LETTERS TO PROGRESS IN PHYSICS

A Re-examination of Kirchhoff's Law of Thermal Radiation in Relation to Recent Criticisms: Reply

Pierre-Marie Robitaille

Department of Radiology, The Ohio State University, 395 W. 12th Ave, Columbus, Ohio 43210, USA.
robitaille.1@osu.edu

Recently, Robert J. Johnson submitted an analysis of my work, relative to Kirchhoff's Law of Thermal Emission (R.J. Johnson, A Re-examination of Kirchhoff's Law of Thermal Radiation in Relation to Recent Criticisms. *Prog. Phys.*, 2016, v. 12, no. 3, 175–183) in which he reached the conclusion that *"Robitaille's claims are not sustainable and that Kirchhoff's Law and Planck's proof remain valid in the situations for which they were intended to apply, including in cavities with walls of any arbitrary materials in thermal equilibrium"*. However, even a cursory review of Johnson's letter reveals that his conclusions are unjustified. No section constitutes a proper challenge to my writings. Nonetheless, his letter is important, as it serves to underscore the impossibility of defending Kirchhoff's work. At the onset, Kirchhoff formulated his law, based solely on thought experiments and, without any experimental evidence (G. Kirchhoff, Über das Verhältnis zwischen dem Emissionsvermögen und dem Absorptionsvermogen. der Körper fur Wärme und Licht. *Pogg. Ann. Phys. Chem.*, 1860, v. 109, 275–301). Thought experiments, not laboratory confirmation, remain the basis on which Kirchhoff's law is defended, despite the passage of 150 years. For his part, Max Planck tried to derive Kirchhoff's Law by redefining the nature of a black body and relying on the use of polarized radiation, even though he realized that heat radiation is never polarized (Planck M. *The Theory of Heat radiation*. P. Blakiston's Son & Co., Philadelphia, PA, 1914). In advancing his proof of Kirchhoff's Law, Max Planck concluded that the reflectivities of any two arbitrary materials must be equal, though he argued otherwise (see P.-M. Robitaille and S. J. Crothers, *"The Theory of Heat Radiation"* Revisited: A Commentary on the Validity of Kirchhoff's Law of Thermal Emission and Max Planck's Claim of Universality. *Prog. Phys., 2015, v. 11, no. 2, 120–132*). Planck's Eq. 40 (ρ=ρ'), as presented in his textbook, constituted a violation of known optics. Planck reached this conclusion, because he did not properly treat absorption and invoked polarized light in his derivation. Planck also made use of a carbon particle, which he characterized as a simple catalyst. This conjecture can be shown to result in a violation of the First Law of Thermodynamics, if indeed, all cavities must contain black radiation. In the end, while Johnson attempts to defend Planck's proof, his arguments fall short. Though the author has argued that Kirchhoff's law lacks both proper theoretical and experimental proof, Johnson avoids advancing any experimental evidence from the literature for his position. It remains the case that experimental data does not support Kirchhoff's claims and no valid theoretical proof exists.

If a space be entirely surrounded by bodies of the same temperature, so that no rays can penetrate through them, every pencil in the interior of the space must be so constituted, in regard to its quality and intensity, as if it had proceeded from a perfectly black body of the same temperature, and must therefore be independent of the form and nature of the bodies, being determined by temperature alone… In the interior therefore of an opake red-hot body of any temperature, the illumination is always the same, whatever be the constitution of the body in other respects.

Gustav Robert Kirchhoff, 1860 [1]

1 Introduction

Nearly two centuries have elapsed since Gustav Kirchhoff formulated his Law of Thermal Emission [1, 2]. In that time, this law has achieved unquestioned acceptance by the physics community, standing at the very foundation of thermodynamics, condensed matter physics, and astronomy. It constitutes the central pillar upon which Max Planck built his blackbody expression and his claims for universal constants [3, 4]. Eddington's theory of the stars, based on ideal gases, depends on Kirchhoff's law, in order to account for stellar spectra [5]. This remains true for stellar physics to this day [6, 7]. Kirchhoff's law constitutes a citadel for modern astronomy, defending not only the ideas that stars are gaseous plasmas devoid of lattice structure [5–7], that white dwarfs and neutron stars are highly compressed objects, and that black holes exist [8], but also the concept that a primordial atom once emitted a thermal spectrum and gave rise to the universe [9, 10]. It is precisely because Planck, Eddington, Chandrashekhar, Penzias, Wilson, Dicke, Peebles, Roll, and Wilkinson [1–10] relied on Kirchhoff's law, that they could ignore the central role of the structural lattice in helping to define the emissivity of an object.* While this could be understood in the days of Gustav Kirchhoff, it can no longer be permitted, in light of the tremendous advances made in condensed matter physics and medicine.

Hence, over the course of the past 15 years, I have turned my attention to Kirchhoff's law [13–18, 20–24, 24–26]. My interest in this law did not arise from any desire to study astronomy, but rather, as a consequence of assembling the first ultra high field magnetic resonance imaging (UHFMRI) scanner, at The Ohio State University [27–29]. It was as a direct result of questioning what it meant to say that nuclear magnetic resonance (NMR) and magnetic resonance imaging (MRI) were thermal processes. This had been highlighted long ago by Felix Bloch (Nobel Prize, physics, 1952) who was concerned with thermal processes linking the lattice and the spins [30].†

The laws of emission [1–4, 31–33], are just beginning to impact upon human medicine, as MRI scanners continue to be pushed to ever higher frequencies [27–29]. Thus, there is much more at stake here than the quest to a better understanding of the universe. Correcting Kirchhoff involves moving to a proper description of all thermal processes, not only in physics and astronomy, but in a field as seemingly remote and unrelated as radiology. I have stated that Planck's blackbody law, although valid, remains unlinked to physical reality [12, 17, 19, 23]. That is precisely because of Kirchhoff's faulty law. The physics community has not provided for thermal radiation what is evident for every other spectroscopic process, namely: 1) the setting under which emission occurs, 2) the nature of the energy levels involved, and 3) the nature of the transition species. Only 4) an equation, and 5) the emission of light, have been described [12]. Yet, in every other spectroscopic process, equations are related to physical reality. It takes a hydrogen atom, for instance, to obtain a Lyman or Balmer line. In that case, the transition species is the electron and the electronic orbitals constitute the energy levels. But, for blackbody radiation, spectra are related only to theory, unrestrained by a particular setting, such as the need to have a structural lattice.

That is how astronomers can justify the creation of blackbody spectra from any object. For instance, they have summed a large number of spectroscopic processes to account for the thermal emission from the Sun (see [34] for a complete discussion of this problem). Yet, not one of these processes can be related to the thermal emission from graphite. They have hypothesized that the Big Bang has generated the microwave monopole which surrounds the Earth [10], but have ignored the hydrogen bond from the water which makes up the oceans bathing our planet [35]. Once again, unrestricted by the need to describe thermal emission using a physical mechanism, astronomy has been left to postulate without any consideration of the central physical question in thermal emission: what causes a thermal photon to be emitted by graphite [19]?

Given all that is involved relative to the validity of Kirchhoff's Law [1, 2], Robert Johnson is to be commended, as the first duty of a scientist is to defend established science against possibly false charges. He has also been forthright in submitting a letter to this journal [36], rather than rely on anonymous attacks through social media.

