A Classical Model of the Photon

Shixing Weng
11 Metzak Dr., Brampton, Ontario L6Z 4N3, Canada
E-mail: wengs2015@gmail.com

A desired solution of the four-potential is presented for free-space photons, obtained with wave equations derived from the Maxwell equations and the Lorenz condition. The solution shows that an electromagnetic field in wave form propagating at the speed of light with a fixed internal phase may exist as a particle taking a limited space at a specific point of time. It reveals the existence of electric charge distributed as an electric capacitor on the parallel cylindrical surface of constant radius to the central axis of the solution. And the charge distribution has a phase change both in the azimuthal angle and along the direction of the wave propagation. The solution is applied to the case of a model photon to determine several parameter values of the solution, which in turn provides a view on the model photon.

1 Introduction

The year of 2015 has been the International Year of Light and Light-Based Technologies, designated by the United Nations Educational, Scientific, Cultural Organization (UNESCO). This designation further emphasizes the importance of light to people's life. As a part of the support for the designation, we present in this paper a theoretical model for the elements of light, photons, based on our knowledge of classical electrodynamics, classical mechanics and mathematical method for quantization rules.

In this paper we consider a single free photon in which photon-photon interactions [1] are neglected. A photon [2] is a quantum of light which is a wave form of the electromagnetic radiation and is characterized by its speed c and wavelength λ. It is known that a photon has both physical properties of wave and particle.

As a particle, the photon has a certain energy and momentum. In the study of the black body radiation [3], Max Planck proposed that the energy ϵ of a radiation oscillator was quantized and each energy was proportional to its vibrational frequency ν as

$$\epsilon = h\nu \,, \tag{1}$$

where h is the Planck constant. Then Einstein applied the idea to the light and proposed that light was made of quanta, inseparable entities, with the energy ϵ in terms of the frequency being given in Eq. (1), which successfully explained the photoelectric effect [4].

The Compton Scattering Experiment [5] further demonstrated that a photon had a certain energy as specified in Eq. (1) as well as a momentum in the direction of its motion. And the magnitude of the momentum p is given by

$$p = \frac{\epsilon}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \,, \tag{2}$$

where the relation $\nu = c/\lambda$ is used.

Furthermore it is known from quantum mechanics [6], that there is an angular momentum difference involved in the magnitude of integral $\hbar$ between the two transitional atomic states, where $\hbar$ is the reduced Planck constant which equals to the Planck constant h divided by 2π. In the case of light emission this angular momentum difference may be transfered to the photon.

On the other hand the Young's two slit experiment [7] shows the wave property of light. In a typical Young's experiment one observes the interference pattern of light from a monochromatic light source of wavelength λ passing through two small-spaced parallel slits, which demonstrates the wave property of light.

It is also known that light is a form of the electromagnetic wave. In the electromagnetism [8], the set of Maxwell equations for vacuum gives relationships among the electric field $\mathbf{E}$, magnetic field $\mathbf{B}$, electric charge density ρ, and electric current density $\mathbf{J}$ as following:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \,, \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0 \,, \tag{4}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \,, \tag{5}$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \,, \tag{6}$$

where ϵ_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum; ∇ represents the differential operator and $\nabla = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}$ in Cartesian coordinates with $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ being unit vectors for the Cartesian coordinates; t represents the time and x, y, z are, respectively, the Cartesian components; the "$\times$" symbol represents the cross operation and the "$\cdot$" represents the dot operation. In this paper we use SI units. And for simplicity we shall consider in the following the medium to be vacuum. For vacuum where $\rho = 0$ and $\mathbf{J} = 0$, the following equations may be obtained for the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ from Eqs. (3) to (6),

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0 \,, \tag{7}$$

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0\,, \tag{8}$$

where c is the speed of light, which is equal to $1/\sqrt{\epsilon_0\mu_0}$ for vacuum, and ∇^2 is the Laplacian operator. Eqs. (7) and (8) are wave equations with the propagation speed equal to the speed of light, which shows the light to be a form of the electromagnetic wave. But we believe that the achieved solution from Eqs. (7) and (8) so far for free-space photon is limited to one-dimension and our current view on the photon is limited.

