X(5) Symmetry to 152Sm

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The excited positive and negative parity states, potential energy surfaces, V(β, γ), electromagnetic transition probabilities, B(E1), B(E2), electric monopole strength X(E0/E2) and staggering effect, ΔJ = 1, were calculated successfully using the interacting boson approximation model IBA-1. The calculated values are compared to the available experimental data and show reasonable agreement. The energy ratios and contour plot of the potential energy surfaces show that 152Sm is an X(5) candidate.

1 Introduction

Phase transition is one of the very interesting topic in nuclear structure physics. The even-even samarium series of isotopes have encouraged many authors to study that area extensively experimentally and theoretically.

Experimentally, authors studied levels energy with their half-lives, transition probabilities, decay schemes, multipole mixing ratios, internal conversion coefficients, angular correlations and nuclear orientation of γ-rays[1-4].

Theoretically, different theoretical models have been applied to that chain of isotopes. One of the very interesting models is the interacting boson approximation model IBA [5-10]. Iachello [11,12] has made an important contribution by introducing the new dynamical symmetries E(5) and X(5), E(5) is the critical point symmetry of phase transition between U(5) and O(6) while X(5) is between U(5) and SU(3) nuclei. The aim of the present work is to calculate:

1. The potential energy surfaces, V(β, γ);
2. The levels energy, electromagnetic transition rates B(E1) and B(E2);
3. The staggering effect, and
4. The electric monopole strength X(E0/E2).

2 IBA-1 model

2.1 Levels energy

The IBA-1 Hamiltonian [13-16] employed on 152Sm in the present calculation is:

\[ H = EPS \cdot n_d + \text{PAIR} \cdot (P \cdot P) + \frac{1}{2} \cdot ELL \cdot (L \cdot L) + \frac{1}{2} \cdot QQ \cdot (Q \cdot Q) + 5 \cdot OCT \cdot (T_3 \cdot T_3) + 5 \cdot HEX \cdot (T_4 \cdot T_4), \]

where

\[ P \cdot P = \frac{1}{2} \left\{ (ss)^0 \cdot (ss)^0 - \sqrt{5} \cdot (dd)^0 \cdot (dd)^0 \right\}, \]

\[ L \cdot L = -10 \sqrt{5} \cdot (dd)^1 \cdot (dd)^1, \]

\[ Q \cdot Q = \sqrt{5} \left\{ (s \cdot s + d \cdot d)^2 - \frac{\sqrt{7}}{2} \cdot (d \cdot d)^2 \right\}, \]

\[ T_3 \cdot T_3 = -\sqrt{7} \cdot (d \cdot d)^2 \cdot (d \cdot d)^2, \]

\[ T_4 \cdot T_4 = 3 \cdot (d \cdot d)^4 \cdot (d \cdot d)^4. \]

In the previous formulas, \( n_d \) is the number of bosons; \( P \cdot P, L \cdot L, Q \cdot Q, T_3 \cdot T_3 \) and \( T_4 \cdot T_4 \) represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions respectively between the bosons; \( EPS \) is the boson energy; and \( \text{PAIR, ELL, QQ, OCT, HEX} \) are the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions respectively (see Table 1).

2.2 Transition rates

The electric quadrupole transition operator employed is:

\[ T^{(E2)} = 2E2D \cdot (s \cdot s + d \cdot d)^2 + \frac{1}{\sqrt{5}} \cdot E2DD \cdot (d \cdot d)^2. \]

\( E2D \) and \( E2DD \) are adjustable parameters.