At the same time, it would be an injustice to fail in one's own defense, when a proper understanding of science rests on the outcome. Therefore, I have decided to provide a point by point discussion of Johnson's letter [36]. I do so with the hope that some members of the physics community will begin to take an interest in Kirchhoff's claims and call into question many of the ideas which have been hypothesized [5–10], as a result of concepts which predate the discovery of the atom.

Before I begin analyzing the contents of Johnson's letter, it is vital to outline the setting under which Max Planck viewed a blackbody, as described in *The Theory of Heat Radiation* [4].

Throughout much of his text, Planck make use of perfectly reflecting walls to construct blackbody cavities. As I mentioned previously, this was *"an interesting approach"* [17, p. 4], precisely because such walls, in Planck's context [4], were *"adiabatic, by definition"* [17, p. 4]. They could not participate in generating, or absorbing, a single photon. Moreover, being adiabatic, they were also immune to all conductive and convective processes.

Conversely, unlike Planck, in thinking about perfectly reflecting cavities, I have invoked silver as a nearly ideal reflector of radiation in the infrared [21, 26]. Furthermore, I have insisted that cavities, constructed from such a perfect reflec-

*Nowhere is this more evident than when Eddington insisted that white dwarfs had to possess a small radius, in order to account for their lack of luminosity [5], given the well-established mass-luminosity relationship. Had Eddington considered the critical role of structure in defining emissivity, he would have seen that white dwarfs simply had a different hydrogen based lattice than the hexagonal planar arrangement shared by the Sun and the stars of the main-sequence (see [11, 12] and references therein). But deprived of the use of a lattice, when he stated that all stars could be viewed as ideal gases, Eddington had no other means of explaining the lower than expected luminosity of the white dwarf. Therefore, he was forced to reduce their radius to unreasonable values [5]. This was the first step towards hypothesizing highly dense objects, including the densities now attributed to neutron stars and black holes [8].

†Suffice it to say that the cavity experiments discussed later in this letter have relevance to both blackbody radiation and MRI. Furthermore, any valid analysis of noise power in MRI will be critically based on properly defining and modeling the processes responsible for thermal emission.

tor, possess a characteristic temperature. They are also subject to conductive and convective heat transfer in the establishment of thermal equilibrium. These are important modifications in properly addressing all thermal processes, including radiation, convection, and conduction. For while Planck properly insists that, at thermal equilibrium, there can be *"no conduction"* [4, § 25], no-one maintains that cavities cannot be subject to conductive processes in reaching thermal equilibrium. Laboratory blackbodies are usually brought to temperature using conduction. This will be important later in this letter.

As for adiabatic walls, they could never be characterized by *any* temperature, as I recently emphasized [23]. Consequently, they could never be in thermal equilibrium with anything. Planck stated *"Hence in a vacuum bounded by perfectly reflecting walls any state of radiation may persist"* [4, § 51]. That was very true. But it is also true that such cavities are devoid of any radiation, unless it had previously been injected by some outside means [15–17, 20–23, 26].

In the initial sections of his text, Planck had insisted that all of the energy could be characterized by the radiation field. In truth, the energy must have, at some time, been associated with his oscillators. Otherwise, no photons could have been produced. Thus, Planck's oscillators could be used to produce the field and set thermal equilibrium, but the energy of the system had to be considered as being irreversibly transfered to the radiation field: *"Accordingly we have frequently . . . pointed out that the simple propagation of free radiation represents a reversible process. An irreversible element is introduced by the addition of emitting and absorbing substances"* [4, § 170].

This irreversibility and the need for the oscillators to have access to energy, in order to produce the photons, was vital to properly understanding this work. In addition, Planck admitted, in the very last section of his text that *"For the oscillators on which the consideration was based influence only the intensities of rays which correspond to their natural vibration, but they are not capable of changing their frequencies, so long as they exert or suffer no other action than emitting or absorbing radiant energy"* [4, § 190].

Planck insisted that he could place a minute particle of carbon within his cavities. He viewed this object as a catalyst [4, § 51–52], converting radiation within the cavity from one form to another: *". . . This change could be brought about by the introduction of a carbon particle, containing a negligible amount of heat as compared with the energy of radiation. This change, of course, refers only to the spectral density of the radiation u_ν, whereas the total density of the energy u remains constant"* [4, § 71]. Planck's particle could only act on the radiation which was already in the cavity. It could not interact with the walls, introduce new energy into the cavity, or set the temperature of the system.

But to interact with the radiation, the carbon particle must have oscillators of its own, functioning over the proper frequency range. Namely, it must be a perfect absorber, characterized by a temperature and part of the thermal equilibrium problem, not a catalyst uncharacterized by any temperature. If devoid of a characteristic temperature, Planck's carbon particle would not contain the proper vibrations to even interact with the radiation in the cavity.

Neither the walls of Planck's perfectly reflecting adiabatic cavities, nor the catalytic carbon particle, could establish temperature. Planck resorted to placing all of the heat within the radiation field. None of the energy could be contained in the walls. He then altered the nature of his walls and removed the requirement that they could not interact with radiation: *"Since, according to this law, we are free to choose any system whatever, we now select from all possible emitting and absorbing systems the simplest conceivable one, namely, one consisting of a large number N of similar stationary oscillators. . . "* [4, § 135]. Note from this quotation, that Planck could advance no mechanism by which oscillators can actually alter the radiation distribution within the cavity. Planck's oscillators cannot convert the radiation from one form to another, as would be required in the action of Planck's carbon particle were simply catalytic. It remains the case that the radiation contained within a cavity can only be characterized by the nature of the oscillators which produced it. For all these reasons, Planck's carbon particle could never by considered as a catalyst. Indeed, if this particle is attributed with only a catalytic function, it can easily introduce a violation of the First Law of Thermodynamics, as will be seen in § 9 below.

At this point, it is time to address Johnson's submission [36]. In order to maintain the same section numbers, I begin immediately with a review of his introduction [36, § 1].

Johnson's first errors occur in his opening statement, wherein he asserts that I have *". . . challenged the validity of Kirchhoff's Law of thermal emission and Planck's derivation of the mathematical form of the universal function of spectral radiance absorbed and emitted by a blackbody"*. There are actually two problems with this statement.

First, I never questioned the mathematical validity of Planck's expression, in the context of an actual blackbody. Rather, I have stated repeatedly that Planck's solution for a blackbody was correct (see e.g. [12, 16, 17, 25]). For instance, in [16, § 1] it is explicitly written that *"The accuracy of Planck's equation has been established beyond question"*. Along with Crothers, I state that *"Fortunately, in Planck's case, the validity of his equation is preserved, but only within the strict confines of the laboratory blackbody"* [25, § 4].*

Secondly, the absorbance of a blackbody does not have a functional form, contrary to Johnson assertion. When Kirch-

*It is troubling that Johnson has misrepresented my position on this matter. My concern has been exclusively centered on Kirchhoff's formulation of a law extending to objects which are not solids and which are constructed from materials lacking a good absorber [13–18, 20–24, 24–26]. I have never questioned the validity of Planck's equation in the case of proper blackbodies.

hoff formulated his law, he defined $E/A = e$ and immediately set A to 1 [1]. This enables the function "e" to have units. Johnson failed to understand, at the onset, Kirchhoff's formulation. Such errors continue throughout his letter [36].