As we know that an electric field or a magnetic field has energy. And the total energy density η is equal to the sum of the electric field energy density $\eta_\mathbf{E}$ and the magnetic field energy density $\eta_\mathbf{B}$ and is given by

$$\eta = \eta_\mathbf{E} + \eta_\mathbf{B} = \frac{1}{2}\,\epsilon_0|\mathbf{E}|^2 + \frac{1}{2\mu_0}|\mathbf{B}|^2\,, \tag{9}$$

where $|\mathbf{E}|$ is the magnitude of the electric field and $|\mathbf{B}|$ the magnitude of the magnetic field.

The Poynting vector $\mathbf{S}$, which is the energy current density of the electromagnetic wave, is given by

$$\mathbf{S} = \frac{1}{\mu_0}\,\mathbf{E}\times\mathbf{B}\,. \tag{10}$$

The Poynting vector is perpendicular to both $\mathbf{E}$ and $\mathbf{B}$ vectors and is in the direction of the thumb while using the right-hand-rule turning fingers from $\mathbf{E}$ to $\mathbf{B}$.

Both the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ can be expressed in terms of the four-potential, a scalar electric potential ψ plus a magnetic vector potential $\mathbf{A}$ as following,

$$\mathbf{B} = \nabla\times\mathbf{A}\,, \tag{11}$$

$$\mathbf{E} = -\nabla\psi - \frac{\partial\mathbf{A}}{\partial t}\,. \tag{12}$$

The Lorenz condition [9], named after the Danish mathematician and physicist, L. V. Lorenz, provides a covariant form of the four-potential and is given by

$$\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{\partial\psi}{\partial t} = 0\,. \tag{13}$$

Eq. (13) appears similar to the continuity equation and may represent a "local form" of the conservation of electric potential energy for a point charge in the electromagnetic field. With the Lorenz condition, both the scalar potential ψ and the vector potential $\mathbf{A}$ satisfy the following equations, respectively,

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi = \frac{\rho}{\epsilon_0}\,, \tag{14}$$

$$\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \nabla^2\mathbf{A} = \mu_0\mathbf{J}\,. \tag{15}$$

The purpose of the paper is to present a model view of the photon by obtaining a three-dimensional solution from Eqs. (14) and (15) for vacuum without external electric charge nor external electric current. The three-dimensional solution hence is theoretical analyzed to reveal its physics meaning. It is finally applied to the case of a model photon to gain a deep insight into the photon, which is new since we are not aware of such a report in the literatures.

This paper is organized as these: Introduction, Solution, Discussions, and Conclusion. The Introduction section provides a brief overview on our fundamental understandings of light and photon. In the Solution section, two expressions of the four-potential as a solution for three-dimensional space are presented, which are obtained from Eqs. (14) and (15) for vacuum without external electric charge nor external electric current. The characteristic of the solution shows that its quantities are in limited space at a specific point of time, which is desirable for photons. In the Discussions section, expressions for the electric field and the magnetic field are derived from the four-potential solution. An analysis of the electric field reveals the existence of electric charge distributed on the parallel cylindrical surface of constant radius to the central axis of the solution. The solution then is applied to the case of a model photon to determine the constant parameter values of the solution from physical quantities of the photon, which in turn provides a view on the model photon. The Conclusion section provides a brief summary of the paper together with some comments.

