# Scaling of Body Masses and Orbital Periods in the Solar System

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The paper shows that the sequence of sorted by value body masses of planets and largest planetoids is connected by a constant scaling exponent with the sequence of their sorted by value orbital periods.

## 1 Introduction

In [1] we have shown that the observable mass distribution of large celestial bodies in the Solar system continues the mass distribution of elementary particles that can be understood as contribution to the fundamental link between quantum- and astrophysics via scaling.

Within the last ten years several articles [2–6] were published which confirm our statement that scaling is a widely distributed phenomenon. Possibly, natural oscillations of matter generate fractal distributions of physical properties in very different processes. Fractal scaling models [7] of oscillation processes in chain systems are not based on any statements about the nature of the link or interaction between the elements of the oscillating system. Therefore, the model statements are quite general, that opens a wide field of possible applications.

In this paper we will show, that the connection between the body mass distribution and the distribution of orbital periods of planets and largest planetoids in the solar system can be described by the scaling law (1):

$$M = \mu \cdot T^{\mathrm{D}}, \qquad (1)$$

where $M$ is a celestial body mass, $T$ is a celestial body orbital period and $\mu$ and $D$ are constants.

We will show, that for sorted by value couples of a body mass $M$ and an orbital period $T$ the exponent D is quite constant and is closed to 3/2. Furthermore, for $M$ in units of the proton rest mass $m_p \approx 1.67 \times 10^{-27}$ kg [8] and $T$ in units of the proton oscillation period $\tau_p = \hbar/m_p c^2 \approx 7.02 \times 10^{-25}$ s [9], the constant $\mu = 1$.

## 2 Methods

Already in the eighties the scaling exponent 3/2 was found in the distribution of particle masses [10]. In [11] we have shown that the scaling exponent 3/2 arises as consequence of natural oscillations in chain systems of harmonic oscillators.

Within our fractal model [1] of matter as a chain system of oscillating protons and under the consideration of quantum oscillations as model mechanism of mass generation [9], we interpret the exponent D in (1) as a Hausdorff [12] fractal dimension of similarity (2):

$$D = \frac{\ln M/m_p}{\ln T/\tau_p}. \qquad (2)$$

The ratio $M/m_p$ is the number of model protons, the ratio $T/\tau_p$ is the number of model proton oscillation cycles.

## 3 Results

If we sort by value the body masses and the orbital periods of planets and largest planetoids of the Solar system, then we can see that for sequently following couples of a body mass $M$ and an orbital period $T$ the fractal dimension D is quite constant and closed to the model value of 3/2.

Table 1 contains properties of planets and of the most massive planetoids in the Solar system. On the left side the bodies are sorted by their masses, on the right side the bodies are sorted by their orbital periods. Within the Solar system the average empiric value $\mathrm{D} \approx 1.527$ is a little bit larger then the model value of 3/2.

Based on the empiric value $\mathrm{D} \approx 1.527$, Table 2 continues the Table 1 until the Jupiter body mass. The orbital period of Eris corresponds well to the Uranus body mass, but the smaller transneptunian orbits, occupied by Pluto, Haumea and Makemake, ask for additional bodies. Possibly, the three vacant body masses and the three vacant orbital periods in Table 2 are properties of bodies which are still to be discover.

## 4 Resume

Celestial bodies are compressed matter which consist of nucleons over 99%. Possibly, the model approximation of $\mathrm{D} = 3/2$ and $\mu = 1$ in (1) for proton units is a macroscopic quantum physical property, which is based on the baryon nature of normal matter, because $\mu = 1$ means that $M/T^{\mathrm{D}} = m_p/\tau^{\mathrm{D}}$.

The scaling law (1) seems a true system property, because it describes a connection between masses and orbital periods of different celestial bodies (Mercury and Jupiter, Earth and Neptune, etc.) within the Solar system.