2 Robitaille's earlier papers

2.1 Kirchhoff's law and Planck's proof

Johnson affirms in § 2, relative to Planck's law, that … *"Planck's derivation is seen as proof of Kirchhoff's law"*. This is not correct.

There are actually several questions addressed in Planck's treatment. This was made clear in the manner in which Planck wrote his book, *The Theory of Heat Radiation* [4].

Planck was primarily concerned with two issues. First, did all arbitrary cavities contain black radiation? This is addressed in the first two chapters [4, § 1–52]. Secondly, Planck was focused on providing the functional form for the blackbody spectrum, through much of the remainder of his presentation. He did so by reviewing the laws of emission advanced by Wein [31] and Stefan [32].* He then discussed Boltzmann and entropy, and presented his oscillators and the blackbody function. In fact, the derivation of the blackbody function itself was completely independent of the derivation of Kirchhoff's law, since when setting $A = 1$, one obtains $E = e$ from Kirchhoff. The functional form of the blackbody spectrum can be obtained, without insisting that all cavities contain black radiation. Planck derived Kirchhoff's Law in the first section of his text solely because of his desire to confer, upon the blackbody expression, universal implications.

If Kirchhoff's Law would be found invalid, as it will eventually become, then Planck does not lose the functional form he supplied describing the radiation of a blackbody, as I have stated repeatedly [12,16,17,25]. However, it would imply that arbitrary cavities are not necessarily blackbodies and that the universality of the constants h and k does not hold [13–18, 20–24, 24–26].

In the next sentence, Johnson writes [36, § 2]: *"However, Robitaille points out that the above definition of Kirchhoff's Law is not complete and furthermore Robtaille maintains that the statement above should be called Stewart's Law as it was originally propounded by Stewart in 1858"*. How could Johnson make such claims?

He begins by omitting an important concept when citing my work. The complete citation is as follows: *"All too frequently, the simple equivalence between apparent spectral absorbance and emission is viewed as a full statement of Kirchhoff's law, adding further confusion to the problem. Kirchhoff's law must always be regarded as extending much beyond this equivalence. It states that the radiation within all true cavities made from arbitrary walls is black. The law of equivalence is Stewart's"* [17]. Importantly, in this citation, I had also included references wherein Kirchhoff's Law was described, solely in the context of the Law of Equivalence, and not within its full scope relative to claiming that a universal function existed. In any event, I never claimed that Kirchhoff's Law was not complete. What I did state was that people often give credit to Kirchhoff for the Law of Equivalence which properly belongs to Stewart [33]. As for Kirchhoff's Law, it is incorrect. Johnson does not seem to understand the fundamental differences between Stewart's Law [33] and Kirchhoff's [1].

The Law of Equivalence [33] simply affirms that, at thermal equilibrium, the radiation emitted by a surface will be equal to the radiation it absorbs, emissivity, ϵ, is equal to absorptivity, α. Stewart did not insist that the radiation inside all cavities was black. That is the reason Kirchhoff's Law [1] does not belong to Stewart [33]. This is an important point, as Johnson falsely asserts, throughout his letter, that Stewart recognized that cavity radiation must always be black. Rather, Stewart recognized that all cavities could become black if they could be driven (see [16] for further discussion). The problem, of course, is that cavities constructed from low emissivity materials cannot be properly driven [15–17, 22–25].

Stewart, while aware of mathematical arguments which might lead to such a conclusion, left the discussion to a footnote [33]. The reason was clear. Stewart recognized, as an experimentalist, that he was not able to prove, in the laboratory, that all arbitrary cavities were black. The experiments described in his work dealt with emission from plates and surfaces [33], not cavities [36]. That was precisely why he did not make a law for cavities, as *The Laws of Physics must be experimentally verified*. In his rebuke of Kirchhoff, Stewart had made the point plainly *"nor did I omit to obtain the best possible experimental verification of my views, or to present this to men of science as the chief feature, grounding theory upon experiment, rather than deducing the experiments from the theory"* (cited in [16]). Stewart never presented any experiments on cavities and therefore, he never made a law related to cavities, as Johnson claims I stated [36, § 2.1].

This was a central difference between the work of Stewart [32] and Kirchhoff [1, 2]. Johnson could have easily come to learn the distinction had he studied the historical review by Seigel [37], which I had cited in [16]. Siegel highlighted that … *"Kirchhoff himself never performed any experiments which could be construed as attempts at quantitative experimental verification of his law"* [37, p. 588]. Seigel went on to state what Kirchhoff believed: *"… Kirchhoff was rightly pointing out that in this instance neither Stewart's experiments nor his own experiments sufficed to establish a quantitative law, and the burden of the priority claims would therefore have to rest on theoretical proof"* [37, p. 588]. Unfortunately, Kirchhoff was not right. Stewart's experiments were more than adequate to establish the Law of Equivalence. It was with the treatment of cavities that experimental confirmation was lacking.

*Planck never addressed the contributions of Balfour Stewart [33].

In any event, experiments take precedence over theory when it comes to formulating a new law, as our theories are not able to define nature. Furthermore, it is all too easy to accidentally omit a critical element from a theoretical discussion, as has happened when Kirchhoff and Planck unknowingly ignored the energy trapped within the walls of cavities. Such energy can remain forever unavailable to thermal emission. That is why Kirchhoff's Law is invalid. It also provides an illustration of the danger of inferring the laws of physics from theory.

In the end, Seigel also highlighted the difference between Stewart's law and Kirchhoff's claims: *"Stewart's conclusion was correspondingly restricted and did not embrace the sort of connection between the emissive and absorptive powers of different materials, through a universal function of wavelength and temperature, which Kirchhoff established"* [37, p. 84]. It is clear that Stewart's Law did not encompass the universal nature of cavity radiation which Kirchhoff sought, as Johnson attempts to inappropriately claim throughout his letter.

This section closes with Johnson quoting from § 51 of Planck's text [4] and insisting that by placing an *"arbitrary small quantity of matter"* in a perfectly reflecting cavity that *"Planck had thereby demonstrated that all cavities either containing some arbitrary matter, or equivalently having walls made of some arbitrary matter, must also contain black radiation when at thermal equilibrium"*. Yet, in § 51, Planck was placing a small particle of carbon in the cavity. The carbon particle was not an arbitrary material. It was acting as a perfect absorber. I have discussed the inappropriate introduction of a perfect absorber into cavities in detail [16] and will return to the question, once again, in § 2.2, § 2.5 and § 2.9 of this letter.

2.2 Black radiation in a perfectly reflecting cavity

As Johnson opens the third section of his letter, he objects to my conclusion that Planck's statement, *"Hence in a vacuum bounded by perfectly reflecting walls any state of radiation may persist"* [4, § 51], constituted an implicit admission against the validity of Kirchhoff's law.

In trying to defend Planck, Johnson writes: *"However, Planck's statement should perhaps be more properly viewed as a situation to which Kirchhoff's law does not apply because there is no matter present which could absorb or emit radiation."* However, Kirchhoff's law was meant to be independent of the nature of the walls, by definition. Planck associated the temperature of a cavity solely with the radiation it contained, not with any material particles.*. If Kirchhoff was correct, what difference should it make if matter was present to absorb or emit radiation? Nothing in Kirchhoff's law required this restriction and that was precisely the problem. Kirchhoff's law was devoid of all link to actual materials and nature. It was only concerned with hypothetical cavities.