The reduced electric quadrupole transition rates between \( I_i \rightarrow I_f \) states are given by:

\[ B(E2, I_i \rightarrow I_f) = \frac{(I_f, \{ (E2^{(s)} \| I_i \}^2)}{2I_i + 1}. \]

3 Results and discussion

In this section we review and discuss the results.
3.1 The potential energy surfaces

The potential energy surfaces [17], $V(\beta, \gamma)$, as a function of the deformation parameters $\beta$ and $\gamma$ are calculated using:

$$E_{\text{IBA}}(\beta, \gamma) = \langle N_\pi N_\nu | H_{\text{IBA}} | N_\pi N_\nu \rangle =$$

$$= \zeta_d(N_\pi N_\nu)|^{\beta^2(1 + \beta^2) + \beta^2(1 + \beta^2)^2} \times$$

$$\times \bigl[ k N_\pi N_\nu[4 - (\bar{X}_\rho \bar{X}_\rho)\beta \cos 3\gamma] +$$

$$+ \left[ \bar{X}_\rho \bar{X}_\rho \beta^2 \right] + N_\nu(N_\nu - 1) \left( \frac{1}{10} c_0 + \frac{1}{7} c_2 \right) \beta^2 \bigr],$$

where

$$X_\rho = \left( \frac{2}{7} \right)^{0.5} X_\rho = \pi \text{ or } \nu. \quad (10)$$

The calculated potential energy surfaces, $V(\beta, \gamma)$, are presented in Figures 1, 2, 3. $^{152}$Sm lies between $^{150}$Sm which is a vibrational like nucleus, $SU(5)$, Fig. 1, while $^{154}$Sm is a rotational like, $SU(3)$, nucleus, Fig. 3. So, $^{150}$Sm can be an X(5) candidate where levels energy, transition probability ratios as well as the potential energy surfaces are supporting that assumption (see Table 2).

3.2 Energy spectra and electric transition rates

The energy of the positive and negative parity states of $^{152}$Sm isotope are calculated using computer code PHINT [19]. A comparison between the experimental spectra [18] and our calculations, using values of the model parameters given in Table 1 for the ground state, $\beta_1$, $\beta_2$ and $\gamma$ bands are illustrated in Fig. 4. The agreement between the calculated levels energy and their corresponding experimental values are fair, but they are slightly higher especially for the higher excited states in $\beta_1$, $\beta_2$ and $\gamma$ bands. We believe this is due to the change of the projection of the angular momentum which is due mainly to band crossing. Fig. 5 shows the position of $X(5)$ and $E(5)$ between the other types of nuclei.

Unfortunately there are no available measurements of electromagnetic transition rates $B(E1)$ for $^{152}$Sm nucleus, Table 3, while some of $B(E2)$ are measured. The measured $B(E2, 2^+_1 \rightarrow 0^+_1)$ is presented, in Table 4, for comparison with the calculated values [20]. The parameters $E2SD$ and $E2DD$ displayed in Table 1 are used in the computer code NPBEM [19] for calculating the electromagnetic transition rates and the calculated values are normalized to $B(E2, 2^+_1 \rightarrow 0^+_1)$. No new parameters are introduced for calculating electromagnetic transition rates $B(E1)$ and $B(E2)$ of intraband and interband.

<table>
<thead>
<tr>
<th>nucleus</th>
<th>EPS</th>
<th>PAIR</th>
<th>ELL</th>
<th>QO</th>
<th>OCT</th>
<th>HEX</th>
<th>E2SD(eb)</th>
<th>E2DD(eb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{152}$Sm</td>
<td>0.3840</td>
<td>0.0000</td>
<td>0.0084</td>
<td>-0.0244</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.1450</td>
<td>-0.4289</td>
</tr>
</tbody>
</table>

Table 1: Parameters used in IBA-1 Hamiltonian (all in MeV).
nucleus\hspace{1cm} E_{4^+}/E_{2^+}\hspace{1cm} E_{6^+}/E_{2^+}\hspace{1cm} E_{8^+}/E_{2^+}\hspace{1cm} E_{6^+}/E_{8^+}\hspace{1cm} E_{6^+}/E_{10^+}\hspace{1cm} BE2(4^+_1 - 2^+_0)/BE2(2^+_1 - 0^+_0)
\hline
^{152}\text{Sm}\hspace{1cm} 3.02\hspace{1cm} 5.83\hspace{1cm} 9.29\hspace{1cm} 5.66\hspace{1cm} 1.03\hspace{1cm} 8.92\hspace{1cm} 1.53
X(5)\hspace{1cm} 3.02\hspace{1cm} 5.83\hspace{1cm} 9.29\hspace{1cm} 5.65\hspace{1cm} 1.53\hspace{1cm} 6.03\hspace{1cm} 1.58
\hline
\end{tabular}

Table 2: Energy and transition probability ratios.