2 Solution

In vacuum where electric charge density $\rho = 0$ and electric current density $\mathbf{J} = 0$, Eqs. (14) and (15) are reduced respectively to

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi = 0\,, \tag{16}$$

$$\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \nabla^2\mathbf{A} = 0\,. \tag{17}$$

Eqs. (16) and (17) are wave equations and their solutions for one-dimensional space are easily obtained and are known as a traveling wave,

$$\psi = \psi_0\sin(kx - \omega t)\,, \tag{18}$$

$$A = A_0\sin(kx - \omega t)\,, \tag{19}$$

where ψ_0 represents the amplitude of the scalar potential, A_0 the amplitude of the vector potential, ω is the angular frequency which equals to $2\pi\nu$ and ν is the wave frequency, and k is the wavenumber and $k = \omega/c = 2\pi/\lambda$. The reason to choose the sine function instead of the cosine function here is arbitrary, but with no difference, since the sine and cosine functions are different by a phase difference of $\pi/2$, they may represent the same physical wave. Also as we know that the electric potential is a measurable quantity which is real, we shall restrict the solution to the real number domain in this paper.

In the following, Eqs. (16) and (17) are solved for three-dimensional space to reveal more features of the solution. First we choose the circular cylindrical coordinates (or cylindrical polar coordinates) as in Fig. 1 for our coordinate system [10]. Here we use the r symbol to represent the polar axis since the ρ symbol is used for the electric charge density. And ϕ represents the azimuthal angle and z represents the central axis and is the same as the Cartesian z axis. Their respective unit vectors are $\hat{\mathbf{r}}$, $\hat{\phi}$, and $\hat{\mathbf{z}}$ as in Fig. 1.

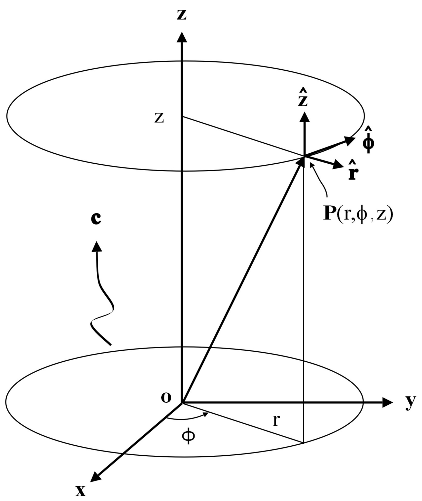

Fig. 1: A drawing of the circular cylindrical coordinate system with respect to the Cartesian coordinates, where $\hat{\mathbf{r}}$, $\hat{\phi}$, and $\hat{\mathbf{z}}$ are unit vectors for the coordinate system. The wave symbol represents a photon moving in the direction of the positive z axis at the speed of light c.

The Laplacian operator ∇^2 in the cylindrical coordinates is expressed as

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}\,, \tag{20}$$

and hence we get a solution of the four-potential from Eqs. (16) and (17) as following

$$\psi = \psi_0 \sin(kz + m\phi - \omega t) \begin{cases} \left(\dfrac{r}{r_0}\right)^m & r < r_0\,, \\ \left(\dfrac{r_0}{r}\right)^m & r > r_0\,, \end{cases} \tag{21}$$

$$\mathbf{A} = \hat{\mathbf{z}} A_0 \sin(kz + m\phi - \omega t) \begin{cases} \left(\dfrac{r}{r_0}\right)^m & r < r_0\,, \\ \left(\dfrac{r_0}{r}\right)^m & r > r_0\,, \end{cases} \tag{22}$$

where we choose the wave to propagate along the positive z axis, ψ_0 is a strength constant for the scalar potential and A_0 is a strength constant for the vector potential whose direction is in that of the wave propagation, r_0 is a constant polar radius to be determined in the next section by the wavelength of the photon, m is a positive integer to satisfy the 2π periodic boundary condition of the azimuthal angle. Here m is a quantum number which may be associated with the angular momentum of the wave. Again the choice of the sine function instead of the cosine function here is arbitrary but has no physics difference. The solution at r_0 is not defined but has finite quantities. r_0 is a boundary of the solution and in the following treatment we shall let the boundary thickness to approach to zero so the solution is approximately defined at r_0.