In considering Kirchhoff's law, we can simply examine mathematical limits, as defined by the opaque perfectly absorbing wall (absortivity, $\alpha_\nu = 1$; reflectivity, $\rho_\nu = 0$) and the opaque perfectly reflecting wall ($\alpha_\nu = 0$; $\rho_\nu = 1$). Yet, the second condition led to an undefined expression for Kirchhoff's law, as Planck himself recognized [4, § 48]. It was not possible to claim that a law applies to all materials, when one of its limits was undefined.

Johnson goes on to cite Planck's §51 stating that the radiation within all cavities will always be black, even though Planck, in the very same section, has just introduced a particle of carbon in this cavity, which Johnson recognizes as being a *"perfect absorber and emitter at all frequencies"* [36]. But, Planck viewed the carbon particle as a catalyst [4, § 51–51]. Johnson then writes, in speaking of Planck: *"Note that the quoted statement covers both the situation where the object absorbs or emits over all frequencies, and the situation where some frequencies are not absorbed or emitted at all"* [36].

Planck reached his conclusion by inserting a particle of carbon. This ensured absorption and emission over all frequencies. Planck never demonstrated that this applied to *situations where some frequencies are not absorbed or emitted at all"*, as Johnson claims [36]. Planck placed the carbon particle within the cavity and then claimed that it acted only as a catalyst. He sidestepped the fact that this particle was acting as a perfect absorber, and thereby controlled the entire problem. I have already demonstrated this fact mathematically and the reality that arbitrary cavities, at thermal equilibrium, do not contain black radiation [15]. Importantly, Johnson's letter fails to address these simple algebreic proofs that Kirchhoff's Law cannot be valid [15].† Again, I will return to the question of the carbon particle in § 2.5 and § 2.9.

2.3 The approach to equilibrium

Johnson opens this section by pondering what was correct: Do all cavities contain black radiation, as Kirchhoff and Planck held, or do arbitrary cavities contain arbitrary radiation, as Robitaille asserted? The question was simple enough to answer, as blackbodies are always constructed from good absorbers.

In fact, had Johnson considered the history of blackbody radiation, he would have recognized that arbitrary cavities are never black. That is why those who provided Max Planck with the data used to verify his equation worked so hard to

*"Still, even Planck recognized that material objects were required to establish a temperature, *"But the temperature of a radiation cannot be determined unless it be brought into thermodynamic equilibrium with a system of molecules or oscillators, the temperature of which is known from other sources"* [4, § 144]

†Reference [15] contains a detailed analysis of some of the problems with Kirchhoff's logical arguments in advancing his proofs [1,2]. It also contains simple proofs of Stewart's Law of Equivalence [33] and clear demonstrations that arbitrary cavities, under conditions of thermal equilibrium, are not black. Johnson cannot ignore these proofs in his letter, if he wishes to honestly evaluate my work.

construct laboratory blackbodies which provided the proper functional form [38–40]. These papers, especially the review by Hoffmann [38], are important to study, because they highlight the complexity of building proper blackbodies.

As a simple example, the problem can be viewed to involve, to some extent, the behavior of graphite itself. In the visible range, some forms of graphite, which are mined, can be relatively good absorbers, but others, surprisingly, can be rather poor, as can be ascertained by examining emissivity tables [41]. However, as one becomes increasingly interested in the region towards the infrared, graphite begins to fail. This has been known since the days of Langley at the end of the 19th century [16, § 2.1]. That is why materials like the metal blacks are utilized, in this region of the electromagnetic spectrum, to assemble blackbodies [42–45].

We already have the experimental proof, but most people simply ignore these laboratory realities. For, if Kirchhoff's Law was valid, there would be no need for metal blacks in building laboratory blackbodies and German scientists would not have used rolled platinum and specialized mixtures of chromium, nickel, and cobalt oxide to blacken the interior of their cavities [38, p. 57]. Such mixtures indicate that their was nothing arbitrary in the construction of blackbodies.

This remains a specialized field and such objects are *always* sophisticated devices unavailable when Kirchhoff extended his law to all cavities.*

In this same section of his letter, Johnson goes on to consider what would happen to the radiation, within an arbitrary cavity, if the initial radiation was less than the maximal hypothesized by Kirchoff's law. The arguments he advanced are flawed at a fundamental level.

Johnson first places an opaque object within a perfectly reflecting cavity and defines that the intensity of the radiation is 100 within the cavity, the proper value for black radiation. He assumes that the object has an emissivity of 0.8 and then states that when radiation within the cavity interacts with the object, 80 units will be absorbed/re-emitted and 20 units being reflected. Johnson notes that the radiation within such a cavity will remain black at 100 units. Of course, the experiment is false, as an object with an emissivity of only 0.8 could never fill the cavity with black radiation in the first place. The radiation would have to be increased by some other means.† Deviations from this case are only permitted if thermal equilibrium has been violated, after the cavity and the object reached the temperature of interest, or if the perfectly reflecting cavity has otherwise been filled with black radiation [16, 17]. It is important to recall, that even the sampling of a cavity with a detector can act to fill it with black radiation [17, § 2]. Therefore, this situation, as described by Johnson, does not lend any support to Kirchhoff's claims. It was simply ill-conceived.

At this point, Johnson considers another scenario wherein an object with an emissivity of 0.8 can only emit 80 units initially into the cavity. These 80 units then strike the wall and reflect back towards the object, where now he claims that only 64 units are absorbed (since the emissivity is 0.8), and 16 units are reflected. Johnson notes that the object *"... was bound by its initial temperature to continue emitting 80 units"* [36, § 2.3]. He notes the shortfall in the total amount of radiation absorbed by the object, and claims that this can only be rectified by lowering the temperature of the object. The errors in logic are striking.

First, Johnson fails to recognize that it is the total radiation coming off the object at thermal equilibrium which matters. That total radiation is equal to 64 units emitted and 16 units reflected at the onset, because the cavity and the object are already at thermal equilibrium, by definition. Johnson does not get to say that the object must emit 80 units to begin his experiment and then state that only 64 units are absorbed and re-emitted. He can only sample the total radiation coming off the object. He has no means of distinguishing what was, in fact, reflected and what was emitted. He only knows that 80 units came off the object. These are then reflected off the wall and travel towards the object, where 64 units will be absorbed, then re-emitted, while 16 units will be reflected. Johnson also fails to understand that he cannot allow the temperature of the object to drop, as this is a violation of the zeroth law of thermodynamics. For my part, I would not disallow thermal equilibrium between the cavity and the object, as Johnson asserts.‡

Relative to the last experiment, it is interesting to note what Johnson has actually done. At first, he ignored reflection, stating that all 80 units leaving the object were emitted. Then, on absorption, he now considered reflection, permitting only 64 units to be absorbed and the remaining 16 to be reflected. So what has happened?

Note, for instance, that when Max Planck derived the first section of his proof of Kirchhoff's Law, he also ignored reflection (see [25, § 4.2] for a complete description of what Planck did in this instance). Robitaille and Crothers note that Planck was allowed to ignore reflection, as these terms, if retained, could be canceled out [25, § 4.2]. They also demonstrate that the full treatment retaining reflection can lead to additional insight, relative to this problem [25, § 4.2].

