

Chrome of Baryons

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Chromes of quarks are changed under the Cartesian turns. And the Lorentz's transformations change chromes and grades of quarks. Baryons represent one of ways of elimination of these noninvariancy.

Introduction

According to the quark model [1], the properties of hadrons are primarily determined by their so-called valence quarks. For example, a proton is composed of two up quarks and one down quark. Although quarks also carry color charge, hadrons must have zero total color charge because of a phenomenon called color confinement. That is, hadrons must be "colorless" or "white". These are the simplest of the two ways: three quarks of different colors, or a quark of one color and an antiquark carrying the corresponding anticolor. Hadrons with the first arrangement are called baryons, and those with the second arrangement are mesons.

1 Cartesian rotation

Let α be any real number and

$$\begin{aligned} x'_0 &:= x_0, \\ x'_1 &:= x_1 \cos(\alpha) - x_2 \sin(\alpha); \\ x'_2 &:= x_1 \sin(\alpha) + x_2 \cos(\alpha); \\ x'_3 &:= x_3; \end{aligned} \quad (1)$$

Since j_A is a 3+1-vector then from [2, p. 59]:

$$\begin{aligned} j'_{A,0} &= -\varphi^\dagger \beta^{[0]} \varphi, \\ j'_{A,1} &= -\varphi^\dagger (\beta^{[1]} \cos(\alpha) - \beta^{[2]} \sin(\alpha)) \varphi; \\ j'_{A,2} &= -\varphi^\dagger (\beta^{[1]} \sin(\alpha) + \beta^{[2]} \cos(\alpha)) \varphi; \\ j'_{A,3} &= -\varphi^\dagger \beta^{[3]} \varphi. \end{aligned} \quad (2)$$

Hence if for φ' :

$$\begin{aligned} j'_{A,0} &= -\varphi'^\dagger \beta^{[0]} \varphi', \\ j'_{A,1} &= -\varphi'^\dagger \beta^{[1]} \varphi'; \\ j'_{A,2} &= -\varphi'^\dagger \beta^{[2]} \varphi'; \\ j'_{A,3} &= -\varphi'^\dagger \beta^{[3]} \varphi', \end{aligned}$$

and

$$\varphi' := U_{1,2}(\alpha) \varphi$$

then

$$\begin{aligned} U_{1,2}^\dagger(\alpha) \beta^{[0]} U_{1,2}(\alpha) &= \beta^{[0]}, \\ U_{1,2}^\dagger(\alpha) \beta^{[1]} U_{1,2}(\alpha) &= \beta^{[1]} \cos \alpha - \beta^{[2]} \sin \alpha; \\ U_{1,2}^\dagger(\alpha) \beta^{[2]} U_{1,2}(\alpha) &= \beta^{[2]} \cos \alpha + \beta^{[1]} \sin \alpha; \\ U_{1,2}^\dagger(\alpha) \beta^{[3]} U_{1,2}(\alpha) &= \beta^{[3]}; \end{aligned} \quad (3)$$

from [2, p. 62]: because

$$\rho_A = \varphi^\dagger \varphi = \varphi'^\dagger \varphi',$$

then

$$U_{1,2}^\dagger(\alpha) U_{1,2}(\alpha) = 1_4. \quad (4)$$

If

$$U_{1,2}(\alpha) := \cos \frac{\alpha}{2} \cdot 1_4 - \sin \frac{\alpha}{2} \cdot \beta^{[1]} \beta^{[2]}$$

i.e.:

$$U_{1,2}(\alpha) = \begin{bmatrix} e^{-i\frac{1}{2}\alpha} & 0 & 0 & 0 \\ 0 & e^{i\frac{1}{2}\alpha} & 0 & 0 \\ 0 & 0 & e^{-i\frac{1}{2}\alpha} & 0 \\ 0 & 0 & 0 & e^{i\frac{1}{2}\alpha} \end{bmatrix} \quad (5)$$

then $U_{1,2}(\alpha)$ fulfils to all these conditions (3), (4).

Then let

$$\begin{aligned} x'_0 &:= x_0, \\ x'_1 &:= x_1 \cos(\alpha) - x_3 \sin(\alpha), \\ x'_2 &:= x_2, \\ x'_3 &:= x_1 \sin(\alpha) + x_3 \cos(\alpha). \end{aligned} \quad (6)$$

Let

$$U_{1,3}(\alpha) := \cos \frac{\alpha}{2} \cdot 1_4 - \sin \frac{\alpha}{2} \cdot \beta^{[1]} \beta^{[3]}.$$

In this case:

$$U_{1,3}(\alpha) = \begin{bmatrix} \cos \frac{1}{2}\alpha & \sin \frac{1}{2}\alpha & 0 & 0 \\ -\sin \frac{1}{2}\alpha & \cos \frac{1}{2}\alpha & 0 & 0 \\ 0 & 0 & \cos \frac{1}{2}\alpha & \sin \frac{1}{2}\alpha \\ 0 & 0 & -\sin \frac{1}{2}\alpha & \cos \frac{1}{2}\alpha \end{bmatrix} \quad (7)$$

and

$$\begin{aligned} U_{1,3}^\dagger(\alpha) \beta^{[0]} U_{1,3}(\alpha) &= \beta^{[0]}, \\ U_{1,3}^\dagger(\alpha) \beta^{[1]} U_{1,3}(\alpha) &= \beta^{[1]} \cos \alpha - \beta^{[3]} \sin \alpha, \\ U_{1,3}^\dagger(\alpha) \beta^{[2]} U_{1,3}(\alpha) &= \beta^{[2]}, \\ U_{1,3}^\dagger(\alpha) \beta^{[3]} U_{1,3}(\alpha) &= \beta^{[3]} \cos \alpha + \beta^{[1]} \sin \alpha. \end{aligned} \quad (8)$$

