Extended Analysis of the Casimir Force

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There are several arguments for the conventional form of the Zero Point Energy frequency spectrum to be put in doubt. It has thus to be revised into that of a self-consistent system in statistical equilibrium where the total energy density and the equivalent pressure become finite. An extended form of the Casimir force is thereby proposed to be used as a tool for determining the local magnitude of the same pressure. This can be done in terms of measurements on the force between a pair polished plane plates consisting of different metals, the plates having very small or zero air gaps. This corresponds to the largest possible Casimir force. Even then, there may arise problems with other adhering forces, possibly to be clarified in further experiments.

1 Introduction

The vacuum is not merely an empty space. Due to quantum theory, there is a non-zero level of the ground state, the Zero Point Energy (ZPE) as described by Schiff [1] among others. An example of the related spectrum of vacuum fluctuations was given by Casimir [2], who predicted that two metal plates will attract each other when being separated by a sufficiently small air gap. This prediction was first confirmed experimentally by Lamoreaux [3].

In a number of investigations the author has called attention to the importance of ZPE in connection with fundamental physics, on both the microscopic and the macroscopic scales. This applies to revised quantum electrodynamics and its relation to massive elementary particle models [4–6], as well as to attempts of explaining the concepts of dark energy and dark matter of the expanding universe [7, 8].

This paper presents an extended analysis of the ZPE frequency spectrum and its effect on the Casimir force, thereby leading to proposed experimental investigations on the features of the same spectrum.

2 Frequency spectrum of the Zero Point Energy

The local Zero Point Energy density has to become derivable from the frequency spectrum of an ensemble of ZPE photons. Such a procedure has to be conducted in the same standard way as for statistical systems in general, as described by Terletskii [9] and Kennard [10] among others.

For a "gas" of ZPE photons the number of field oscillations per unit volume in the range $(\nu, \nu + d\nu)$ becomes

$$dn = \frac{8\pi}{c^3}\, \nu^2\, d\nu. \tag{1}$$

This number can also be conceived to represent the various "rooms" to be populated by the photon frequency distribution.

In finding the corresponding self-consistent and fully determined contribution to the ZPE energy density, two points have to be taken into account:

- The quantized energy of every single photon is $E_0 = \frac{1}{2}h\nu$.
- The photon population of the frequency states has to be adapted to a statistical equilibrium, under the constraint of a finite and given total energy density. The latter corresponds to an average energy $\bar{E}_0 = \frac{1}{2}h\bar{\nu}$ per photon with a related average frequency $\bar{\nu}$.

Due to these points, the contribution to the energy density within the range $(\nu, \nu + d\nu)$ becomes [7, 8]

$$du = \frac{4\pi h}{c^3}\, \nu^3 \exp\left(-\frac{\nu}{\bar{\nu}}\right). \tag{2}$$

Here the Boltzmann factor

$$P_B = \exp\left(-\frac{E_0}{\bar{E}_0}\right) = \exp\left(-\frac{\nu}{\bar{\nu}}\right) \tag{3}$$

is due to the probability of the various photon states in statistical equilibrium.

In the present isotropic state, the contribution to the pressure becomes $dp = du/3$. The local ZPE pressure then has the total integrated value

$$p_0 = \frac{8\pi h \bar{\nu}^4}{c^3} \tag{4}$$

as obtained from relation (2).

In the earlier conventional analysis, the factor (3) has been missing, thus resulting in an infinite total ZPE energy density and pressure. Several investigators, such as Riess and Turner [11] as well as Heitler [12], have thrown doubt upon such an outcome. Attempts to circumambulate this irrelevant result by introducing cutoff frequencies either at the Planck length or at an arbitrary energy of 100 GeV, are hardly acceptable. This omission does not only debouch into a physically unacceptable result, but also represents an *undetermined* and not self-consistent statistical system [7, 8].