If Planck was allowed to ignore reflection, perhaps this can be most easily explained by examining the Law of Equivalence itself [33]. I have already highlighted that Stewart's Law can be written either as, $\epsilon_\nu = \alpha_\nu$, or as, $\epsilon_\nu + \rho_\nu = \alpha_\nu + \rho_\nu$ [15]. The use of either form will lead to the correct answer. However, what Johnson has done was to mix the two forms of Stewart's law, inventing a scenario wherein he sets $\epsilon_\nu = \alpha_\nu + \rho_\nu$,

*The author has reviewed laboratory blackbodies in [16, 17].

†I have already demonstrated mathematically, that the radiation in the cavity, in this case, will not be black but will have an intensity appropriate for the emissivity of the object it contained [15].

‡I also reject all of Johnson's other deductions relative to how I would view an experiment which I never even described in my papers.

which is clearly false.

At this point, Johnson once again tries to state that I have attributed Kirchhoff's Law to Balfour Stewart. In this, he misses the central point. Stewart's footnote does not make a law of physics. It presents a mathematical argument. Stewart recognized that, if he wanted a blackbody spectrum from a cavity, he must have recourse to lampblack. Johnson believes that Stewart was specific on this point, arguing for a *"theoretical leap"* [36]. But in so thinking, he failed to recognize what Stewart understood: cavities can only be *demonstrated* to be black experimentally if they contain a good emitter. Stewart did hypothesize extensively about bandied radiation, well after 1858 (see [16] for a complete discussion), and conjectured that cavities of low emissivity can be made to appear black. The thesis was never proven and with good reason [15–17, 22–25]. Stewart stated a theoretical idea, not a law. The point has been made clearly by Seigel, as noted in § 2.1 above [37]. That is why I wrote in my initial paper: *"Stewart realizes that the lampblack surface within the enclosure is essential"* [16]. Stewart might have had a theoretical argument, but he did not have data. It is in this aspect that he was much more prudent than Kirchhoff when he presented his work [33]. That is why I have always acknowledged this Scottish scientist. Stewart exercised wisdom in 1858 [33] and Johnson shall not deprive him of this quality.

2.4 Stewart's treatment of reflection

Johnson then goes on to describe, in detail, Stewart's footnote, as if this was central to the idea which Stewart was conveying. Stewart's paper deals with the Law of Equivalence, not with cavity radiation and universality [33]. The argument which Johnson resurrects is contained in a footnote, precisely because this constitutes its proper position in the paper. Stewart makes us aware that he understands a mathematical argument previously advanced by others (see references contained in [17]), but he does not raise them to a central part of this thesis, because these ideas were not supported by laboratory data.

In considering the bandied radiation, Johnson makes the claim that the energy required to fill the cavity can be extracted from the walls in order to drive *"Stewart's mechanism"*. In this aspect of his letter, Johnson is actually repeating ideas from my own papers on cavity radiation, wherein such processes have already been discussed in detail [21, 23, 26].* Johnson adds nothing new to this discussion. He also fails to understand, at a fundamental level, that it is by invoking the energy retained in the wall that Kirchhoff's Law can be proven to be false [21, 23, 26]. Planck specifically used an adiabatic wall which could not be characterized by any temperature to build his perfect reflector, because he wanted all of the energy to be contained in the field, not in the wall [4]. Since adiabatic walls are detached from all thermal processes (i.e. radiation, conduction, convection), they cannot be characterized by any temperature [21].

Johnson analyzes Stewart's experiments [33] with low emissivity plates in obtaining the same functional form as if the plate had been black. Yet, it is not solely a question of time elapsed, as he attempts to argue. For instance, he permits the temperature of one of his plates to drop in clear violation of the Zeroth Law of thermodynamics *"... the only difference in this case is that during the initial period the partially absorbing plates is absorbing less radiation than it is emitting; it is therefore cooling down and part of its initial energy is being used to increase the radiation density between the plates, or, in Robitaille's terms, in "driving the reflection"* [36, § 2.4]. I never permitted an object temperature to drop, in order to drive the reflection.

My papers are concerned with a law defined under thermal equilibrium, not the approach to equilibrium. I have highlighted that one cannot create photons from nothing. Scientists are not permitted to violate the First Law. What has happened in this letter is that Johnson permits the temperature to drop in order to avoid violating the First Law, as he knows that he must get photons from somewhere. The arguments are all invalid, as we are concerned with a system in thermal equilibrium, not the approach to such equilibrium.

While Johnson understands that the idea of driving a cavity is an important concept, he continues to ignore its consequences. For instance, such processes rely on access to a perfect absorber, or some temporary violation of thermal equilibrium [15–17, 22–25]. They are also prone to introduce a violation of the First Law of Thermodynamics, as energy must come from somewhere. Also, energy cannot be destroyed.

The question, relative to thought experiments, relates to the origin of the energy entering a cavity once it is already at thermal equilibrium. Provided that the cavity walls are not adiabatic, but can be represented by graphite, or silver, then there are three scenarios to consider: 1) energy enters the system from outside, 2) energy travels reversibly out of the walls of the cavity to irreversibly fill the cavity [21, 26], and 3) energy is irreversbilty trapped within the cavity walls [26]. None of these possibilities have ever been considered by Planck. They arise from the assembly of work which is currently being challenged by Johnson.

Let us assume that the energy came from outside the system. Then, once it reaches the cavity walls, it must be allowed to either 1) help fill the cavity with additional photons, or 2) dissipate additional energy into the walls of the cavity. However, the cavity walls are already at a given temperature. To permit additional energy to enter would alter this value. As such, no energy can be allowed to enter the walls, as this would violate the zeroth law. Thus, if any energy enters the cavity walls from outside the system, it must simultaneously leave and produce additional photons in the interior. It is clear that, with such a scenario, if the walls are fully reversible

*The author published [26] just a few days before Johnson submitted his letter and he was made aware of this work.

stores of energy, the cavity will become filled with radiation. The problem becomes, when does one stop? Obviously, the experimentalist can place any amount of photons in the cavity, given enough available energy from the outside and no concern for the First Law. If the radiation intensity within the cavity becomes too great, then he can simply affirm that thermal equilibrium has been violated and that the cavity must now be represented by a higher temperature.

As for the idea that the energy contained within the walls can be reversibly used to fill the cavity with even more radiation, I have already considered the concept on two occasions [21, 26]. In reality, such processes are likely to be physically impossible. Thermodynamically, the concept is allowed, but the problem is that, if the energy of the walls if fully available to build up photons in the interior, the cavity would already be black, unless specialized means are used to isolate this energy [26]. In reality, every material which is not a perfect emitter will actually possess at least some energy which is irreversibly trapped in the walls [26] relative to the ability to support emission. That is the central reason why arbitrary cavities are never black. Planck had considered that only the production of the radiation field was irreversible, as I discussed in the introduction. This may have been everyone's major stumbling block relative to cavity radiation and Kirchhoff's Law. Prior to 1906, when Planck's lectures where written [4, p. xi], neither he, nor Kirchhoff, understood that some of the energy which enters a metal will be trapped in its conduction band electrons and forever remain unavailable to emission [26].* We shall return to *"Stewart's mechanism"* in § 2.5, § 2.6, § 2.7, and § 2.9.