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| Bodies, sorted by $M$ | Body mass $M$, kg | $\ln(M/m_p)$ | D | $\ln(T/\tau_p)$ | Orbital period $T$, years | Bodies, sorted by $T$ |
|---|---|---|---|---|---|---|
| Ceres | $9.5000 \times 10^{20}$ | 109.9584 | 1.5387 | 71.4603 | 0.2408 | Mercury |
| Makemake | $2.1000 \times 10^{21}$ | 110.7516 | 1.5298 | 72.3980 | 0.6152 | Venus |
| Haumea | $4.0100 \times 10^{21}$ | 111.3985 | 1.5284 | 72.8839 | 1.0000 | Earth |
| Pluto | $1.3000 \times 10^{22}$ | 112.5746 | 1.5313 | 73.5156 | 1.8808 | Mars |
| Eris | $1.7000 \times 10^{22}$ | 112.8429 | 1.5165 | 74.4099 | 4.6000 | Ceres |
| Mercury | $3.3020 \times 10^{23}$ | 115.8094 | 1.5368 | 75.3573 | 11.8626 | Jupiter |
| Mars | $6.4191 \times 10^{23}$ | 116.4741 | 1.5272 | 76.2665 | 29.4475 | Saturn |
| Venus | $4.8690 \times 10^{24}$ | 118.5003 | 1.5327 | 77.3149 | 84.0168 | Uranus |
| Earth | $5.9742 \times 10^{24}$ | 118.7049 | 1.5221 | 77.9885 | 164.7913 | Neptune |

Table 1: For sorted by value couples of a body mass $M$ and an orbital period $T$ the fractal dimension D(2) is quite constant and closed to the model value 3/2. Data come from [8, 13–16].

| Bodies, sorted by $M$ | Body mass $M$, kg | $\ln(M/m_p)$ | D2 | $\ln(T/\tau_p)$ | Orbital period $T$, years | Bodies, sorted by $T$ |
|---|---|---|---|---|---|---|
| vacant | $1.6358 \times 10^{25}$ | 119.7122 | 1.5270 | 78.3970 | 247.9207 | Pluto |
| vacant | $2.0281 \times 10^{25}$ | 119.9271 | 1.5270 | 78.5378 | 285.4000 | Haumea |
| vacant | $2.2999 \times 10^{25}$ | 120.0529 | 1.5270 | 78.6201 | 309.9000 | Makemake |
| Uranus | $8.6849 \times 10^{25}$ | 121.3816 | 1.5325 | 79.2064 | 557.0000 | Eris |
| Neptun | $1.0244 \times 10^{26}$ | 121.5467 | 1.5270 | 79.5984 | 824.2881 | vacant |
| Saturn | $5.6851 \times 10^{26}$ | 123.2605 | 1.5270 | 80.7207 | 2532.1227 | vacant |
| Jupiter | $1.8987 \times 10^{27}$ | 124.4664 | 1.5270 | 81.5104 | 5577.7204 | vacant |

Table 2: Continues Table 1 until the Jupiter body mass. The masses and orbital periods for vacant bodies are calculated, based on the empiric average value $D \approx 1.527$.

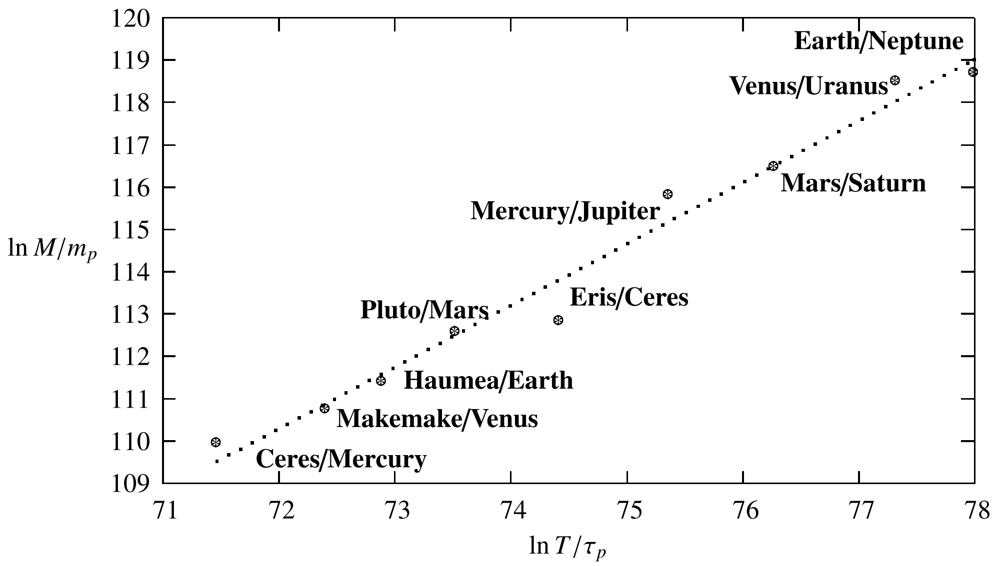


Fig. 1: Graphic representation of Table 1. For sorted by value couples of a body mass $M$ and an orbital period $T$ the fractal dimension D is quite constant. The doted line is drawed for the average $D \approx 1.527$.

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