$$r'_{crit} = r_{crit}\left(\frac{1}{2}-\frac{1}{6\sqrt{3}}+\sqrt{\frac{1}{4}\left(1-\frac{2}{3\sqrt{3}}\right)}\right) \approx 0.8\,r_{crit}\,. \tag{6}$$

This reduction in radius is accomplished by the forces generated by a material component of the gravitational field.

5 VLBI measurements

Expected advances in submillimetre very long baseline interferometry are expected to be able to soon resolve the optical shadow of the compact radio source Sagittarius A*, based on the size expected by general relativity [5]. The factor of 0.8 is close enough to unity that the flat teleparallel approximation should produce a visible shadow under similar assumptions.

Observing the optical shadow is confounded by several known issues, much less measuring the radius. The optical properties of the medium surrounding Sagittarius A*, the 20 percent uncertainty in the mass of and distance to Sagittarius A* and the 10 percent uncertainty introduced by the dependency on the optical shadow on the rotation of Sagittarius A* are three significant issues [5].

A successful imaging of the shadow could help to determine some of these uncertainties, allowing a determination to be made between general relativity and any potential teleparallel theory including a material gravitational field.

While other effects can account for an optical shadow larger than r_{crit} within general relativity, an optical shadow less than $r_{crit} = 30 \pm 7\mu_{as}$ cannot be reconciled with general relativity [5].

6 Conclusion

Given that the event horizon in the flat teleparallel approximation is a quarter of that predicted by general relativity, the reduction in optical shadow of 20 percent is a disappointingly small change. This is less than the expected uncertainty in the optical shadow of Sagittarius A* due to its uncertain mass and possible rotation.

This also means that the same assumptions for observing the shadow expected for general relativity using VLBI can be applied to the flat teleparallel approximation. Such measurements can be expected on the order of years, not centuries.

This reduction would vary for different models of the material gravitational field, possibly resulting in a smaller optical radius. This would confound the ability to observe the optical shadow but simplify the ability to distinguish the predictions of general relativity and the model in question.

While other effects can account for an optical shadow larger than r_{crit} within general relativity, an optical shadow less than r_{crit} would indicate that gravity is not determined by space-time curvature alone.

The teleparallel equivalent of general relativity phrases the effects of gravity as due to an index of refraction in a flat space-time. If this is not acting alone, it is simpler to view this index of refraction as a property of the material field required to explain the super compact optical shadow.

If a super compact optical shadow is demonstrated, space-time curvature should then be abandoned in favour of a material, refractive gravitational field in a flat or DeSitter space-time.

Submitted on June 19, 2013 / Accepted on June 20, 2013

References

1. Will C. M. 2006 Confrontation between GR and experiment. *Living Rev. Relativity*, 2006, v.9(3), (cited on 6/5/2013: http://relativity.livingreviews.org/Articles/lrr-2006-3/)
2. Aldrovandi R., Pereira J.G. An Introduction to Teleparallel Gravity. Springer, New York, 2013
3. D.H. et al. 2013 Criticism of the theory of relativity. *Wikipedia* (cited on 6/5/2013, oldid=551162068)
4. Buchdahl H. A. Isotropic coordinates and Schwarzschild metric. *Int. J. Theor. Phys.*, 1985, v.24, 731–739.
5. Falcke H., Melia F., Agol E., Viewing the Shadow of the Black Hole at the Galactic Center. *Astrophys. J.*, 2000, v.528, L13–L16.