2.5 Planck's particle of carbon

Johnson then moves to try to defend Planck's use of a carbon particle as a simple catalyst. I have already spoken extensively on this issue: Planck's carbon particle is not a catalyst [16]. It is a perfect absorber/emitter. Planck uses carbon, not a particle of some other material, and with good reason. He needs a perfect absorber. It is not simply a question of having a particle which can absorb over all frequencies of interest. In fact, a quick study of emissivity tables would demonstrate that, if this were the case, Max Planck had many other materials available to him [41]. He wanted a perfect absorber and, when he placed it in his cavity, as I have said previously, it was as if he had coated the entire inner surface with lampblack. Otherwise, what does it mean to be "perfect"? As I stated in the introduction, the reality remains that Planck's carbon particle must have access to oscillators, otherwise it cannot even interact with the radiation. It must also be characterizable with a temperature, such its oscillators could operate over the entire range of frequencies required to make the cavity radiation black at the proper temperature. The need for this temperature directly implies that the carbon particle is a perfect absorber, not a catalyst.

Johnson claims that there is a difference between *"the nature of the black radiation and the quantity of it"*. He then argues that Planck has made the particle small such that its energy content can be neglected relative to filling the cavity with radiation. Planck's position and Johnson's defense are not well-reasoned in that they neglect that the particle and cavity must be allowed to come to thermal equilibrium. This is one of the reasons why Planck's use of an adiabatic wall to build a perfectly reflecting cavity is not appropriate. Planck also attempted to deprive the carbon particle of a specific temperature. In so doing, he was overlooking the very detail which was critical to obtaining the proper answer (see also § 2.9). Johnson states, *"...By definition, therefore, the carbon particle cannot increase the radiation density in the cavity to the level commensurate with the black body temperature; in Robitaille's terms, the particle cannot "drive the reflection", and therefore this cannot be the reason why Planck included it. Furthermore, if the radiation density is being increased at all frequencies by Stewart's mechanism then there is no need to include the carbon particle"* [36, § 2.5]. Unfortunately, for Johnson, he cannot resort to *"Stewart's mechanism"*, as he cannot practically demonstrate its validity in the context of a perfect reflector. Even Stewart, cannot generate photons from a perfectly reflecting cavity. The issue at hand is the carbon particle, not *"Stewart's mechanism"*.

As such, let us first consider the proper way of viewing the carbon particle, then return to Planck and Johnson, both in this section and in § 2.9.

The simplest means of addressing this problem is to consider that whatever light is reflected off the walls of a perfectly reflecting cavity can strike the particle. The particle must then transform the radiation and return this light back towards the cavity walls [15]. The temperature of both the cavity and the particle must be the same and the temperature of the latter must not be allowed to drop in order to respect the zeroth law. Under this condition, full equilibrium between the walls and the particle would exist and the cavity could easily be demonstrated to contain black radiation [15]. Herein was the power of equilibrium arguments.

In order to further clarify the point, let us consider what was physically occurring within the cavity when Planck introduced his small particle of carbon. Since the cavity was perfectly reflecting, we can assume that it can be best approximated by polished silver [23,26], not by an adiabatic wall [4]. The emissivity of the cavity must be 0 and it initially contains no photons. Let us surround the cavity with an adiabatic wall, in order to isolate the system.

As a result, the temperature of the cavity in this case is defined by the energy content of its walls. When the carbon particle is introduced into such a system, even if it contains no appreciable heat on its own, it also comes into thermal contact with the wall of the perfectly reflecting cavity. At this point,

*The energy can still be removed from the wall through conduction and convection.

thermal energy will become available to the carbon particle from the cavity wall. This particle can then transform the energy which would be otherwise irreversibly trapped in the walls [26] and fill the cavity with radiation. In this sense, the carbon particle acted as a transformer, converting phonon energy and/or energy associated with thermal conduction in the silver wall into photons. It was not a catalyst, as it was critical to conversion occurring. I have always modeled perfect reflectors using silver [23, 26], not using adiabatic walls. Without the carbon particle, the cavity would remain devoid of any radiation and all of its energy would remain forever trapped in its walls.

As for the case considered by Max Planck, an adiabatic wall contained no energy. Therefore, the carbon particle, devoid of significant heat, could never fill such a cavity with radiation at any temperature.

Contrary to Johnson's claim, neither Crothers, nor I, have said that the carbon particle cannot increase the radiation inside the cavity. Rather, my papers provided the only means for the carbon particle to fill the perfectly reflecting cavity. As for Johnson, he must adopt Planck's position, and remain forever unable to consider the content of the walls and the ability of the carbon particle to transform this energy into photons. He cannot be permitted to jump between my model and Planck's, as this is the entire basis of this discussion.

If Planck stated that *"Hence in a vacuum bounded by totally reflecting walls any state of radiation may persist"* [4, §51], it was because he recognized that Kirchhoff's law became undefined when A=0. But that does not mean that the cavity in this case contains forms of radiation which are blackbody, unless such radiation has been introduced by some outside mechanism.* In fact, the perfectly reflecting cavity must be considered empty, because it had no means of producing a photon and all of its energy content was trapped in its walls before the introduction of the carbon particle [26]. Johnson argues that, if the spectrum is indeterminate at any frequency, it is impossible to set a temperature. Again this is false, as the walls also contain energy [26]. Max Planck also ignored this fact, a critical error in selecting adiabatic walls.

Johnson then cites Planck's discussion [4, § 11] that all objects show significant reflection at sufficiently long wavelengths, except perfect blackbodies. He concludes that this is why Planck introduced the carbon particle [36]. But, if that was true, then Planck's introduction of the carbon particle would be acting to make all cavities perfect blackbodies, a point which supports my position.

In closing this section, Johnson makes the charge that I now accept, at least in principle, *"Stewart's mechanism"* for building up the reflection within a cavity. He alleges a remarkable *"volte-face"* on my part when I published a paper with Crothers [25].

Such a conclusion is not reasonable, as my papers have always considered Stewart's hypothesis (see e.g. [15–17, 22–24] all of which precede [25]). Furthermore, Crothers and I have restated, in no uncertain terms, that *"Stewart's mechanism"* does not work [25, § 2, 3].

Over the years, I have dealt consistently with the problem of thermal emission and have always held the position that arbitrary cavities are not black. I have examined numerous questions including 1) perfectly absorbing cavities, 2) perfectly reflecting cavities, 3) perfectly reflecting cavities containing a carbon particle, 4) perfectly reflecting cavities containing an arbitrary object, 5) perfectly reflecting cavities containing two arbitrary objects, 6) perfectly absorbing cavities containing a perfectly reflecting cavity, 7) two cavity problems (both for the reversible and the irreversible cases), 8) actual laboratory blackbodies, 9) Kirchhoff's two faulty initial proofs, 10) Planck's faulty proof of Kirchhoff's Law, 11) proper equations governing cavity radiation and 12) effects of driving the reflection term. Nowhere have I ever stated that *"Stewart's mechanism"*, as Johnson refers to bandied reflection, can ever lead to black radiation in all cavities, despite repeatedly addressing the question [15–17, 22–25]. What I have stated is that, if one tries to drive a cavity made from materials with a low emissivity, in order to build up black radiation in its interior, it is likely that the cavity will simply prefer to move to a higher temperature [23]. That is because any energy introduced into the cavity must also be available to the walls. If those walls cannot easily emit a photon, they will simply increase their temperature. Moreover, I have emphasized that the use of bandied radiation, even if possible, could only lead to filling a cavity with black radiation, in the ideal that the walls were capable of Lambertian reflection [23]. No specular reflection must have taken place and all reflection must have been diffuse. Otherwise, one risks generating standing waves, as I have previously highlighted [16] (see also § 2.6). Johnson ignores all these points when he addresses bandied radiation.