If

$$\varphi' := U_{1,3}(\alpha) \varphi$$

and

$$j'_{A,k} := \varphi'^{\dagger} \beta^{[k]} \varphi'$$

where $(k \in \{0, 1, 2, 3\})$ then

$$j'_{A,0} = j_{A,0}, \tag{9}$$

$$j'_{A,1} = j_{A,1} \cos \alpha - j_{A,3} \sin \alpha, \tag{10}$$

$$j'_{A,2} = j_{A,2},$$

$$j'_{A,3} = j_{A,3} \cos \alpha + j_{A,1} \sin \alpha.$$

Then let

$$\begin{aligned} x'_0 &:= x_0, \\ x'_1 &:= x_1, \\ x'_2 &= \cos \alpha \cdot x_2 + \sin \alpha \cdot x_3, \\ x'_3 &= \cos \alpha \cdot x_3 - \sin \alpha \cdot x_2. \end{aligned} \tag{11}$$

Let

$$U_{3,2}(\alpha) = \cos \frac{\alpha}{2} \cdot 1_4 - \sin \frac{\alpha}{2} \cdot \beta^{[3]} \beta^{[2]}$$

In this case:

$$U_{3,2}(\alpha) = \begin{bmatrix} \cos \frac{1}{2}\alpha & i \sin \frac{1}{2}\alpha & 0 & 0 \\ i \sin \frac{1}{2}\alpha & \cos \frac{1}{2}\alpha & 0 & 0 \\ 0 & 0 & \cos \frac{1}{2}\alpha & i \sin \frac{1}{2}\alpha \\ 0 & 0 & i \sin \frac{1}{2}\alpha & \cos \frac{1}{2}\alpha \end{bmatrix}, \tag{12}$$

and

$$\begin{aligned} U_{3,2}^{\dagger}(\alpha) \beta^{[0]} U_{3,2}(\alpha) &= \beta^{[0]}, \\ U_{3,2}^{\dagger}(\alpha) \beta^{[1]} U_{3,2}(\alpha) &= \beta^{[1]}, \\ U_{3,2}^{\dagger}(\alpha) \beta^{[0]} U_{3,2}(\alpha) &= \beta^{[0]} \cos \alpha + \beta^{[3]} \sin \alpha, \\ U_{3,2}^{\dagger}(\alpha) \beta^{[3]} U_{3,2}(\alpha) &= \beta^{[3]} \cos \alpha - \beta^{[2]} \sin \alpha \end{aligned} \tag{13}$$

If

$$\varphi' := U_{3,2}(\alpha) \varphi$$

and

$$j'_{A,k} := \varphi'^{\dagger} \beta^{[k]} \varphi'$$

where $(k \in \{0, 1, 2, 3\})$ then

$$\begin{aligned} j'_{A,0} &= j_{A,0}, \\ j'_{A,1} &= j_{A,1}, \\ j'_{A,2} &= j_{A,2} \cos \alpha + j_{A,3} \sin \alpha, \\ j'_{A,3} &= j_{A,3} \cos \alpha - j_{A,1} \sin \alpha. \end{aligned} \tag{14}$$

2 Lorentzian rotation

Let v be any real number such that $-1 < v < 1$.

And let:

$$\alpha := \frac{1}{2} \ln \frac{1-v}{1+v}.$$

In this case:

$$\begin{aligned} \cosh \alpha &= \frac{1}{\sqrt{1-v^2}}, \\ \sinh \alpha &= -\frac{v}{\sqrt{1-v^2}}. \end{aligned} \tag{15}$$

Let

$$\begin{aligned} x'_0 &:= x_0 \cosh \alpha - x_1 \sinh \alpha, \\ x'_1 &:= x_1 \cosh \alpha - x_0 \sinh \alpha, \\ x'_2 &:= x_2, \\ x'_3 &:= x_3. \end{aligned} \tag{16}$$

Let

$$U_{1,0}(\alpha) = \cosh \frac{\alpha}{2} \cdot 1_4 - \sinh \frac{\alpha}{2} \cdot \beta^{[1]} \beta^{[0]}.$$

That is:

$$U_{1,0}(\alpha) := \begin{bmatrix} \cosh \frac{1}{2}\alpha & \sinh \frac{1}{2}\alpha & 0 & 0 \\ \sinh \frac{1}{2}\alpha & \cosh \frac{1}{2}\alpha & 0 & 0 \\ 0 & 0 & \cosh \frac{1}{2}\alpha & -\sinh \frac{1}{2}\alpha \\ 0 & 0 & -\sinh \frac{1}{2}\alpha & \cosh \frac{1}{2}\alpha \end{bmatrix}. \tag{17}$$

In this case:

$$\begin{aligned} U_{1,0}^{\dagger}(\alpha) \beta^{[0]} U_{1,0}(\alpha) &= \beta^{[0]} \cosh \alpha - \beta^{[1]} \sinh \alpha, \\ U_{1,0}^{\dagger}(\alpha) \beta^{[1]} U_{1,0}(\alpha) &= \beta^{[1]} \cosh \alpha - \beta^{[0]} \sinh \alpha, \\ U_{1,0}^{\dagger}(\alpha) \beta^{[2]} U_{1,0}(\alpha) &= \beta^{[2]}, \\ U_{1,0}^{\dagger}(\alpha) \beta^{[3]} U_{1,0}(\alpha) &= \beta^{[3]}. \end{aligned} \tag{18}$$

If

$$\varphi' := U_{1,0}(\alpha) \varphi$$

and

$$j'_{A,k} := \varphi'^{\dagger} \beta^{[k]} \varphi'$$

where $(k \in \{0, 1, 2, 3\})$ then

$$\begin{aligned} j'_{A,0} &= j_{A,0} \cosh \alpha - j_{A,1} \sinh \alpha, \\ j'_{A,1} &= j_{A,1} \cosh \alpha - j_{A,0} \sinh \alpha, \\ j'_{A,2} &= j_{A,2}, \\ j'_{A,3} &= j_{A,3}. \end{aligned} \tag{19}$$

Then let

$$\begin{aligned} x'_0 &:= x_0 \cosh \alpha - x_2 \sinh \alpha, \\ x'_1 &:= x_1, \\ x'_2 &:= x_2 \cosh \alpha - x_0 \sinh \alpha, \\ x'_3 &:= x_3. \end{aligned} \tag{20}$$

