Scaling of Moon Masses and Orbital Periods in the Systems of Saturn, Jupiter and Uranus

Hartmut Müller
Advanced Natural Research Institute in memoriam Leonhard Euler, Munich, Germany
E-mail: admin@anr-institute.com

The paper shows, that the sequence of sorted by value masses of the largest moons in the systems of Saturn, Jupiter and Uranus is connected by constant scaling exponents with the sequence of their sorted by value orbital periods.

1 Introduction

In [1] we have shown, that the connection between the body mass distribution and the distribution of orbital periods of planets and planetoids in the Solar System can be described by the scaling law:

$$M = \mu \cdot T^{\mathrm{D}}, \tag{1}$$

where M is a celestial body mass, T is a celestial body orbital period and μ and D are constants. We have shown, that for sorted by value couples of a body mass M and an orbital period T the exponent D is quite constant and is closed to the model value 3/2. Furthermore, for M in units of the proton rest mass $m_p \approx 1.67 \times 10^{-27}$ kg [2] and T in units of the proton oscillation period $\tau_p = \hbar / m_p c^2 \approx 7.02 \times 10^{-25}$ s, the constant $\mu = 1$.

In this paper we will show, that the scaling law (1) describes also the distribution of masses and orbital periods in the moon systems of Saturn, Jupiter and Uranus.

2 Methods

In [3] we have shown that the scaling exponent 3/2 arises as consequence of natural oscillations in chain systems of harmonic oscillators.

Within our fractal model [4] of matter as a chain system of oscillating protons and under the consideration of quantum oscillations as model mechanism of mass generation [5], we interpret the exponent D in (1) as a Hausdorff [6] fractal dimension of similarity (2):

$$\mathrm{D} = \frac{\ln M/m_p}{\ln T/\tau_p}. \tag{2}$$

The ratio M/m_p is the number of model protons, the ratio T/τ_p is the number of model proton oscillation cycles.

Already in the eighties the scaling exponent 3/2 was found in the distribution of particle masses [7]. Possibly, the model approximation of $\mathrm{D} \approx 3/2$ and $\mu = 1$ in (1) for proton units is a macroscopic quantum physical property, which is based on the baryon nature of normal matter, because $\mu = 1$ means that:

$$M/T^{\mathrm{D}} = m_p/\tau_p^{\mathrm{D}} \tag{3}$$

In [1] we have shown, that for planets and the most massive planetoids the average empiric value $\mathrm{D} \approx 1.527$ is a little bit larger then the model value 3/2. If we interpret the deviation of the empiric value $\mathrm{D} \approx 1.527$ in comparison with the model value 3/2 as a consequence of the fractality of the mass distribution in the system, then we can represent (1) in the form:

$$M^{\Delta}/T^2 = 1 \tag{4}$$

where $\Delta = 2/\mathrm{D}$ is the fractal dimension of the mass distribution, the constant of proportionality is 1 for proton units m_p and τ_p. The model value of Δ is 2/(3/2) = 4/3.

3 Results

The tables 1-3 contain properties of the largest moons of the Saturn, Juputer and Uranus systems. Always on the left side the moons are sorted by their masses, on the right side the moons are sorted by their orbital periods. The tables show, that within each moon system the fractal dimension Δ (4) is quite constant, but different from the average empiric value $\Delta = 2/D = 2/1.527 \approx 1.31$ for planets and planetoids [1]. This fact we interpret as criterion of different levels of fractality of the mass distribution in these systems. Furthermore, the tables show, that for the systems of Saturn and Uranus the fractal dimension Δ is nearly of the same average value, which is quite different of Δ for the system of Jupiter.

4 Resume

Within our fractal model [8], the scaling law (4) arises in chain systems of many harmonic oscillators and can be understood as fractal equivalent of the Hooke law. The scaling law (4) is valid for sorted by value couples of system properties. The Saturn system shows, that the scaling law (4) can be valid for one and the same body. The Jupiter and Uranus systems shows, that the scaling law (4) can be valid also for couples of different bodies. This may mean, that in general, the orbital period of each body does not depend only on its own mass, but depends on the body mass distribution in the system.

5 Acknowledgements

I'm thankful to my friend Victor Panchelyuga, my son Erwin and my partner Leili for the great experience to work with them, for the deep discussions and permanent support.