The presence of (+ve) and (−ve) parity states has encouraged us to study the staggering effect \cite{21-23} for \(^{152}\text{Sm}\) isotope using staggering function equations (11, 12) with the help of the available experimental data \cite{18}.

\[
S_t(I) = 6\Delta E(I) - 4\Delta E(I - 1) - 4\Delta E(I + 1) + \\
+ \Delta E(I + 2) + \Delta E(I - 2),
\]
with
\[
\Delta E(I) = E(I + 1) - E(I).
\]

The calculated staggering patterns are illustrated in Fig. 6 and show an interaction between the (+ve) and (−ve) parity states for the ground state band of \(^{152}\text{Sm}\).

3.4 Electric monopole transitions
The electric monopole transitions, \(E0\), are normally occurring between two states of the same spin and parity by transferring energy and zero unit of angular momentum. The strength of the electric monopole transition, \(X_{i'f}(E0/E2)\), \cite{24} can be calculated using equations (13, 14) and presented in Table 5.

\[
X_{i'f}(E0/E2) = \frac{B(E0, I_i - I_f)}{B(E2, I_i - I_f)},
\]

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\]
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\[
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\]
Table 4: Calculated $B(E2)$ in $^{152}$Sm (* from Ref.[20])

<table>
<thead>
<tr>
<th>$I_f^+ I_i^+$</th>
<th>$B(E2)Exp^*$</th>
<th>$B(E2)IBA-1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^+ 0^+$</td>
<td>0.670(15)</td>
<td>0.6529</td>
</tr>
<tr>
<td>$3^+ 1^+$</td>
<td>-----</td>
<td>0.0168</td>
</tr>
<tr>
<td>$4^+ 2^+$</td>
<td>1.017(4)</td>
<td>1.0014</td>
</tr>
<tr>
<td>$5^+ 3^+$</td>
<td>1.179(33)</td>
<td>1.1304</td>
</tr>
<tr>
<td>$6^+ 4^+$</td>
<td>0.176(1)</td>
<td>0.3363</td>
</tr>
<tr>
<td>$2^- 1^-$</td>
<td>0.0258(26)</td>
<td>0.0610</td>
</tr>
<tr>
<td>$2^- 1^-$</td>
<td>0.091(11)</td>
<td>0.1057</td>
</tr>
<tr>
<td>$2^- 1^-$</td>
<td>0.0035(35)</td>
<td>0.0003</td>
</tr>
<tr>
<td>$2^- 1^-$</td>
<td>0.037(23)</td>
<td>0.0458</td>
</tr>
<tr>
<td>$2^- 1^-$</td>
<td>0.0163(11)</td>
<td>0.0141</td>
</tr>
<tr>
<td>$2^- 1^-$</td>
<td>0.0417(42)</td>
<td>0.0125</td>
</tr>
<tr>
<td>$2^- 1^-$</td>
<td>0.0416(32)</td>
<td>0.0296</td>
</tr>
<tr>
<td>$2^- 1^-$</td>
<td>0.0035(13)</td>
<td>0.0038</td>
</tr>
<tr>
<td>$2^- 1^-$</td>
<td>0.0037(13)</td>
<td>0.0084</td>
</tr>
<tr>
<td>$3^- 2^-$</td>
<td>-----</td>
<td>0.1235</td>
</tr>
<tr>
<td>$3^- 2^-$</td>
<td>-----</td>
<td>0.0070</td>
</tr>
<tr>
<td>$3^- 2^-$</td>
<td>-----</td>
<td>0.1110</td>
</tr>
<tr>
<td>$3^- 2^-$</td>
<td>-----</td>
<td>0.6418</td>
</tr>
<tr>
<td>$8^+ 6^+$</td>
<td>-----</td>
<td>1.1681</td>
</tr>
<tr>
<td>$8^+ 6^+$</td>
<td>-----</td>
<td>0.0376</td>
</tr>
<tr>
<td>$10^+ 8^+$</td>
<td>-----</td>
<td>1.1421</td>
</tr>
</tbody>
</table>

Table 5: $X_{f'f}(E0/E2)$ ratios in $^{152}$Sm (* from Ref [20]).