Let

$$U_{2,0}(\alpha) := \cosh \frac{\alpha}{2} \cdot 1_4 - \sinh \frac{\alpha}{2} \cdot \beta^{[2]}\beta^{[0]}. \quad (21)$$

That is:

$$U_{2,0}(\alpha) = \begin{bmatrix} \cosh \frac{1}{2}\alpha & -i \sinh \frac{1}{2}\alpha & 0 & 0 \\ i \sinh \frac{1}{2}\alpha & \cosh \frac{1}{2}\alpha & 0 & 0 \\ 0 & 0 & \cosh \frac{1}{2}\alpha & i \sinh \frac{1}{2}\alpha \\ 0 & 0 & -i \sinh \frac{1}{2}\alpha & \cosh \frac{1}{2}\alpha \end{bmatrix}.$$

In this case:

$$\begin{aligned} U_{2,0}^\dagger(\alpha)\beta^{[0]}U_{2,0}(\alpha) &= \beta^{[0]}\cosh\alpha - \beta^{[2]}\sinh\alpha, \quad (22) \\ U_{2,0}^\dagger(\alpha)\beta^{[1]}U_{2,0}(\alpha) &= \beta^{[1]}, \\ U_{2,0}^\dagger(\alpha)\beta^{[2]}U_{2,0}(\alpha) &= \beta^{[2]}\cosh\alpha - \beta^{[0]}\sinh\alpha, \\ U_{2,0}^\dagger(\alpha)\beta^{[3]}U_{2,0}(\alpha) &= \beta^{[3]}. \end{aligned}$$

If

$$\varphi' := U_{2,0}(\alpha)\varphi$$

and

$$j'_{A,k} := \varphi'^\dagger \beta^{[k]} \varphi'$$

where $(k \in \{0, 1, 2, 3\})$ then

$$\begin{aligned} j'_{A,0} &= j_{A,0} \cosh\alpha - j_{A,1} \sinh\alpha, \quad (23) \\ j'_{A,1} &= j_{A,1}, \\ j'_{A,2} &= j_{A,2} \cosh\alpha - j_{A,0} \sinh\alpha, \\ j'_{A,3} &= j_{A,3}. \end{aligned}$$

Then let

$$\begin{aligned} x'_0 &:= x_0 \cosh\alpha - x_3 \sinh\alpha, \quad (24) \\ x'_1 &:= x_1, \\ x'_2 &:= x_2, \\ x'_3 &:= x_3 \cosh\alpha - x_0 \sinh\alpha. \end{aligned}$$

Let

$$U_{3,0}(\alpha) := \cosh \frac{\alpha}{2} \cdot 1_4 - \sinh \frac{\alpha}{2} \cdot \beta^{[3]}\beta^{[0]}.$$

That is:

$$U_{3,0}(\alpha) = \begin{bmatrix} e^{\frac{1}{2}\alpha} & 0 & 0 & 0 \\ 0 & e^{-\frac{1}{2}\alpha} & 0 & 0 \\ 0 & 0 & e^{-\frac{1}{2}\alpha} & 0 \\ 0 & 0 & 0 & e^{\frac{1}{2}\alpha} \end{bmatrix}. \quad (25)$$

In this case:

$$\begin{aligned} U_{3,0}^\dagger(\alpha)\beta^{[0]}U_{3,0}(\alpha) &= \beta^{[0]}\cosh\alpha - \beta^{[3]}\sinh\alpha, \quad (26) \\ U_{3,0}^\dagger(\alpha)\beta^{[1]}U_{3,0}(\alpha) &= \beta^{[1]}, \\ U_{3,0}^\dagger(\alpha)\beta^{[2]}U_{3,0}(\alpha) &= \beta^{[2]}, \\ U_{3,0}^\dagger(\alpha)\beta^{[3]}U_{3,0}(\alpha) &= \beta^{[3]}\cosh\alpha - \beta^{[0]}\sinh\alpha. \end{aligned}$$

If

$$\varphi' := U_{3,0}(\alpha)\varphi$$

and

$$j'_{A,k} := \varphi'^\dagger \beta^{[k]} \varphi'$$

where $(k \in \{0, 1, 2, 3\})$ then

$$\begin{aligned} j'_{A,0} &= j_{A,0} \cosh\alpha - j_{A,3} \sinh\alpha, \quad (27) \\ j'_{A,1} &= j_{A,1}, \\ j'_{A,2} &= j_{A,2}, \\ j'_{A,3} &= j_{A,3} \cosh\alpha - j_{A,0} \sinh\alpha. \end{aligned}$$

3 Equation of motion

Function φ submits to the following equation [2, p. 82]:

$$\begin{aligned} \frac{1}{c} \partial_t \varphi - (i\Theta_0 \beta^{[0]} + i\Upsilon_0 \beta^{[0]}\gamma^{[5]}) \varphi = \\ = \left(\sum_{v=1}^3 \beta^{[v]} (\partial_v + i\Theta_v + i\Upsilon_v \gamma^{[5]}) + \right. \\ \left. + iM_0 \gamma^{[0]} + iM_4 \beta^{[4]} - \right. \\ \left. - iM_{\zeta,0} \gamma_{\zeta}^{[0]} + iM_{\zeta,4} \zeta^{[4]} - \right. \\ \left. - iM_{\eta,0} \gamma_{\eta}^{[0]} - iM_{\eta,4} \eta^{[4]} + \right. \\ \left. + iM_{\theta,0} \gamma_{\theta}^{[0]} + iM_{\theta,4} \theta^{[4]} \right) \varphi. \end{aligned}$$

That is:

$$\begin{aligned} \left(\sum_{v=0}^3 \beta^{[v]} (\partial_v + i\Theta_v + i\Upsilon_v \gamma^{[5]}) + \right. \\ \left. + iM_0 \gamma^{[0]} + iM_4 \beta^{[4]} - \right. \\ \left. - iM_{\zeta,0} \gamma_{\zeta}^{[0]} + iM_{\zeta,4} \zeta^{[4]} - \right. \\ \left. - iM_{\eta,0} \gamma_{\eta}^{[0]} - iM_{\eta,4} \eta^{[4]} + \right. \\ \left. + iM_{\theta,0} \gamma_{\theta}^{[0]} + iM_{\theta,4} \theta^{[4]} \right) \varphi = 0. \quad (28) \end{aligned}$$