Eqs. (21) and (22) represent a traveling wave propagating along the positive z axis. The solution by the two expressions is desirable since its quantities are limited in the polar axis. It is worthwhile to mention that this solution may be for individual photons free from interactions with each other. The study of photon interactions is out of the scope of this paper. In the following section we will analyze the solution to reveal its physics meaning.

3 Discussions

Applying the Lorenz condition, Eq. (13), to Eqs. (21) and (22), we have

$$A_0 = \frac{\psi_0}{c}\,. \tag{23}$$

Hence the vector potential and the scalar potential are related to each other, only one of them is independent.

Now applying Eqs. (11) and (12) to the solution Eqs. (21) and (22) and using Eq. (23), we may have for the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ as following:

$$\mathbf{E} = -m\psi_0 \begin{cases} \dfrac{r^{m-1}}{r_0{}^m}\Big(\hat{\mathbf{r}}\sin(kz + m\phi - \omega t) + \\ \quad + \hat{\phi}\cos(kz + m\phi - \omega t)\Big) & r < r_0\,, \\ \dfrac{r_0{}^m}{r^{m+1}}\Big(-\hat{\mathbf{r}}\sin(kz + m\phi - \omega t) + \\ \quad + \hat{\phi}\cos(kz + m\phi - \omega t)\Big) & r > r_0\,, \end{cases} \tag{24}$$

$$\mathbf{B} = mA_0 \begin{cases} \dfrac{r^{m-1}}{r_0{}^m}\Big(\hat{\mathbf{r}}\cos(kz + m\phi - \omega t) - \\ \quad - \hat{\phi}\sin(kz + m\phi - \omega t)\Big) & r < r_0\,, \\ \dfrac{r_0{}^m}{r^{m+1}}\Big(\hat{\mathbf{r}}\cos(kz + m\phi - \omega t) + \\ \quad + \hat{\phi}\sin(kz + m\phi - \omega t)\Big) & r > r_0\,, \end{cases} \tag{25}$$

where $\hat{\mathbf{r}}$ is the unit vector for the polar axis, $\hat{\phi}$ is the unit vector for the azimuthal angle. From Eqs. (24) and (25) we know that both the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ are traveling in the direction of the positive z axis and are perpendicular to the direction of the wave propagation. Furthermore we have $\mathbf{E}\cdot\mathbf{B} = 0$, meaning that the electric field and the magnetic field are perpendicular to each other, which is consistent with the basic electromagnetic theory for free-space.

For better understanding of the fields, in the following discussions we shall restrict ourself to the case of the angular momentum number $m = 1$, which may correspond to the case of the photon we know. For general case of $m > 1$, following treatments are similarly applicable. Hence Eqs. (24) and (25) become

$$\mathbf{E} = -\psi_0 \begin{cases} \dfrac{1}{r_0}\Big(\hat{\mathbf{r}}\sin(kz+\phi-\omega t) + \\ +\hat{\phi}\cos(kz+\phi-\omega t)\Big) & r < r_0\,, \\ \dfrac{r_0}{r^2}\Big(-\hat{\mathbf{r}}\sin(kz+\phi-\omega t) + \\ +\hat{\phi}\cos(kz+\phi-\omega t)\Big) & r > r_0\,, \end{cases} \tag{26}$$

$$\mathbf{B} = A_0 \begin{cases} \dfrac{1}{r_0}\Big(\hat{\mathbf{r}}\cos(kz+m\phi-\omega t) - \\ -\hat{\phi}\sin(kz+m\phi-\omega t)\Big) & r < r_0\,, \\ \dfrac{r_0}{r^2}\Big(\hat{\mathbf{r}}\cos(kz+m\phi-\omega t) + \\ +\hat{\phi}\sin(kz+m\phi-\omega t)\Big) & r > r_0\,. \end{cases} \tag{27}$$

From Eqs. (26) and (27), for $r > r_0$ both field strengths are inversely proportional to r^2 and approach to zero as r goes to infinity, which is a desirable result because a photon takes a limited space at a specific point of time. The electric field **E** at r_0, or on the parallel cylindrical surface in a three-dimensional view, is not continue in the radial direction, meaning charge may exist on the surface. To derive an expression for the surface charge density σ, apply Eq. (3) to Eq. (26), we have

$$\sigma = 2\epsilon_0\psi_0\frac{1}{r_0}\sin(kz+\phi-\omega t)\,. \tag{28}$$

Hence the charge density is also in the form of a traveling wave, moving uniformly in the direction of the positive z axis with a fixed internal phase both in the azimuthal angle and along the z axis.