Submitted on February 15, 2015 / Accepted on February 17, 2015

Saturn moons, sorted by M	Body mass M, kg	$\ln(M/m_p)$	Δ	$\ln(T/\tau_p)$	Orbital period T, years	Saturn moons sorted by T
Mimas	3.7493×10^{19}	106.7277	1.2541	66.9235	0.9420	Mimas
Enceladus	1.0802×10^{20}	107.7858	1.2487	67.2983	1.3702	Enceladus
Tethys	6.1745×10^{20}	109.5291	1.2347	67.6187	1.8878	Tethys
Dione	1.0955×10^{21}	110.1024	1.2350	67.9901	2.7369	Dione
Iapetus	1.8056×10^{21}	110.6022	1.2385	68.4914	4.5182	Rhea
Rhea	2.3065×10^{21}	110.8470	1.2585	69.7524	15.9450	Titan
Titan	1.3452×10^{23}	114.9130	1.2419	71.3568	79.3215	Iapetus

Table 1: For sorted by value couples of a body mass M and an orbital period T the fractal dimension $\Delta(4)$ is quite constant within the Saturn moon system. The Saturn moon system average $\Delta = 1.2445$. Data comes from [9].

Jupiter moons, sorted by M	Body mass M, kg	$\ln(M/m_p)$	Δ	$\ln(T/\tau_p)$	Orbital period T, years	Jupiter moons sorted by T
Europa	4.7998×10^{22}	113.8824	1.1864	67.5538	1.7691	Io
Io	8.9319×10^{22}	114.5035	1.1921	68.2506	3.5512	Europa
Callisto	1.0759×10^{23}	114.6896	1.2024	68.9510	7.1546	Ganymede
Ganymede	1.4819×10^{23}	115.0098	1.2138	69.7980	16.6890	Callisto

Table 2: For sorted by value couples of a body mass M and an orbital period T the fractal dimension $\Delta(4)$ is quite constant within the Jupiter moon system. The Jupiter moon system average $\Delta = 1.1987$. Data comes from [12].

Uranus moons, sorted by M	Body mass M, kg	$\ln(M/m_p)$	Δ	$\ln(T/\tau_p)$	Orbital period T, years	Uranus moons sorted by T
Miranda	6.5900×10^{19}	107.2916	1.2551	67.3294	1.4135	Miranda
Umbriel	1.1720×10^{21}	110.1700	1.2328	67.9076	2.5200	Ariel
Ariel	1.3530×10^{21}	110.3136	1.2402	68.4050	4.1440	Umbriel
Oberon	3.0140×10^{21}	111.1145	1.2446	69.1473	8.7062	Titania
Titania	3.5270×10^{21}	111.2717	1.2507	69.5833	13.4632	Oberon

Table 3: For sorted by value couples of a body mass M and an orbital period T the fractal dimension $\Delta(4)$ is quite constant within the Uranus moon system. The Uranus moon system average $\Delta = 1.2447$. Data comes from [10, 11].

References

1. Müller H. Scaling of Body Masses and Orbital Periods in the Solar System. *Progress in Physics*, 2015, v. 2, 133–135.
2. Physical constants. Particle Data Group. www.pdg.lbl.gov
3. Müller H. Scaling Models of Natural Oscillations in Chain Systems and the Mass Distribution of Particles. *Progress in Physics*, 2010, v. 3, 61-66.
4. Müller H. Scaling models of Natural Oscillations in Chain Systems and the Mass Distribution of the Celestial Bodies in the Solar System. *Progress in Physics*, 2010, v. 1, 62–66.
5. Müller H. Emergence of Particle Masses in Fractal Scaling Models of Matter. *Progress in Physics*, 2012, v. 4, 44–47.
6. Hausdorff F. Dimension und äußeres Maß, 1919, v. 79.
7. Kolombet V. Macroscopic fluctuations, masses of particles and discrete space-time. *Biofizika*, 1992, v. 36, 492–499.
8. Müller H. Fractal scaling models of Resonant Oscillations in Chain systems of Harmonic Oscillators. *Progress in Physics*, 2009, v. 2, 72-76.
9. Bordi J., Jacobson R., Antreasian P. et al. The Gravity Field of the Saturnian System from Satellite Observations and Spacecraft Tracking Data. *Astronomical Journal*, 2006, v. 132, 2520–2526.
10. Uranus System Nomenclature Table Of Contents. Gazetteer of Planetary Nomenclature. USGS Astrogeology, 2009.
11. Taylor A., Synnott S., Jacobson R., Campbell J. The masses of Uranus and its major satellites from Voyager tracking data and Earth based Uranian satellite data. *Astronomical Journal*, 1992, v. 103, 2068–2078.
12. Malhotra R., Showman A. The Galilean Satellites. *Science*, 1999, v. 286, 77–84.