Like coordinates x_5 and x_4 [2, p. 83] here are entered new coordinates $y^\beta, z^\beta, y^\zeta, z^\zeta, y^\eta, z^\eta, y^\theta, z^\theta$ such that

$$\begin{aligned} -\frac{\pi c}{h} \leq y^\beta \leq \frac{\pi c}{h}, -\frac{\pi c}{h} \leq z^\beta \leq \frac{\pi c}{h}, \\ -\frac{\pi c}{h} \leq y^\zeta \leq \frac{\pi c}{h}, -\frac{\pi c}{h} \leq z^\zeta \leq \frac{\pi c}{h}, \\ -\frac{\pi c}{h} \leq y^\eta \leq \frac{\pi c}{h}, -\frac{\pi c}{h} \leq z^\eta \leq \frac{\pi c}{h}, \\ -\frac{\pi c}{h} \leq y^\theta \leq \frac{\pi c}{h}, -\frac{\pi c}{h} \leq z^\theta \leq \frac{\pi c}{h}. \end{aligned}$$

and like $\tilde{\varphi}$, [2, p. 83] let:

$$\begin{aligned} [\varphi](t, \mathbf{x}, y^\beta, z^\beta, y^\zeta, z^\zeta, y^\eta, z^\eta, y^\theta, z^\theta) := \quad (29) \\ := \varphi(t, \mathbf{x}) \times \exp\left(i(y^\beta M_0 + z^\beta M_4 + y^\zeta M_{\zeta,0} + z^\zeta M_{\zeta,4} + \right. \\ \left. + y^\eta M_{\eta,0} + z^\eta M_{\eta,4} + y^\theta M_{\theta,0} + z^\theta M_{\theta,4})\right). \end{aligned}$$

In this case if

$$\begin{aligned}
 &([\varphi], [\chi]) := \\
 &:= \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dy^\beta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dz^\beta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dy^\zeta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dz^\zeta \times \\
 &\times \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dy^\eta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dz^\eta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dy^\theta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dz^\theta \times \\
 &\quad \times [\varphi]^\dagger [\chi]
 \end{aligned} \tag{30}$$

then

$$\begin{aligned}
 ([\varphi], [\varphi]) &= \rho_{\mathcal{A}}, \\
 ([\varphi], \beta^{[s]} [\varphi]) &= -\frac{j_{\mathcal{A},k}}{c},
 \end{aligned} \tag{31}$$

and in this case from (28):

$$\begin{aligned}
 &\left(\sum_{\nu=0}^3 \beta^{[\nu]} (\partial_\nu + i\Theta_\nu + i\Upsilon_\nu \gamma^{[5]}) + \right. \\
 &\quad + \gamma^{[0]} \partial_y^\beta + \beta^{[4]} \partial_z^\beta - \\
 &\quad - \gamma_\zeta^{[0]} \partial_y^\zeta + \zeta^{[4]} \partial_z^\zeta - \\
 &\quad - \gamma_\eta^{[0]} \partial_y^\eta - \eta^{[4]} \partial_z^\eta + \\
 &\quad \left. + \gamma_\theta^{[0]} \partial_y^\theta + \theta^{[4]} \partial_z^\theta \right) [\varphi] = 0.
 \end{aligned} \tag{32}$$

Because

$$\gamma_\eta^{[0]} = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, \eta^{[4]} = i \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}; \tag{33}$$

$$\gamma_\theta^{[0]} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \theta^{[4]} = i \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}; \tag{34}$$

$$\gamma_\zeta^{[0]} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \zeta^{[4]} = i \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}; \tag{35}$$

then from (32):

$$\begin{aligned}
 &\sum_{\nu=0}^3 \beta^{[\nu]} (\partial_\nu + i\Theta_\nu + i\Upsilon_\nu \gamma^{[5]}) [\varphi] + \\
 &\quad + \gamma^{[0]} \partial_y^\beta [\varphi] + \beta^{[4]} \partial_z^\beta [\varphi] + \\
 &\quad + \left(\begin{bmatrix} 0 & 0 & -\partial_y^\theta & \partial_y^\zeta - i\partial_y^\eta \\ 0 & 0 & \partial_y^\zeta + i\partial_y^\eta & \partial_y^\theta \\ -\partial_y^\theta & \partial_y^\zeta - i\partial_y^\eta & 0 & 0 \\ \partial_y^\zeta + i\partial_y^\eta & \partial_y^\theta & 0 & 0 \end{bmatrix} + \right. \\
 &\quad \left. i \begin{bmatrix} 0 & 0 & \partial_z^\theta & \partial_z^\zeta + i\partial_z^\eta \\ 0 & 0 & \partial_z^\zeta - i\partial_z^\eta & -\partial_z^\theta \\ -\partial_z^\theta & -\partial_z^\zeta - i\partial_z^\eta & 0 & 0 \\ -\partial_z^\zeta + i\partial_z^\eta & \partial_z^\theta & 0 & 0 \end{bmatrix} \right) \\
 &\quad \times [\varphi] = 0.
 \end{aligned} \tag{36}$$

Let a Fourier transformation of

$$[\varphi] (t, \mathbf{x}, y^\beta, z^\beta, y^\zeta, z^\zeta, y^\eta, z^\eta, y^\theta, z^\theta)$$

be the following:

$$\begin{aligned}
 &[\varphi] (t, \mathbf{x}, y^\beta, z^\beta, y^\zeta, z^\zeta, y^\eta, z^\eta, y^\theta, z^\theta) = \\
 &= \sum_{w, p_1, p_2, p_3, n^\beta, s^\beta, n^\zeta, s^\zeta, n^\eta, s^\eta, n^\theta, s^\theta} c(w, p_1, p_2, p_3, n^\beta, s^\beta, \\
 &\quad n^\zeta, s^\zeta, n^\eta, s^\eta, n^\theta, s^\theta) \times \\
 &\quad \times \exp \left(-i \frac{h}{c} (wx_0 + p_1 x_1 + p_2 x_2 + p_3 x_3 + \right. \\
 &\quad + n^\beta y^\beta + s^\beta z^\beta + n^\zeta y^\zeta + s^\zeta z^\zeta + \\
 &\quad \left. + n^\eta y^\eta + s^\eta z^\eta + n^\theta y^\theta + s^\theta z^\theta) \right).
 \end{aligned} \tag{37}$$

Let in (36) $\Theta_\nu = 0$ and $\Upsilon_\nu = 0$.