To get a precise sense of the fields and charge distribution, we simplify Eqs. (26), (27), and (28) by letting $z = 0$, and $t = 0$, which allows us to better understand the solution at the specific point of time and space. And hence we have

$$\mathbf{E} = \psi_0 \begin{cases} -\dfrac{1}{r_0}\hat{\mathbf{j}} & r < r_0\,, \\ \dfrac{r_0}{r^2}\Big(\hat{\mathbf{i}}\sin(2\phi) - \hat{\mathbf{j}}\cos(2\phi)\Big) & r > r_0\,, \end{cases} \tag{29}$$

$$\mathbf{B} = A_0 \begin{cases} \dfrac{1}{r_0}\hat{\mathbf{i}} & r < r_0\,, \\ \dfrac{r_0}{r^2}\Big(\hat{\mathbf{i}}\cos(2\phi) + \hat{\mathbf{j}}\sin(2\phi)\Big) & r > r_0\,, \end{cases} \tag{30}$$

and

$$\sigma = 2\epsilon_0\psi_0\frac{1}{r_0}\sin\phi\,, \tag{31}$$

where $\hat{\mathbf{i}}$ is the unit vector for the x axis and $\hat{\mathbf{j}}$ is the unit vector for the y axis. In deriving Eqs. (29) and (30), we use the following relations for unit vector transformations between the polar and Cartesian coordinates

$$\hat{\mathbf{r}} = \hat{\mathbf{i}}\cos\phi + \hat{\mathbf{j}}\sin\phi\,, \tag{32}$$

$$\hat{\phi} = -\hat{\mathbf{i}}\sin\phi + \hat{\mathbf{j}}\cos\phi\,. \tag{33}$$

The electric field **E**, magnetic field **B**, and the surface charge density σ at $z = 0$ and $t = 0$ are shown in Fig. 2.

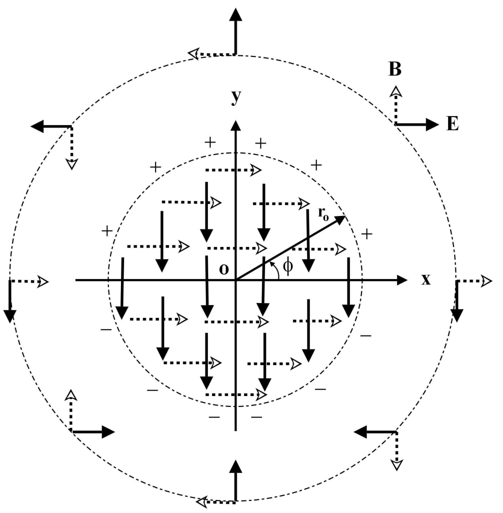

Fig. 2: A schematic diagram of the electric field **E** (solid lines), magnetic field **B** (dash lines), and charge distribution ("+" for positive charge and "−" for negative charge) on an imaging cylindrical surface ($r = r_0$) of the solution in the x-y plane, where $z = 0, t = 0$. The wave is propagating along the positive z axis (pointing out of the x-y plane). r_0 is the constant radius, and ϕ is the azimuthal angle.