<table>
<thead>
<tr>
<th>$I_f^+ I_i^+$</th>
<th>$X(E0/E2)Exp^*$</th>
<th>$X(E0/E2)IBA-1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+ 0^+$</td>
<td>0.7(0.1)</td>
<td>0.85</td>
</tr>
<tr>
<td>$0^+ 0^+$</td>
<td>-----</td>
<td>3.68</td>
</tr>
<tr>
<td>$0^+ 0^+$</td>
<td>-----</td>
<td>0.72</td>
</tr>
<tr>
<td>$0^+ 0^+$</td>
<td>-----</td>
<td>4.39</td>
</tr>
<tr>
<td>$0^+ 0^+$</td>
<td>-----</td>
<td>0.64</td>
</tr>
<tr>
<td>$0^+ 0^+$</td>
<td>-----</td>
<td>1.27</td>
</tr>
<tr>
<td>$2^+ 1^+$</td>
<td>4.5(0.5)</td>
<td>3.52</td>
</tr>
<tr>
<td>$2^+ 1^+$</td>
<td>-----</td>
<td>12.23</td>
</tr>
<tr>
<td>$2^+ 1^+$</td>
<td>-----</td>
<td>11.19</td>
</tr>
<tr>
<td>$4^+ 2^+$</td>
<td>-----</td>
<td>1.76</td>
</tr>
<tr>
<td>$4^+ 2^+$</td>
<td>-----</td>
<td>1.40</td>
</tr>
<tr>
<td>$4^+ 2^+$</td>
<td>-----</td>
<td>0.44</td>
</tr>
<tr>
<td>$4^+ 2^+$</td>
<td>-----</td>
<td>3.15</td>
</tr>
<tr>
<td>$4^+ 2^+$</td>
<td>6.6(2.10)</td>
<td>2.02</td>
</tr>
<tr>
<td>$6^+ 4^+$</td>
<td>-----</td>
<td>1.46</td>
</tr>
<tr>
<td>$8^+ 6^+$</td>
<td>-----</td>
<td>1.20</td>
</tr>
<tr>
<td>$10^+ 8^+$</td>
<td>-----</td>
<td>1.07</td>
</tr>
</tbody>
</table>

where $I_f = I_f = 0, I_f = 2$ and $I_f = I_f \neq 0, I_f = I_f$.

$$X_{f'f}(E0/E2) = \left(2.54 \times 10^9\right) A^{3/4} \times \frac{E_2^3(E0)}{\Omega_{KL}} \frac{\alpha(E2)}{E'} \frac{T(E0, I_i - I_f)}{T(E2, I_i - I_f)} \ ,$$

where:

- $A$ : mass number;
- $I_f$ : spin of the initial state where $E0$ and $E2$ transitions are depopulating it;
- $I_f$ : spin of the final state of $E0$ transition;
- $E_f$ : gamma ray energy;
- $\Omega_{KL}$ : electronic factor for $K,L$ shells [25];
- $\alpha(E2)$ : conversion coefficient of the $E2$ transition;
- $T(E0, I_i - I_f)$ : absolute transition probability of the $E0$ transition between $I_i$ and $I_f$ states, and
- $T(E2, I_i - I_f)$ : absolute transition probability of the $E2$ transition between $I_i$ and $I_f$ states.

### 3.5 Conclusions

The $IBA-1$ model has been applied successfully to the $^{152}$Sm isotope and:

1. Levels energy are successfully reproduced;
2. Potential energy surfaces are calculated and show $X(5)$ characters to $^{152}$Sm;
3. Electromagnetic transition rates $B(E2)$ and $B(E2)$ are calculated;
4. Staggering effect has been calculated and beat pattern observed which show an interaction between the $(-ve)$ and $(+ve)$ parity states, and
5. Strength of the electric monopole transitions $X_{f'f}(E0/E2)$ are calculated.

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### References