Let us design:

$$\begin{aligned}
 G_0 := &\left(\sum_{\nu=0}^3 \beta^{[\nu]} \partial_\nu + \gamma^{[0]} \partial_y^\beta + \beta^{[4]} \partial_z^\beta - \right. \\
 &\quad - \gamma_\zeta^{[0]} \partial_y^\zeta + \zeta^{[4]} \partial_z^\zeta - \\
 &\quad - \gamma_\eta^{[0]} \partial_y^\eta - \eta^{[4]} \partial_z^\eta + \\
 &\quad \left. + \gamma_\theta^{[0]} \partial_y^\theta + \theta^{[4]} \partial_z^\theta \right).
 \end{aligned} \tag{38}$$

that is:

$$\begin{aligned}
 &G_0 = \\
 &\begin{bmatrix} -\partial_0 + \partial_3 & \partial_1 - i\partial_2 & \partial_y^\beta - \partial_y^\theta & \partial_y^\zeta - i\partial_y^\eta \\ \partial_1 + i\partial_2 & -\partial_0 - \partial_3 & \partial_y^\zeta + i\partial_y^\eta & \partial_y^\beta + \partial_y^\theta \\ \partial_y^\beta - \partial_y^\theta & \partial_y^\zeta - i\partial_y^\eta & -\partial_0 - \partial_3 & -\partial_1 + i\partial_2 \\ \partial_y^\zeta + i\partial_y^\eta & \partial_y^\beta + \partial_y^\theta & -\partial_1 - i\partial_2 & -\partial_0 + \partial_3 \end{bmatrix} \\
 &+ i \begin{bmatrix} 0 & 0 & \partial_z^\beta + \partial_z^\theta & \partial_z^\zeta + i\partial_z^\eta \\ 0 & 0 & \partial_z^\zeta - i\partial_z^\eta & \partial_z^\beta - \partial_z^\theta \\ -\partial_z^\beta - \partial_z^\theta & -\partial_z^\zeta - i\partial_z^\eta & 0 & 0 \\ -\partial_z^\zeta + i\partial_z^\eta & -\partial_z^\beta + \partial_z^\theta & 0 & 0 \end{bmatrix}
 \end{aligned} \tag{39}$$

$$\begin{aligned}
 G_0 [\varphi] = &-i \frac{h}{c} \sum_{w, p_1, p_2, p_3, n^\beta, s^\beta, n^\zeta, s^\zeta, n^\eta, s^\eta, n^\theta, s^\theta} \check{g}(w, \\
 &p_1, p_2, p_3, n^\beta, s^\beta, n^\zeta, s^\zeta, n^\eta, s^\eta, n^\theta, s^\theta) \\
 &\sum_{k=0}^3 c_k(w, p_1, p_2, p_3, n^\beta, s^\beta, n^\zeta, s^\zeta, n^\eta, s^\eta, n^\theta, s^\theta) \times \\
 &\times \exp \left(-i \frac{h}{c} (wx_0 + p_1 x_1 + p_2 x_2 + p_3 x_3 + \right. \\
 &\quad + n^\beta y^\beta + s^\beta z^\beta + n^\zeta y^\zeta + s^\zeta z^\zeta + \\
 &\quad \left. + n^\eta y^\eta + s^\eta z^\eta + n^\theta y^\theta + s^\theta z^\theta) \right).
 \end{aligned} \tag{40}$$

Here

$$c_k(w, p_1, p_2, p_3, n^\beta, s^\beta, n^\zeta, s^\zeta, n^\eta, s^\eta, n^\theta, s^\theta)$$

is an eigenvector of

$$\check{g}(w, p_1, p_2, p_3, n^\beta, s^\beta, n^\zeta, s^\zeta, n^\eta, s^\eta, n^\theta, s^\theta)$$

and

$$\begin{aligned} \check{g}(w, p_1, p_2, p_3, n^\beta, s^\beta, n^\zeta, s^\zeta, n^\eta, s^\eta, n^\theta, s^\theta) := & (41) \\ := & \beta^{[0]}w + \beta^{[1]}p_1 + \beta^{[2]}p_2 + \beta^{[3]}p_3 + \\ & + \gamma^{[0]}n^\beta + \beta^{[4]}s^\beta - \gamma_\zeta^{[0]}n^\zeta + \zeta^{[4]}s^\zeta - \\ & - \gamma_\eta^{[0]}n^\eta - \eta^{[4]}s^\eta + \gamma_\theta^{[0]}n^\theta + \theta^{[4]}s^\theta. \end{aligned}$$

Here

$$\{c_0, c_1, c_2, c_3\}$$

is an orthonormalized basis of the complex4-vectors space.

Functions

$$\begin{aligned} c_k(w, p_1, p_2, p_3, n^\beta, s^\beta, n^\zeta, s^\zeta, n^\eta, s^\eta, n^\theta, s^\theta) \times & (42) \\ \times \exp \left(-i \frac{\hbar}{c} (wx_0 + p_1x_1 + p_2x_2 + p_3x_3 + \right. & \\ \left. + n^\beta y^\beta + s^\beta z^\beta + \right. & \\ \left. + n^\zeta y^\zeta + s^\zeta z^\zeta + n^\eta y^\eta + s^\eta z^\eta + n^\theta y^\theta + s^\theta z^\theta) \right) & \end{aligned}$$

are eigenvectors of operator G_0 .