As we know from Eqs. (29) and (30), both the electric field **E** and the magnetic field **B** are constant inside of the circle r_0; For outside of the r_0 both fields decreases as the radius squared, r^2, increases, and the field direction changes two times as fast as the azimuthal angle ϕ (Fig. 2). The distribution of the surface charge density σ is described by the sine function of the azimuthal angle, and the total charge by the r_0 circle is zero. Referring to Fig. 2, the charge distribution is polarized, i.e., the positive charge on its corresponding half-circle at r_0 is distributed symmetrically to the negative charge on the other half-circle, or vice versa. The total charge distribution appears as an electric capacitor made of circularly distributed electric dipoles.

In the following discussions we apply the solution to a model photon and shall use the physical quantities of the photon to determine the values of the constants used in the solution.

For $z \neq 0$ and $t = 0$ the electric field **E**, the magnetic field **B** and the surface electric charge density σ are distributed around the central axis z with a certain phase. And the phase change depends on both the azimuthal angle ϕ and the z axis. We show the charge distribution for $z < 0$ and $t = 0$ in Fig. 3.

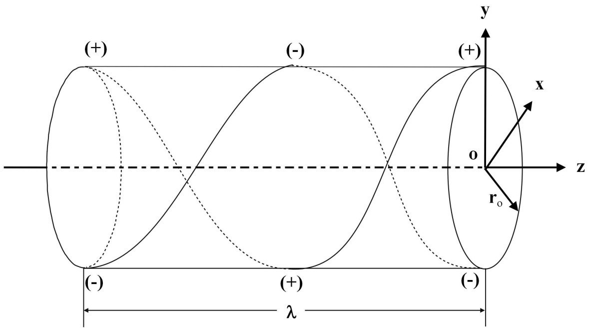

Fig. 3: A schematic diagram showing the surface charge distribution("+" for positive charge and "−" for negative charge) on the surface of $r = r_0$ in the z axis direction for one wavelength λ, where $t = 0$, the model photon is moving along the positive z axis at the speed of light c and r_0 is the constant radius. For clarity we only show two lines of charges here.

The charge distribution appears as a circularly distributed electric dipole "twisted" in the azimuthal angle and along the z axis. The twisting phase change is exactly the same as that of the photon (one cycle of the charge phase change by one wavelength λ). The model photon picture in Fig. 3 represents a "frozen" view at $t = 0$. For $t \neq 0$, by the phase analysis of the sine wave (Eq. (28)), the model photon is doing a displacement along the positive z without changing its internal phase. Now imaging that if we place an observer facing the incoming photon at a fixed z position, it may see the circularly distributed charge rotating counter-clockwise (in the direction of the azimuthal angle) around the photon's central axis. Since this rotation represents a certain angular momentum, the photon may carry an angular momentum in the phase of the charge distribution.

In the following we shall assume that the length of the model photon, l, equals to $n\lambda$, where n is a positive integer to satisfy the periodic condition in the propagation direction. Here n may be considered as a quantum number and its minimum value is one, which makes a minimum complete cycle.

Now applying Eq. (23) to Eqs. (26) and (27), we find that the electric field energy density $\eta_{\mathbf{E}}$ and the magnetic field energy density $\eta_{\mathbf{B}}$ (Eq. (9)) are equal to each other for the photon. And we have the total energy density η as following

$$\eta = \epsilon_0 |\mathbf{E}|^2 = \epsilon_0 {\psi_0}^2 \begin{cases} \dfrac{1}{{r_0}^2} & r < r_0\,, \\ \dfrac{{r_0}^2}{r^4} & r > r_0\,, \end{cases} \tag{34}$$

where $|\mathbf{E}|$ is the magnitude of the electric field. The energy density is constant for $r < r_0$ and is inversely proportional to r^4 for $r > r_0$. The photon energy (Eq. (1)) may be equal to the integration value of Eq. (34) in the photon space at time $t = 0$. The integration path for r is 0 to r_0 and r_0 to ∞, for z is -$n\lambda$ to 0, and for ϕ is 0 to 2π. And hence we find the ψ_0 to have the following relationship

$$\psi_0 = \sqrt{\frac{\hbar c}{\epsilon_0 n}}\,\frac{1}{\lambda}\,. \tag{35}$$

In deriving Eq. (35) we used Eq. (1). It is interesting to note that the potential strength constant, ψ_0, is inversely proportional to the wavelength λ.