3 Experimental possibilities

The average frequency $\bar{\nu}$ appearing in the factor (3) is an important but so far not determined basic parameter. It may have a characteristic value in the environment of the Earth, or even of our galaxy. It should therefore be investigated if this parameter can be determined from experiments. This would require earlier experiments on the Casimir force to be extended. Two options are here proposed for such investigations, all using polished plane metal plates:

- Air gaps of a smaller width than those in earlier experiments, but being larger than the electromagnetic skin depth of the plates, would extend the measurable range. Thereby the insertion of insulating material of very small thickness may be tested.
- The largest possible Casimir force is expected to occur at a vanishing air gap. In this case the skin depth of the plates acts as an equivalent air gap. Even at this maximum Casimir force, other surface and sticking mechanisms such as by Van der Waals' forces may interfere with the measurements. To eliminate at least part of these difficulties, any magnetic alloy should be avoided as plate material in the first place. Further, as pointed out by N. Abramson [13] and G. Brodin [14], plates of different materials should be chosen to avoid microscopic matching of the metal structures. Possible choices of plate material are Ag, Cu, Au, Al, Mg, Mo, W, Zn, Ni, Cd, Sb, and Bi in order of decreasing electric conductivity.

As a device for measurement of the Casimir force, a weighting machine with two horizontal plates is proposed, in which the weight of the upper plate is outbalanced and a vertical Casimir force can be recorded.

4 The Casimir force

The Casimir force arises from the difference in pressure on the out- and insides of the metal plates. Whereas the full ZPE pressure acts at their outsides, there is a reduced pressure acting on their insides, due to the boundary condition which sorts out all frequencies below a limit $\hat{\nu}$. The latter corresponds to wavelengths larger than $\hat{\lambda} = c/\hat{\nu}$, as being further specified for the two options defined in Sec. 3. The net Casimir pressure thus becomes

$$\hat{p} = \int_0^\infty \mathrm{d}p - \int_{\hat{\nu}}^\infty \mathrm{d}p = \frac{4\pi h}{3c^3}\int_0^{\hat{\nu}} \nu^3 \exp\left(-\frac{\nu}{\bar{\nu}}\right) \mathrm{d}\nu \qquad (5)$$

due to the distribution (2). With $x = \nu/\bar{\nu}$ and $\hat{x} = \hat{\nu}/\bar{\nu}$ expression (5) obtains the form

$$\hat{p} = p_0 \, \Pi\,(\hat{x}) \qquad (6)$$

where p_0 is given by (4) and

$$\Pi = \int_0^{\hat{x}} x^3 \exp(-x)\, \mathrm{d}x = 1 - \left(1 + \hat{x} + \frac{1}{2}\hat{x}^2 + \frac{1}{6}\hat{x}^3\right)\exp(-\hat{x})\,. \qquad (7)$$

4.1 Plates with an air gap

The first option concerns an air gap of the width a, being substantially larger than the skin depth of the plates at relevant frequencies. Then the frequencies smaller than $\hat{\nu} = c/2a$ and wavelengths larger than $\hat{\lambda} = 2a$ are excluded. In the limit of $\hat{x} \ll 1$, Π then approaches the value $\hat{x}^4/24$, and the net pressure becomes

$$\hat{p} \cong \frac{\pi hc}{48a^4} \qquad (8)$$

being proportional to $1/a^4$ as earlier shown by Casimir [2].

For arbitrary values of $\hat{x} = c/2a\bar{\nu}$, the Casimir pressure (6) can then for various gap widths be studied as a function of $\bar{\nu}$. The set of obtained values of $\hat{p}$ then leads to information about the average frequency $\bar{\nu}$, within the limits of application of this option.

4.2 Plates with zero air gap

With the second option of a vanishing air gap, the sum of the skin depths at each plate plays the rôle of a total air gap. Using two plates of different metals having the electric conductivities σ_1 and σ_2, their skin depths at the frequency ν become [15]

$$(\delta_1, \delta_2) = \frac{1}{\sqrt{\pi\mu_0\nu}}\left(\frac{1}{\sqrt{\sigma_1}}, \frac{1}{\sqrt{\sigma_2}}\right). \qquad (9)$$

The total skin depth can then be written as

$$\delta_1 + \delta_2 = \frac{2}{\sqrt{\pi\mu_0\nu}}\frac{1}{\sqrt{\sigma_{12}}} \qquad (10)$$

where

$$\sigma_{12} = \frac{4\,\sigma_1\sigma_2}{\sigma_1 + \sigma_2 + 2\sqrt{\sigma_1\sigma_2}}. \qquad (11)$$