4 Chromes under Lorentz's and Cartesian transformations

$$\varphi_y^\zeta := c(w, \mathbf{p}, f) \exp \left(-i \frac{\hbar}{c} (wx_0 + \mathbf{p}\mathbf{x} + \gamma_\zeta^{[0]} f y^\zeta) \right)$$

is a red lower chrome function,

$$\varphi_z^\zeta := c(w, \mathbf{p}, f) \exp \left(-i \frac{\hbar}{c} (wx_0 + \mathbf{p}\mathbf{x} - i\zeta^{[4]} f z^\zeta) \right)$$

is a red upper chrome function,

$$\varphi_y^\eta := c(w, \mathbf{p}, f) \exp \left(-i \frac{\hbar}{c} (wx_0 + \mathbf{p}\mathbf{x} + \gamma_\eta^{[0]} f y^\eta) \right)$$

is a green lower chrome function,

$$\varphi_z^\eta := c(w, \mathbf{p}, f) \exp \left(-i \frac{\hbar}{c} (wx_0 + \mathbf{p}\mathbf{x} - i\eta^{[4]} f z^\eta) \right)$$

is a green upper chrome function,

$$\varphi_y^\theta := c(w, \mathbf{p}, f) \exp \left(-i \frac{\hbar}{c} (wx_0 + \mathbf{p}\mathbf{x} + \gamma_\theta^{[0]} f y^\theta) \right)$$

is a blue lower chrome function,

$$\varphi_z^\theta := c(w, \mathbf{p}, s^\theta) \exp \left(-i \frac{\hbar}{c} (wx_0 + \mathbf{p}\mathbf{x} - i\theta^{[4]} f z^\theta) \right)$$

is a blue upper chrome function.

Operator $-\partial_y^\zeta \partial_y^\zeta$ is called a red lower chrome operator, $-\partial_z^\zeta \partial_z^\zeta$ is a red upper chrome operator, $-\partial_y^\eta \partial_y^\eta$ is called a green lower chrome operator, $-\partial_z^\eta \partial_z^\eta$ is a green upper chrome operator, $-\partial_y^\theta \partial_y^\theta$ is called a blue lower chrome operator, $-\partial_z^\theta \partial_z^\theta$ is a blue upper chrome operator.

For example, if φ_z^ζ is a red upper chrome function then

$$\begin{aligned} -\partial_y^\zeta \partial_y^\zeta \varphi_z^\zeta &= -\partial_y^\eta \partial_y^\eta \varphi_z^\zeta = -\partial_z^\eta \partial_z^\eta \varphi_z^\zeta = \\ &= -\partial_y^\theta \partial_y^\theta \varphi_z^\zeta = -\partial_z^\theta \partial_z^\theta \varphi_z^\zeta = 0 \end{aligned}$$

but

$$-\partial_z^\zeta \partial_z^\zeta \varphi_z^\zeta = -\left(\frac{\hbar}{c} f\right)^2 \varphi_z^\zeta.$$

Because

$$G_0[\varphi] = 0$$

then

$$UG_0U^{-1}U[\varphi] = 0.$$

If $U = U_{1,2}(\alpha)$ then $G_0 \rightarrow U_{1,2}(\alpha)G_0U_{1,2}^{-1}(\alpha)$ and $[\varphi] \rightarrow U_{1,2}(\alpha)[\varphi]$.

In this case:

$$\partial_1 \rightarrow \partial'_1 := (\cos \alpha \cdot \partial_1 - \sin \alpha \cdot \partial_2),$$

$$\partial_2 \rightarrow \partial'_2 := (\cos \alpha \cdot \partial_2 + \sin \alpha \cdot \partial_1),$$

$$\partial_0 \rightarrow \partial'_0 := \partial_0,$$

$$\partial_3 \rightarrow \partial'_3 := \partial_3,$$

$$\partial_y^\beta \rightarrow \partial_y^{\beta'} := \partial_y^\beta,$$

$$\partial_z^\beta \rightarrow \partial_z^{\beta'} := \partial_z^\beta,$$

$$\partial_y^\zeta \rightarrow \partial_y^{\zeta'} := (\cos \alpha \cdot \partial_y^\zeta - \sin \alpha \cdot \partial_y^\eta),$$

$$\partial_y^\eta \rightarrow \partial_y^{\eta'} := (\cos \alpha \cdot \partial_y^\eta + \sin \alpha \cdot \partial_y^\zeta),$$

$$\partial_z^\zeta \rightarrow \partial_z^{\zeta'} := (\cos \alpha \cdot \partial_z^\zeta + \sin \alpha \cdot \partial_z^\eta),$$

$$\partial_z^\eta \rightarrow \partial_z^{\eta'} := (\cos \alpha \cdot \partial_z^\eta - \sin \alpha \cdot \partial_z^\zeta),$$

$$\partial_y^\theta \rightarrow \partial_y^{\theta'} := \partial_y^\theta,$$

$$\partial_z^\theta \rightarrow \partial_z^{\theta'} := \partial_z^\theta.$$

Therefore,

$$-\partial_z^{\zeta'} \partial_z^{\zeta'} \varphi_z^\zeta = \left(f \frac{\hbar}{c} \cos \alpha\right)^2 \cdot \varphi_z^\zeta,$$

$$-\partial_z^{\eta'} \partial_z^{\eta'} \varphi_z^\zeta = \left(-\sin \alpha \cdot f \frac{\hbar}{c}\right)^2 \varphi_z^\zeta.$$

If $\alpha = -\frac{\pi}{2}$ then

$$-\partial_z^{\zeta'} \partial_z^{\zeta'} \varphi_z^\zeta = 0,$$

$$-\partial_z^{\eta'} \partial_z^{\eta'} \varphi_z^\zeta = \left(f \frac{\hbar}{c}\right)^2 \varphi_z^\zeta.$$

That is under such rotation the red state becomes the green state.

If $U = U_{3,2}(\alpha)$ then $G_0 \rightarrow U_{3,2}(\alpha) G_0 U_{3,2}^{-1}(\alpha)$ and $[\varphi] \rightarrow U_{3,2}(\alpha) [\varphi]$.