By using Eqs. (10), (26), and (27), the Poynting vector is

$$\mathbf{S} = \hat{\mathbf{z}}\,\frac{\psi_0 A_0}{\mu_0} \begin{cases} \dfrac{1}{{r_0}^2} & r < r_0\,, \\ \dfrac{{r_0}^2}{r^4} & r > r_0\,. \end{cases} \tag{36}$$

According to Eq. (36), the photon energy flows in the direction of the positive z axis, which is consistent with the photon direction of motion. The total energy by the Poynting vector for the photon is $h\nu$, which may be calculated by integrating out the Poynting vector, Eq. (36), for the photon and using Eqs. (23) and (35). This is an expected result.

Since the charge is distributed in the r_0 cylindrical surface, which generates a surface electric current by the displacement of the photon at the speed of light, the density of the photon self energy may also be expressed in the following relationship,

$$\eta' = \frac{1}{2}\,\sigma\psi + \frac{1}{2}\,\mathbf{A}\cdot\mathbf{J}'\,, \tag{37}$$

where η' represents the surface energy density, σ the surface charge density, ψ the electric potential, $\mathbf{A}$ the vector potential, and $\mathbf{J}'$ represents the surface electric current density. For the photon, $\mathbf{A}\cdot\mathbf{J}' = AJ'$ and $J' = \sigma c$, the second term is equal to the first term on the right hand side of Eq. (37) and we have.

$$\eta' = \sigma\psi\,. \tag{38}$$

Using Eqs. (28), (21) for $m = 1$, and (35), we may calculate the photon energy ϵ by integrating out Eq. (38) on the r_0 cylindrical surface of length $n\lambda$,

$$\begin{aligned} \epsilon &= \int_{-n\lambda}^{0}\int_{0}^{2\pi} \eta' dS = \int_{-n\lambda}^{0} dz \int_{0}^{2\pi} \sigma\psi r_0\, d\phi \\ &= \int_{-n\lambda}^{0} dz \int_{0}^{2\pi} 2\epsilon_0\psi_0^2 \sin^2(kz+\phi)\, d\phi \\ &= \int_{-n\lambda}^{0} dz \int_{kz}^{kz+2\pi} 2\epsilon_0\psi_0^2 \sin^2(\phi')\, d\phi' \\ &= n\lambda 2\epsilon_0\psi_0^2\pi = h\nu\,, \end{aligned} \tag{39}$$

where dS represents an infinite small area on the r_0 cylindrical surface, the time $t = 0$, and a variable change, $kz + \phi = \phi'$. Hence we get that the energy is $h\nu$. This result indicates that it is equivalent to consider the photon energy being stored in the r_0 cylindrical surface.

Now we evaluate the value of the constant length of the polar radius, r_0, of the model photon. We first assume that r_0 is proportional to the wavelength λ as

$$r_0 = \frac{\lambda}{2\pi}\,. \tag{40}$$

Then we support it by two reasons. The first reason is that with this assumption the phase velocity of the charge distribution on the r_0 cylindrical surface is equal to the speed of light c, i.e., $\omega r_0 = 2\pi\nu\lambda/2\pi = \nu\lambda = c$. This is consistent with the nature of the photon. This velocity may be physically experienced by an electron in an atom as in light absorption.

The second reason is that the angular momentum carried by the photon is $\hbar$, which is consistent with the angular momentum number $m = 1$. To evaluate the angular momentum, we use following expression

$$d\mathbf{J} = \mathbf{r}_0 \times d\mathbf{P}\,, \tag{41}$$

where we consider the angular momentum to be generated in the r_0 cylindrical surface, $d\mathbf{J}$ represents an infinite small quantity of angular momentum, $d\mathbf{P}$ represents an infinite small quantity of momentum in the cylindrical surface, and $\mathbf{r}_0$ is the polar radius vector pointing to the cylindrical surface where the small momentum is considered. Referring to Fig. 3, an observer like an electron in an atom may experience a rotational force from the photon, which corresponds to a momentum in the direction of the azimuthal angle ϕ. This momentum may generate an angular momentum in the direction of the positive z axis.

