Flow of Viscous Fluid between Two Parallel Porous Plates with Bottom Injection and Top Suction

Hafeez Y. Hafeez\(^1\) and Chifu E. Ndikilar\(^2\)

\(^1\)Physics Department, Federal University Dutse, Nigeria. E-mail: hafeeyusufhafeez@gmail.com
\(^2\) Physics Department, Federal University Dutse, Nigeria. E-mail: ebenechifu@yahoo.com

This paper deals with the problem of steady laminar flow of viscous incompressible fluid between two parallel porous plates with bottom injection and top suction. The flow is driven by a pressure gradient \(\frac{dp}{dx}\) and uniform vertical flow is generated i.e. the vertical velocity is constant everywhere in the field flow i.e. \(v = \dot{v}_w = \text{constant}\). Also a solution for the small and large Reynolds number is discussed and the graph of velocity profile for flow between parallel plates with the bottom injection and top suction for different values of Reynolds numbers is drawn.

1 Introduction

The two dimensional steady laminar flow in channels with porous walls has numerous application in field of Science and Engineering through boundary layer control, transpiration cooling and biomedical engineering.

Berman (1953) was the first researcher who studied the problem of steady flow in an incompressible viscous fluid through a porous channel with rectangular cross section, when the Reynolds number is low and the perturbation solution assuming normal wall velocity to be equal was obtained [1].

Sellars (1955), extended problem studied by Berman by using very high Reynolds numbers [2].

Yuan (1956) [3] and Terrill (1964) [4] analysed the same problem assuming different normal velocity at the wall.

Terrill and Shrestha (1965) analysed the problem for a flow in a channel with walls of different permeabilities [5].

Green (1979) studied the flow in a channel with one porous wall [7].

In this paper, we considered the flow of an incompressible viscous fluid between two parallel porous plates with bottom injection and top suction and assume that the wall velocity is uniform.

2 Formulation of the problem

The study laminar flow of an incompressible viscous fluid between two parallel porous plates with bottom injection and top suction at walls and uniform cross flow velocity is considered. The well known governing equations of the flow are:

Continuity equation:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}
\]

Momentum equations (without body force):
\[
u \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right). \tag{2}
\]
\[
u \frac{\partial v}{\partial x} + u \frac{\partial v}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right). \tag{3}
\]

For particular integral of eqn. (5), we set
\[
u \frac{\partial u}{\partial y} = ay^2 + by + c, \tag{8}
\]
where a, b, and c are constants.
Substituting eqn. (9) in eqn. (5) we get
\( \left( 2a - \frac{v_w}{v} b \right) - 2a \frac{v_w y}{v} = \frac{1}{\mu} \frac{dp}{dx}. \)

Comparing the co-efficients, we get
\[ a = 0 \Rightarrow b = -\frac{v}{v_w} \frac{1}{\mu} \frac{dp}{dx}. \]

Now, eqn. (8) becomes
\[ u(y) = -\frac{v}{v_w} \frac{1}{\mu} \frac{dp}{dx} y + c. \]

The final solution forms by adding eqn. (7) and eqn. (11)
\[ u(y) = D + Be \frac{v}{v_w} y - \frac{v}{v_w} \frac{1}{\mu} \frac{dp}{dx} y. \]

Since \( v_w \) is constant, the equation is linear. We retain the no-slip condition for the main flow.

\[ u(+h) = u(-h) = 0 \]
\[ u(h) = D + Be \frac{v}{v_w} \frac{1}{\mu} \frac{dp}{dx} h \]
\[ u(-h) = D + Be \frac{v}{v_w} \frac{1}{\mu} \frac{dp}{dx} h. \]

Subtracting eqn. (14) from eqn. (13), we get
\[ B = \frac{2 \frac{v}{v_w} \frac{h dp}{dx} e^{\frac{v}{v_w} h}}{e^{\frac{v}{v_w} h} - e^{-\frac{v}{v_w} h}} = \frac{2 \frac{v}{v_w} \frac{h dp}{dx}}{2 \sinh \left( \frac{v}{v_w} h \right)} = \frac{\frac{v}{v_w} \frac{h dp}{dx}}{\sinh \left( \frac{v}{v_w} h \right)}. \]