In this case:

$$\begin{aligned} \partial_0 &\rightarrow \partial'_0 := \partial_0, \\ \partial_1 &\rightarrow \partial'_1 := \partial_1, \\ \partial_2 &\rightarrow \partial'_2 := (\cos \alpha \cdot \partial_2 + \sin \alpha \cdot \partial_3), \\ \partial_3 &\rightarrow \partial'_3 := (\cos \alpha \cdot \partial_3 - \sin \alpha \cdot \partial_2), \\ \partial_y^\beta &\rightarrow \partial_{y'}^\beta := \partial_y^\beta, \\ \partial_y^\zeta &\rightarrow \partial_{y'}^\zeta := \partial_y^\zeta, \\ \partial_y^\eta &\rightarrow \partial_{y'}^\eta := (\cos \alpha \cdot \partial_y^\eta - \sin \alpha \cdot \partial_y^\theta), \\ \partial_y^\theta &\rightarrow \partial_{y'}^\theta := (\cos \alpha \cdot \partial_y^\theta + \sin \alpha \cdot \partial_y^\eta), \\ \partial_z^\beta &\rightarrow \partial_{z'}^\beta := \partial_z^\beta, \\ \partial_z^\zeta &\rightarrow \partial_{z'}^\zeta := \partial_z^\zeta, \\ \partial_z^\eta &\rightarrow \partial_{z'}^\eta := (\cos \alpha \cdot \partial_z^\eta - \sin \alpha \cdot \partial_z^\theta), \\ \partial_z^\theta &\rightarrow \partial_{z'}^\theta := (\cos \alpha \cdot \partial_z^\theta + \sin \alpha \cdot \partial_z^\eta). \end{aligned}$$

Therefore, if φ_y^η is a green lower chrome function then

$$\begin{aligned} -\partial_{z'}^{\eta'} \partial_{z'}^{\eta''} \varphi_y^\eta &= \left(\frac{\hbar}{c} \cos \alpha \cdot f\right)^2 \cdot \varphi_y^\eta, \\ -\partial_{y'}^{\theta'} \partial_{y'}^{\theta''} \varphi_y^\eta &= \left(\frac{\hbar}{c} \sin \alpha \cdot f\right)^2 \cdot \varphi_y^\eta. \end{aligned}$$

If $\alpha = \pi/2$ then

$$\begin{aligned} -\partial_{z'}^{\eta'} \partial_{z'}^{\eta''} \varphi_y^\eta &= 0, \\ -\partial_{y'}^{\theta'} \partial_{y'}^{\theta''} \varphi_y^\eta &= \left(\frac{\hbar}{c} f\right)^2 \cdot \varphi_y^\eta. \end{aligned}$$

That is under such rotation the green state becomes blue state.

If $U = U_{3,1}(\alpha)$ then $G_0 \rightarrow U_{3,1}(\alpha) G_0 U_{3,1}^{-1}(\alpha)$ and $[\varphi] \rightarrow U_{3,1}(\alpha) [\varphi]$.

In this case:

$$\begin{aligned} \partial_0 &\rightarrow \partial'_0 := \partial_0, \\ \partial_1 &\rightarrow \partial'_1 := (\cos \alpha \cdot \partial_1 - \sin \alpha \cdot \partial_3), \\ \partial_2 &\rightarrow \partial'_2 := \partial_2, \\ \partial_3 &\rightarrow \partial'_3 := (\cos \alpha \cdot \partial_3 + \sin \alpha \cdot \partial_1), \\ \partial_y^\beta &\rightarrow \partial_{y'}^\beta := \partial_y^\beta, \\ \partial_y^\zeta &\rightarrow \partial_{y'}^\zeta := (\cos \alpha \cdot \partial_y^\zeta + \sin \alpha \cdot \partial_y^\theta), \\ \partial_y^\eta &\rightarrow \partial_{y'}^\eta := \partial_y^\eta, \\ \partial_y^\theta &\rightarrow \partial_{y'}^\theta := (\cos \alpha \cdot \partial_y^\theta - \sin \alpha \cdot \partial_y^\zeta), \\ \partial_z^\beta &\rightarrow \partial_{z'}^\beta := \partial_z^\beta, \\ \partial_z^\zeta &\rightarrow \partial_{z'}^\zeta := (\cos \alpha \cdot \partial_z^\zeta - \sin \alpha \cdot \partial_z^\theta), \\ \partial_z^\eta &\rightarrow \partial_{z'}^\eta := \partial_z^\eta, \\ \partial_z^\theta &\rightarrow \partial_{z'}^\theta := (\cos \alpha \cdot \partial_z^\theta + \sin \alpha \cdot \partial_z^\zeta). \end{aligned}$$

Therefore,

$$-\partial_{z'}^{\zeta'} \partial_{z'}^{\zeta''} \varphi_z^\zeta = -\left(f \frac{\hbar}{c} \cos \alpha\right)^2 \cdot \varphi_z^\zeta,$$

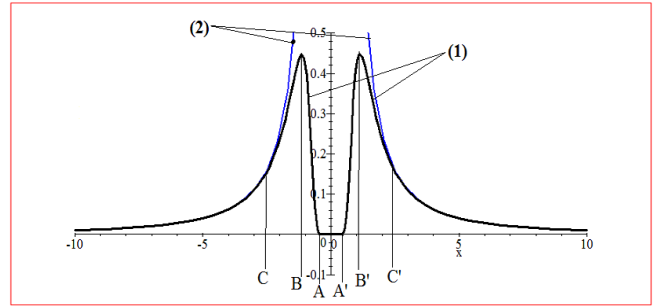


Fig. 1:

$$-\partial_{z'}^{\theta'} \partial_{z'}^{\theta''} \varphi_z^\zeta = -\left(\sin \alpha \cdot f \frac{\hbar}{c}\right)^2 \varphi_z^\zeta.$$

If $\alpha = \pi/2$ then

$$\begin{aligned} -\partial_{z'}^{\zeta'} \partial_{z'}^{\zeta''} \varphi_z^\zeta &= 0, \\ -\partial_{y'}^{\theta'} \partial_{y'}^{\theta''} \varphi_z^\zeta &= -\left(f \frac{\hbar}{c}\right)^2 \varphi_z^\zeta. \end{aligned}$$

That is under such rotation the red state becomes the blue state. Thus at the Cartesian turns chrome of a state is changed.

