Geometric Distribution of Path and Fine Structure

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Previously (*Progr. Phys.*, v.2, 105–106) one predicted the exact value of the inverse fine structure constant respecting the double surface concept on Bohr orbit. In this paper one extends the same principle on the geometric distribution of the frequencies of the path of the electron in the ground state of Hydrogen atom. The inverse fine structure constant reflects the kind of the distribution and the later increases the constant in the range of the fifth decimal from $\alpha^{-1}_{0-sided} > 137.036006$ to $\alpha^{-1}_{\infty-sided} < 137.036018$.

1 Theoretical background

The number 137 expresses the translation component n of the path s of the electron on Bohr orbit [1]. Let us consider other translations n around this value are also possible. Each of them has its own frequency:

$$f_z = f(z), \quad \text{where } z = n - 137,\ n \in \mathbb{Z}. \tag{1}$$

It is also reasonable to assume the sum of the frequencies of all translations equals the unit which is the frequency of the whole translation of the path:

$$F_z = \sum f_z = 1. \tag{2}$$

The two-sided distribution ranges from the translation $n = -\infty$ to $n = 137$ on Bohr orbit and further from there to $n = \infty$. Overall interval is opened since the frequencies at the infinite ends $f_{\pm\infty}$ equal zero and can be ignored. There are also possible even-sided distributions provided on the arbitrary number of two-sided dimensions. From this point of view the non-distribution at $n = 137$ is regarded as zero-sided.

Each translation n belongs to its path s_n so the frequency of the former is identical to the frequency of the later. Product of the given frequency of the path f_z and the path s_n itself is the pondered partial path $f_z \cdot s_n$ inside the whole distribution of the path:

$$s_{whole} \cdot F_z = s_{whole} = \sum f_z \cdot s_n\,. \tag{3}$$

The inverse fine structure constant reflecting the whole distribution of the path [2] can be then expressed as:

$$\alpha^{-1}_{distributed} = \sum f_z \cdot s_n. \tag{4}$$

According to the double-surface concept [2] the value of the path s_n depends on the translation n:

$$s_n = n\left(2 - 1\Big/\sqrt{1 + \pi^2/n^2}\right), \quad \text{where } n \in \mathbb{Z}. \tag{5}$$

Knowing the type of the distribution function of frequencies $f(z)$ the inverse fine structure constant $\alpha^{-1}_{distributed}$ can be calculated. And vice versa, knowing the inverse fine structure constant $\alpha^{-1}_{distributed}$ the type of the distribution function of frequencies $f(z)$ can be speculated.

Our subject of interest in this paper is the geometric distribution of the frequencies of the path with ratio $1/2$ where the jumping of the electron to the non-adjacent positions is not allowed.

2 The two-sided geometric distribution

This is the symmetric distribution of the frequencies of the path provided on and around the zero numbered position z at $n = 137$:

$$f_z = \frac{1}{3}\frac{1}{2^{|z|}}, \quad \text{where } z \in \mathbb{Z}. \tag{6}$$

The sum of the frequencies f_z of all translations n from $-\infty$ to $+\infty$ equals the unit:

$$F_z = \sum_{z=-\infty}^{z=\infty} f_z = 1,$$

since

$$\sum_{z=-\infty}^{z=\infty} \frac{1}{3}\frac{1}{2^{|z|}} = \frac{1}{3}\sum_{z=-\infty}^{z=-1}\frac{1}{2^{|z|}} + \frac{1}{3}\sum_{z=0}^{z=0}\frac{1}{2^{z}} + \frac{1}{3}\sum_{z=1}^{z=\infty}\frac{1}{2^{z}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1. \tag{7}$$

The value of the inverse fine structure constant reflecting the 2-sided geometric distribution of the frequencies of the path of the electron in the ground state of Hydrogen atom can be calculated with the help of equations (1), (4), (5) and (6):

$$\alpha^{-1}_{2-sided} = \frac{1}{3}\sum_{n=-\infty}^{n=\infty} \frac{n\left(2 - 1\Big/\sqrt{1 + \pi^2/n^2}\right)}{2^{|n-137|}}\,. \tag{8}$$

The values of the frequencies of the path f_z rapidly lessen in the negative as well as positive direction from the zero numbered position z on Bohr orbit so the enough accurate value of the constant can be calculated numerically on the appropriate finite interval, for instance $n = [104, 170]$:

$$\alpha^{-1}_{2-sided} \approx 137.036014. \tag{9}$$

3 The even-sided geometric distribution

On the arbitrary number of sides generalized distribution of the frequencies of the path makes sense to be taken into account when some extra two-sided dimensions are proposed to be involved. The distribution on the even number of k sides is then expressed as:

$$f_z = \frac{1}{k+1}\frac{1}{2^{|z|}}, \text{ where } z \in \mathbb{Z} \text{ and } k = 2m,\ m \in \mathbb{N}_0. \quad (10)$$