2.6 Experimental evidence against Kirchhoff's law

This is perhaps the most unusual section of Johnson's letter [36], as he tries to explain why manufacturers do not build blackbodies from arbitrary materials. Rather than concede that this constitutes direct experimental evidence against Kirchhoff's law, as I have stated, Johnson reaches for the indefensible. He argues: *"It is also likely that manufacturers are concerned, as Planck himself apparently was, to ensure that there are no frequencies at which the cavity is a perfect reflector, which would preclude a proper measurement of temperature"*.

Kirchhoff's Law demands that all cavities be black, independent of the nature of the walls. Manufacturers are not concerned with materials acting as perfect reflectors, since most solids emit continuous spectra over a wide range of frequencies. The problem is that many solids are poor emitters,

*I will return to this issue in § 2.9.

not that they are perfect reflectors.

Furthermore, the temperature of a cavity in the laboratory is determined by temperature sensors in its walls. Cavities are heated, conductively or otherwise, the temperature on the sensors in its walls are noted, and thermal equilibrium is defined by those sensors maintaining a stable temperature reading. Establishing the temperature of a laboratory cavity has nothing to do with measuring its radiation field and it would be irrelevant, if some frequencies were absent from the spectrum. This cannot affect the reading of a sensor in the wall of the cavity.* Even Planck recognized that a proper measure of temperature depends on the use of sensors or thermometers: *"But the temperature of a radiation cannot be determined unless it be brought into thermodynamic equilibrium with a system of molecules or oscillators, the temperature of which is known from other sources"* [4, § 144].

Johnson then moves to question any work in microwave cavities. He launches this new challenge precisely because these cavities are known not to contain black radiation, as I have demonstrated experimentally using UHF frequencies near the microwave region [17]. In attempting to dismiss microwave cavities, Johnson cites Planck: *"The last statement excludes from our consideration a number of radiation phenomena such as fluorescence, phosphorescence, electrical and chemical luminosity"* [4, § 7]. Johnson's use of such a quotation relative to microwave cavities demonstrates that he does not fully understand the experimental problem.

Kirchhoff's law allows for the presence of any object within the cavity. Therefore, the resonant elements used in my own work [17] are allowed, as they do not emit a single photon. They build up standing waves. Still, it remains clear that fluorescence, phosphorescence, electrical and chemical luminosity cannot be considered.†

The microwave cavity is not producing radiation by some non-thermal means, like fluorescence, phosphorescence, electrical or chemical luminosity. Rather, it is being subjected to sampling by a network analyzer which is sending microwave energy into the enclosure and noting what energy returns. If the cavity is able to reflect some of this radiation internally, then it can build up standing waves. Alternatively, if the cavity is truly black, then it should be able to absorb all the energy coming from the network analyzer with no returned energy. In any case, such return-loss measurements on cavities are routinely done throughout thermometry (see references cited in [48]).

In the infrared, cavities can be subjected to radiation from a standard blackbody, for instance, in order to verify their absorptivity by noting the returned energy.‡ In the microwave, when testing blackbodies for satellites, the source is often a network analyzer (see references cited in [48, 49]). This is a common measurement in testing the quality of blackbodies at these frequencies.

Johnson must recognize that microwave cavities are utilized on satellites such as COBE [50] and PLANCK [49]. These cavities are tested using return-loss methods, exactly as I have done in [17], when testing an MRI cavity. Many of these cavities are not black, including some which have been claimed as such and launched aboard satellites [49]. Microwave cavities often contain signs of standing waves, as radiation from the network analyzer enters the cavity. The presence of such standing waves provides solid evidence that not all cavities in the microwave contain blackbody radiation. This is an important point to recognize, as Johnson would like to build up arbitrary radiation in cavities with reflection, using *"Stewart's mechanism"*. Standing waves demonstrate that the presence of specular reflection within a cavity is always counter to the interior containing black radiation [48,49]. This highlights yet another problem with *"Stewart's mechanism"*. It is critically dependent on any reflection within a cavity being diffuse and not specular. Otherwise, the radiation will not be Lambertian, as required of a blackbody.

Contrary to Johnson's position, experiments with cavities in MRI provide strong evidence that Kirchhoff's law does not hold (see [17] and references therein). This is especially true given that Kirchhoff's Law has been generalized to treat geometries where diffraction becomes important (see [17] and references therein). Furthermore, microwave studies demonstrate that small cavities, containing only a few centimeters of Ecosorb and conductively anchored to a radiation shield, like the 4K reference loads on the Planck satellite, can never be black [48, 49]. This presents a serious problem for those interested in the LFI data produced by this satellite [49].

Once again, the fact remains that Kirchhoff's Law does not have any valid experimental support. Arbitrary cavities are not black and this reality has consequences which must not be ignored.

*However, for real blackbodies, when the temperature sensors indicate a certain temperature, one can be assured that the radiation sampled will be black.

†Surprisingly however, in Kirchhoff's initial paper [2] he actually insists that even fluorescent material could be included within the cavity and it will still be black: *"It may be observed, by the way, that the proposition demonstrated in this section does not cease to hold good even if some of the bodies are fluorescent. A fluorescent body may be defined as one whose radiating power depends on the rays incident on it for the time being. The equation E/A = e cannot generally be true for such a body; but it is true if the body enclosed in a black covering of the same temperature as itself, since the same considerations that led to the equation in question on the hypothesis that the body C was not fluorescent, avail in this case even if the body C be supposed to be fluorescent."* [2]. These arguments are removed however, without explanation, when Kirchhoff's work is revised several years later [46]. Still, this indicates a flaw in Kirchhoff's initial derivation of his law [2], as he had thought that his derivation applied to fluorescent bodies, which was not correct. There are indeed flaws in Kirchhoff's initial derivation, as the author has independently ascertained [15]. Moreover, Schirrmacher [47] has reviewed the proofs of Kirchhoff's Law before and after Planck [48]. Even in 1912, Hilbert complained that a valid proof a Kirchhoff's law still did not exist [47], even though the Planck's lectures on the subject were given in 1906 [4, p. xi]. Such a proof is lacking, to this day.

‡Note that if Kirchoff's law was correct, there would be no need to have standard blackbodies in order to calibrate other cavities, as all cavities would be black.

2.7 Challenges to Monte Carlo simulations

Johnson then moves to briefly discuss Monte Carlo simulations in a single paragraph stating: *"Apparently, Robitaille's objection to the Monte Carlo simulations is that they rely on Stewart's mechanism for building up the radiation by internal reflection. As Robitaille and Crothers now accept that this mechanism is valid in principle, Robtaille's previous objections to Monte Carlo simulations supporting Kirchhoff's Law should also drop away"*.