In the limiting case where half a wavelength $\lambda/2 = c/2\nu$ is equal to the total skin depth (10), the corresponding frequency limit becomes

$$\hat{\nu} = \frac{\mu_0\pi c^2\sigma_{12}}{16}\,. \qquad (12)$$

Since λ varies as $1/\nu$ and $\delta_1 + \delta_2$ as $1/\sqrt{\nu}$, it is seen that all frequencies ν less than $\hat{\nu}$ are excluded by the boundary condition. Thus $\hat{\nu}$ represents the Casimir frequency limit, as in the analogous case of a nonzero air gap.

With p_0 given by (4), $\hat{p}$ and Π by (6) and (7), $\hat{\nu}$ by (12), and $\hat{x} = \hat{\nu}/\bar{\nu}$, the Casimir pressure $\hat{p}$ is obtained as a function of the average frequency $\bar{\nu}$ for a given effective conductivity (11) of a pair of plates. Examples are given by (Ag/Cu, Ni/Cd, Sb/Bi) for which $\sigma_{12} = (60.5, 14.1, 1.26) \times 10^6$ A/Vm and $\hat{\nu} = (134, 31.2, 2.79) \times 10^{16}$ s^{-1} and $\hat{\lambda} = (2.23, 9.60, 107) \times 10^{-10}$ m, respectively. The dependence of $\hat{p}$ on $\bar{\nu}$ for the three examples of metal plate combinations are demonstrated in Fig. 1. The left-hand part of the figure relates to large values of $\hat{x}$ for which $\hat{p}$ nearly includes the full pressure (4), and

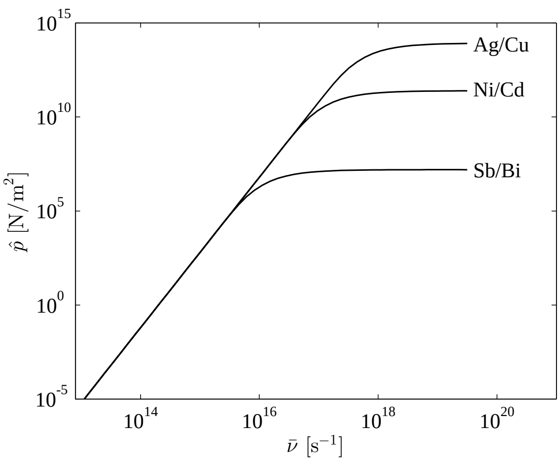

Fig. 1: Casimir pressure $\hat{p}$ as a function of the ZPE average frequency $\bar{\nu}$ for the three metal plate combinations Ag/Cu, Ni/Cd, and Sb/Bi.

for which there is a vanishing difference between the various plate combinations. The right-hand part of the same figure corresponds on the other hand to small $\hat{x}$ for which there is a difference due to the various values of resistivity and $\hat{\nu}$. This part leads to a pressure $\hat{p}$ having the asymptotic limit $(\pi h/3c^3)\,\hat{\nu}^4$ at large $\bar{\nu}$. To extend the range of resistivity dependent Casimir pressures in respect to $\bar{\nu}$, plates with even lower values of σ_{12} would have to be used. Provided that the Casimir force is the dominant one, the measured pressure $\hat{p}$ should thus be related to the same value of the average frequency $\bar{\nu}$, then being independent of the choice of metal combinations. This would, in its turn, lead to an identification of $\bar{\nu}$.

5 Conclusions

There are strong arguments for the frequency spectrum of the Zero Point Energy to be determined by means of a self-consistent system of statistical equilibrium in which there is a finite total pressure and a related finite average frequency. To investigate this state, an extended experimental analysis is proposed, based on the largest possible Casimir force which occurs on a pair of metal plates separated by a very small or even vanishing air gap. Provided that these forces become much stronger than those due to other possible adhering mechanisms, the proposed measurements may give an estimate of the average frequency defined in Section 2.

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