The sum of the frequencies f_z of the infinite number of translations n on all k-sides and one zero position equals the unit:

$$F_z = \sum_{z=-\infty}^{z=\infty} f_z = 1,$$

since

$$\begin{aligned} \sum_{z=-\infty}^{z=\infty} \frac{1}{k+1}\frac{1}{2^{|z|}} &= \frac{k}{2}\sum_{z=-\infty}^{z=-1} \frac{1}{k+1}\frac{1}{2^{|z|}} + \sum_{z=0}^{z=0} \frac{1}{k+1}\frac{1}{2^{z}} + \\ &+ \frac{k}{2}\sum_{z=1}^{z=\infty} \frac{1}{k+1}\frac{1}{2^{z}} = \frac{k}{2}\frac{1}{k+1} + \frac{1}{k+1} + \frac{k}{2}\frac{1}{k+1} = 1. \end{aligned} \quad (11)$$

The value of the inverse fine structure constant reflecting the k-sided geometric distribution of the path of the electron in the ground state of Hydrogen atom is found with the help of equations (1), (4), (5), (10) and (11):

$$\begin{aligned} \alpha^{-1}_{k-sided} &= \frac{k}{2(k+1)} \sum_{n=-\infty}^{n=136} \frac{n\left(2 - 1\big/\sqrt{1+\pi^2/n^2}\right)}{2^{|n-137|}} + \\ &+ \frac{137\left(2 - 1\big/\sqrt{1+\pi^2/137^2}\right)}{k+1} + \\ &+ \frac{k}{2(k+1)} \sum_{n=138}^{n=\infty} \frac{n\left(2 - 1\big/\sqrt{1+\pi^2/n^2}\right)}{2^{n-137}}, \end{aligned} \quad (12)$$

where $n \in \mathbb{Z}$ and $k = 2m$, $m \in \mathbb{N}_0$.

The enough accurate value of the constant can be calculated numerically on the appropriate finite interval. For the acceptable results rounded on the six decimals can be used the finite intervals $n = 137 \pm 33$ instead of the infinite ones $n = 137 \pm \infty$. There is the infinite number of the even-sided distributions available from $k = 0$ to $k = \infty$. The 2-sided distribution at $k = 2$ is only one of them.

4 The non-distribution

Such special distribution of the frequencies of the path of the electron is considered on the zero position and zero sides on Bohr orbit. At $k = 0$ the equation (10) and (11) are simplified to $f_z = F_z = 1$ so the equation (12) takes the known form useful for the calculation of the theoretical inverse fine structure constant[2],(5):

$$\alpha^{-1}_{0-sided} = 137\left(2 - 1\big/\sqrt{1+\pi^2/137^2}\right) > 137.036006. \quad (13)$$

5 The infinite-sided geometric distribution

Such special distribution of the frequencies of the path of the electron takes place on the infinite sides around Bohr orbit. At $k = \infty$ the equation (12) is shortened for the vanished middle term and transformed into the next simplified form useful for the finding the theoretical inverse fine structure constant:

$$\begin{aligned} \alpha^{-1}_{\infty-sided} &= \sum_{n=-\infty}^{n=136} \frac{n\left(2 - 1\big/\sqrt{1+\pi^2/n^2}\right)}{2^{|n-137|+1}} + \\ &+ \sum_{138}^{n=\infty} \frac{n\left(2 - 1\big/\sqrt{1+\pi^2/n^2}\right)}{2^{n-137+1}} < 137.036018. \end{aligned} \quad (14)$$

6 The inverse fine structure reflecting the geometric distribution

The distributed value of the inverse fine structure constant seems to be greater than the non-distributed one since:

$$\begin{aligned} &\alpha^{-1}_{\infty-sided} \approx 137.036018 > \alpha^{-1}_{2-sided} \approx \\ &\approx 137.036014 > \alpha^{-1}_{0-sided} \approx 137.036006. \end{aligned} \quad (15)$$

The answer doesn't lie in the frequency of the path f_z which otherwise equally decreases on both sides of the number 137 but depends on the factor $\left(2 - 1\big/\sqrt{1+\pi^2/n^2}\right)$ which increases more with $n < 137$ than decreases with $n > 137$. The overall effect is thus the increasing value of the distributed inverse fine structure constant inside the range of the fifth decimal.

7 Conclusions

According to the double surface concept the exact inverse fine structure constant reflects the kind of the distribution of the frequencies of the path of the electron in the ground state of Hydrogen atom. The factor $\left(2 - 1\big/\sqrt{1+\pi^2/n^2}\right)$ asymmetrically changes partial values of the constant what results the increasing value of the whole constant. The number of sides of the distribution influences the above change in the range of the fifth decimal. The zero-, two- and infinite-sided geometric distribution of the frequencies of the path yields on the six decimal rounded inverse fine structure constant of 137.036006, 137.036014 and 137.036018, respectively.

Dedication

This fragment is dedicated to my daughters Alenka, Manica and Natalija.

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References

1. Špringer J. Fine structure constant as a mirror of sphere geometry. *Progress in Physics*, 2013, v. 1, 12–14.
2. Špringer J. Double surface and fine structure. *Progress in Physics*, 2013, v. 2, 105–106.