Similar to Eq. (2), the magnitude of the infinite small quantity of momentum dP may be written as

$$dP = \frac{d\epsilon}{c}\,, \tag{42}$$

where $d\epsilon$ represents an infinite small amount of energy in the cylindrical surface and c is the speed of light. Using Eq. (38), we have for the $d\epsilon$,

$$d\epsilon = \eta' dS = \sigma\psi dS\,, \tag{43}$$

where dS represents an infinite small area on the r_0 cylindrical surface. And finally we have for the magnitude of the infinite small quantity of the angular momentum dJ as

$$dJ = \frac{r_0}{c}\,\sigma\psi dS\,, \tag{44}$$

where r_0 is given in Eq. (40). The direction of the angular momentum is in the positive z axis.

By integrating out Eq. (44) for the photon on the r_0 cylindrical surface at the time $t = 0$, as has been done in Eq. (39), we get that the total angular momentum of the photon is indeed $\hbar$. Hence from the second reasoning we prove that the constant radius r_0 of the photon cylindrical surface is $\lambda/2\pi$.

This angular momentum, derived from the classical mechanics, may be considered as the spin angular momentum of the photon since it is generated by the self-rotation around its central axis.

Now based on the solution of Eqs. (21) and (22), we have built a consistent three-dimensional model of the photon: a quantized electromagnetic wave of length $n\lambda$ with a charged cylindrical surface core of radius $\lambda/2\pi$. Such a model may be tested for it is expected that the photon is very hard to pass a pinhole of radius less than $\lambda/2\pi$.

4 Conclusion

Conclusion by summarizing what have been presented in the paper. First a desirable solution was shown in terms of the two expressions, Eqs. (21) and (22), for the four-potential, obtained from wave Eqs. (16) and (17) derived by using the Maxwell equations together with the Lorenz condition. Although we assumed the medium to be vacuum in the solution for simplicity, our solution may be extended to the case of a homogeneous medium by using the medium parameters of the permittivity, permeability, and the speed of light. Also for clarity we limited our consideration in the Discussions section to the case of $\phi \geq 0$ and $t \geq 0$, but the solution itself is equally applicable if we substitute ϕ by $-\phi$ or t by $-t$. In the case of ϕ, the $\pm$ signs respectively may represent the right or left spin state of the photon.

Then the solution was analyzed for understanding its characteristics, which showed that an electromagnetic field in isolated wave form at the speed of light might exist in a limited space at a specific point of time. The solution requires the existence on the r_0 cylindrical surface of electric charge distributed in certain phase with the azimuthal angle ϕ and along the direction of the light propagation. The solution was specifically studied for the case of the angular momentum number $m = 1$.

We then applied the solution to the case of a model photon and determined the constant values of the solution in terms of the photon quantities. By doing that, a detailed theoretical three-dimensional model of the photon was achieved. We showed that the angular momentum of the photon might be considered as coded in the r_0 cylindrical surface by the phase of the charge distribution.

Notice that we have solved a special case of Eqs. (16) and (17) by restricting the angular momentum of the photon in the direction of the light propagation. Furthermore, the length of the photon was assumed to be $n\lambda$, but the upper bound of n was not determined specifically.

Finally it is theoretically interesting to mention that by letting the angular momentum number $m > 1$ in the solution, which could correspond to a photon with spin larger than one, we may get similar results as the spin one photon in terms of the wave taking a limited space at a specific point of time.

Acknowledgements

The author would like to express his thanks to all teachers for his education up to the post-doctoral position. Helpful discussions with Dr Wing-Ki Liu are acknowledged.

Submitted on November 10, 2015 / Accepted on December 7, 2015

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