Substituting eqn. (15) into eqn. (13), we get
\[ D = \frac{\frac{v}{v_w} \frac{h dp}{dx}}{\sinh \left( \frac{v}{v_w} h \right)} + \frac{v}{v_w} \frac{h dp}{dx}. \]

Eqn. (12) reduces to
\[ u(y) = -\frac{\frac{v}{v_w} \frac{h dp}{dx} e^{\frac{v}{v_w} h}}{\sinh \left( \frac{v}{v_w} h \right)} + \frac{v}{v_w} \frac{h dp}{dx} + \frac{\frac{v}{v_w} \frac{h dp}{dx} e^{\frac{v}{v_w} y}}{\sinh \left( \frac{v}{v_w} h \right)} - \frac{v}{v_w} \frac{h dp}{dx} \]

But wall Reynold number is
\[ Re = \frac{\frac{v}{v_w} h}{\frac{v}{v_w}} \Rightarrow \frac{h}{Re} = \frac{v_w}{v}. \]

Re-arranging eqn. (17), we get
\[ u(y) = -\frac{h^2}{Re} \frac{1}{\mu} \frac{dp}{dx} \left[ y - 1 + \frac{e^{Re} - e^{-Re}}{\sinh(Re)} \right]. \]

The final solution of eqn. (5),
\[ \frac{u(y)}{u_{\text{max}}} = \frac{2}{Re} \frac{y}{h} \left( 1 + \frac{e^{Re} - e^{-Re}}{\sinh(Re)} \right). \]

Where \( u_{\text{max}} = \frac{v^2}{2y} \) is the centerline velocity for imporous or poiseuille.

For very small \( Re \) (or very small vertical velocity), then the last terms in the parentheses of eqn. (19) can be expanded in a power series and \( sinh Re \approx Re \) i.e.
\[ \frac{u(y)}{u_{\text{max}}} = \frac{2}{Re} \frac{y}{h} - 1 + \frac{Re^2}{2} + \ldots \]

Eqn. (20) shows that the poiseuille solution recovered.

For very large \( Re \) (or large vertical velocity), eqn. (19) can be written as
\[ \frac{u(y)}{u_{\text{max}}} = \frac{2}{Re} \frac{y}{h} - 1 + 2 \frac{e^{Re} - e^{-Re}}{1 - e^{-2Re}}. \]

For \( Re \to \infty \) and \( \frac{y}{h} \to 1 \), except for \( y = +h \), we get
\[ \frac{u(y)}{u_{\text{max}}} = \frac{2}{Re} \left[ 1 + \frac{y}{h} \right]. \]

so that a straight line variation which suddenly drops to zero at the upper wall.

3 Discussion

The velocity profiles have been drawn for different values of Reynold numbers (i.e. \( Re = 0, 3, 5, 10 \)). From Fig. (1), it is observed that for \( Re \geq 0 \) in the region \( -1 \leq y' \leq 1 \), the shapes change smoothly with Reynold numbers and the average velocity is decreasing and Reynold number increases; i.e. the friction factor increases as we apply more cross flow through the wall.

4 Conclusion

In the above analysis a class of solution of flow of viscous fluid between two parallel porous plates with bottom injection and top suction is presented when a cross flow velocity along the boundary is uniform, the convective acceleration is linear and the flow is derived from pressure gradient.
Fig. 1: Velocity profiles for flow between parallel plates with bottom injection and top suction for different values of Re.

Nomenclature

A,B,C,D: Constants
h: Height of the channel
P: Pressure
x: Axial distance
y: Lateral distance
v_w: Lateral wall velocity
u(x,y): Axial velocity component
v(x,y): Lateral velocity component
y* = y/h: Dimensionless lateral distance
Re = v_w h/μ: Wall Reynolds number

Greek Symbols

μ: Shear viscosity
ν: Kinematic viscosity
ρ: Fluid density

References

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