One of ways of elimination of this noninvariancy consists in the following. Calculations in [2, p. 156] give the grounds to assume that some oscillations of quarks states bend time-space in such a way that acceleration of the bent system in relation to initial system submits to the following law (Fig. 1):

$$g(t, \mathbf{x}) = c\lambda / (\mathbf{x}^2 \cosh^2(\lambda t / \mathbf{x}^2)).$$

Here the acceleration plot is line (1) and the line (2) is plot of λ / \mathbf{x}^2 .

Hence, to the right from point C' and to the left from point C the Newtonian gravitation law is carried out.

AA' is the Asymptotic Freedom Zone.

CB and $B'C'$ is the Confinement Zone.

Let in the potential hole AA' there are three quarks $\varphi_y^\zeta, \varphi_y^\eta, \varphi_y^\theta$. Their general state function is determinant with elements of the following type: $\varphi_y^{\zeta\eta\theta} := \varphi_y^\zeta \varphi_y^\eta \varphi_y^\theta$. In this case:

$$-\partial_{y'}^{\zeta'} \partial_{y'}^{\zeta''} \varphi_y^{\zeta\eta\theta} = \left(\frac{\hbar}{c} f\right)^2 \varphi_y^{\zeta\eta\theta}$$

and under rotation $U_{1,2}(\alpha)$:

$$\begin{aligned} -\partial_{y'}^{\zeta'} \partial_{y'}^{\zeta''} \varphi_y^{\zeta\eta\theta} &= \left(\frac{\hbar}{c} f\right)^2 (\gamma_\zeta^{[0]} \cos \alpha - \gamma_\eta^{[0]} \sin \alpha)^2 (\varphi_y^\zeta \varphi_y^\eta \varphi_y^\theta) \\ &= \left(\frac{\hbar}{c} f\right)^2 \varphi_y^{\zeta\eta\theta}. \end{aligned}$$

That is at such turns the quantity of red chrome remains.

As and for all other Cartesian turns and for all other chromes.

Baryons $\Delta^- = ddd$, $\Delta^{++} = uuu$, $\Omega^- = sss$ belong to such structures.

If $U = U_{1,0}(\alpha)$ then $G_0 \rightarrow U_{1,0}^{-1\dagger}(\alpha) G_0 U_{1,0}^{-1}(\alpha)$ and $[\varphi] \rightarrow U_{1,0}(\alpha) [\varphi]$.

In this case:

$$\partial_0 \rightarrow \partial'_0 := (\cosh \alpha \cdot \partial_0 + \sinh \alpha \cdot \partial_1),$$

$$\partial_1 \rightarrow \partial'_1 := (\cosh \alpha \cdot \partial_1 + \sinh \alpha \cdot \partial_0),$$

$$\partial_2 \rightarrow \partial'_2 := \partial_2,$$

$$\partial_3 \rightarrow \partial'_3 := \partial_3,$$

$$\partial_y^\beta \rightarrow \partial_y^{\beta'} := \partial_y^\beta,$$

$$\partial_y^\zeta \rightarrow \partial_y^{\zeta'} := \partial_y^\zeta,$$

$$\partial_y^\eta \rightarrow \partial_y^{\eta'} := (\cosh \alpha \cdot \partial_y^\eta - \sinh \alpha \cdot \partial_z^\eta),$$

$$\partial_y^\theta \rightarrow \partial_y^{\theta'} := (\cosh \alpha \cdot \partial_y^\theta + \sinh \alpha \cdot \partial_z^\theta),$$

$$\partial_z^\beta \rightarrow \partial_z^{\beta'} := \partial_z^\beta,$$

$$\partial_z^\zeta \rightarrow \partial_z^{\zeta'} := \partial_z^\zeta,$$

$$\partial_z^\eta \rightarrow \partial_z^{\eta'} := (\cosh \alpha \cdot \partial_z^\eta + \sinh \alpha \cdot \partial_y^\eta),$$

$$\partial_z^\theta \rightarrow \partial_z^{\theta'} := (\cosh \alpha \cdot \partial_z^\theta - \sinh \alpha \cdot \partial_y^\theta).$$

Therefore,

$$-\partial_y^{\eta'} \partial_y^{\eta'} \varphi_y^\eta = (1 + \sinh^2 \alpha) \cdot \left(\frac{\hbar}{c} f\right)^2 \varphi_y^\eta,$$

$$-\partial_z^{\theta'} \partial_z^{\theta'} \varphi_y^\eta = \sinh^2 \alpha \cdot \left(\frac{\hbar}{c} f\right)^2 \varphi_y^\eta.$$

Similarly chromes and grades change for other states and under other Lorentz transformation.

One of ways of elimination of this noninvariancy is the following:

Let

$$\varphi_{yz}^{\zeta\eta\theta} := \varphi_y^\zeta \varphi_y^\eta \varphi_y^\theta \varphi_z^\zeta \varphi_z^\eta \varphi_z^\theta,$$

Under transformation $U_{1,0}(\alpha)$:

$$-\partial_z^{\theta'} \partial_z^{\theta'} \varphi_{yz}^{\zeta\eta\theta} = -\left(\frac{\hbar}{c} f\right)^2 \varphi_{yz}^{\zeta\eta\theta}.$$

That is a magnitude of red chrome of this state doesn't depend on angle α .

This condition is satisfied for all chromes and under all Lorentz's transformations.

Pairs of baryons

$$\begin{aligned} \{p = uud, n = ddu\}, \\ \{\Sigma^+ = uus, \Xi^0 = uss\}, \\ \{\Delta^+ = uud, \Delta^0 = udd\} \end{aligned}$$

belong to such structures.

Conclusion

Baryons represent one of ways of elimination of the chrome noninvariancy under Cartesian and under Lorentz transformation.

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