Clearly, I have never accepted *"Stewart's mechanism"* for building up radiation within a cavity. First, such a mechanism, under certain circumstances, constitutes a violation of the First Law of Thermodynamics. Secondly, it is not possible to place energy into the interior of a cavity without also potentially placing energy into the walls. This is never considered by Monte Carlo simulations, and that is why they remain invalid. Such simulations agree with Kirchhoff Law, precisely because they ignore the dynamics going on in the wall and *a priori* forbid the temperature of the wall to rise in lieu of emitting a new photon.

2.8 Super-Planckian emission

Johnson's letter then examines my treatment of metamaterials [23]. He argues that Planck specifically excluded the near field by quoting: *"Throughout the following discussion it will be assumed that the linear dimensions of all parts of space considered, as well as the radii of curvature of all surfaces under consideration, are large compared with the wave lengths of the rays considered"* [4, § 2]. On the surface, this is a good point. Planck is clearly allowed to restrict his derivation. This does not mean, however, that the near field region cannot be considered today, in order to shed additional light on thermal emission.

In this regard, Kirchhoff's law has been generalized to include the limit initially excluded by both Kirchhoff and Planck [17, § 3]. The near field behavior can be considered for additional insight and the point raised by Johnson is weak at best. Science does get to move forward.

Johnson then goes on to claim that the evidence in the far field, is not convincing. He notes from Guo et al. [51] that: *"the presence of an interface is enough to guarantee that the far-field emissivity is limited to 1"* [36]. Guo's statement is noteworthy. However, Johnson neglects to cite the following from Guo's paper: *"The usual upper limit to the blackbody emission is not fundamental and arises since energy is carried to the far-field only by propagating waves emanating from the heated source. If one allows for energy transport in the near-field using evanescent waves, this limit can be overcome"* [51].

It is clear that the study of metamaterials is an area of science which is just beginning to be explored. It is also not established that far-field behavior will always adhere to the limits set forth by Planck's law. This is why I previously highlighted [23] the work by Yu et al. [52] and [53]. Yu et al. removed the claim made in the arXiv version of their paper [53] when they published their *Nature Communications* paper [52]. Here is the exact quotation from my paper on this issue: *"In that case, the spatial extent of the blackbody is enhanced by adding a transparent material above the site of thermal emission. A four-fold enhancement of the far-field emission could thus be produced."* In their *Nature Communications* article, the authors argue that this does not constitute a violation of the Stefan-Boltzmann law, because the effective "emitting surface" is now governed by the transmitter, which is essentially transparent. However, this was not the position advanced when the results were first announced and the authors wrote: *"The aim of our paper here is to show that a macroscopic blackbody in fact can emit more thermal radiation to far field vacuum than $P = \sigma T^4$"* [53].

In Yu's work, the emission is arising from a small blackened disk of material [52, 53]. The photons emitted from this surface greatly exceed anything predicted by Planck. At issue is the assignment of the emitting surface, from a theoretical perspective. Is it the blackened disk, which is the only possible source of photons, or the transparent shield? The key difficulty for blackbody radiation science is that blackbodies were always defined as opaque objects. Hence, it is difficult to conceive why the blackened disk should not be considered as the proper emitting surface in this problem. But assigning the emission to a transparent surface is now the only way of salvaging Kirchhoff's law. Once again, note how Kirchhoff had worded his law in the quotation at the very beginning of this reply. He was referring to opaque objects.

Then, there is the problem that, during Yu et al's experiment, the blackened disk is always heated [52, 53]. This implies that thermal equilibrium does not exist, since conduction of energy, which is heating the disk, must be considered. However, if this is to be used as an argument against these findings, then what of the problem of continuously heating ordinary cavities, in order to maintain their temperature equilibrium? As I previously stated: *"Obviously, modern experiments fall short of the requirements for thermal equilibrium, as the cavities involved are heated to the temperature of operation. But given that all laboratory blackbodies suffer the same shortcomings, the production of super-Planckian emission in the near and far fields cannot be easily dismissed. After all, in order for Planck to obtain a blackbody spectrum in every arbitrary cavity, he had to drive the reflection term, either by injecting a carbon particle or by permitting additional heat to enter the system, beyond that required at the onset of thermal equilibrium"* [23]. Johnson cannot apply his arguments to metamaterial experiments and not make them with regard to regular laboratory cavities. In light of these many considerations, he has not demonstrated that my position, relative to the universality of blackbody radiation, has been overstated.

2.9 Robtaille's thought experiment

In the next section of his paper, Johnson reviews a very simple thought experiment, which I advanced in 2014, illustrating that Kirchhoff's Law cannot be valid [20]. Briefly, the idea involved two cavities. The larger outer cavity was constructed from perfectly emitting and absorbing walls and initially placed in a helium bath. Within this cavity and *in thermal contact with its floor*, rested an inner cavity made from perfectly reflecting walls. Initially, one of the six sides of this latter cavity remained open. As such, both cavities now contained black radiation at 4K, which had been produced by the outer cavity. The open wall of the inner cavity was then closed. It thus contained blackbody radiation at 4K. Then, the helium bath was removed and the system was allowed to rise to room temperature. In that case, the inner cavity still contained radiation associated with a 4K blackbody and the outer cavity contained radiation corresponding to room temperature.

Johnson argues that: *But by making the inner cavity walls perfectly reflecting and closing the last side, Robitaille has created two entirely separate cavities; by definition, the inner cavity walls cannot emit radiation in either direction, whatever the temperature. They therefore act as boundary walls to what has become a "hollow" outer cavity. The outer cavity no longer contains the inner cavity within itself in a thermal sense; Kirchhoff's Law therefore survives this thought experiment"* [36].

There is no validity in this argument. Simply examine the quotation by Kirchhoff which opens this reply: *"...In the interior therefore of an opake red-hot body of any temperature, the illumination is always the same, whatever be the constitution of the body in other respects"*. Obviously, Kirchhoff's statement has been violated and Kirchhoff's law permits the placement of any object within the cavity interior, provided that it does not have the ability to emit photons by non-thermal means. I have not sidestepped the conditions set forth by Kirchhoff. The inner cavity, having perfectly reflecting walls, is linked to the floor of the outer cavity through thermal conduction [20]. The inner cavity is not composed of an adiabatic wall which is unable to contain or transmit heat, as Planck used. Rather, it is made of a perfect reflector, best approached by a material such as silver: *"Since the inner cavity is perfectly reflecting, it will also be highly conducting, as good reflectors tend to be good conductors"* [20, p. 38]. Therefore, conductive heat transfer was allowed [23, 26]. Silver is known to be essentially a perfect reflector in the infrared ($\rho > 0.994$ [54]), as I previously mentioned in the work under question [20]. It also possesses one of the highest electrical conductivities and has a very reasonable thermal conductivity, on the order of 400 W m^{-1} K^{-1} [55]. Johnson cannot argue that: *"The outer cavity no longer contains the inner cavity within itself in a thermal sense."* [36].

Mathematically, adiabatic walls can act as perfect reflectors, but reflectors themselves are not mathematical walls. Silver reflectors can be characterized by temperature, precisely because, though they are ideally immune to capturing radiative energy, they are able to allow energy to enter or leave either through conduction or, when applicable, convection. Johnson will not deny that thermal conduction exists. Conversely, adiabatic walls cannot be characterized by any temperature, as they are fully immune to energy transfer by radiation, conduction, and convection.

In the case of a perfect reflector, all of the energy of the system can be trapped in its walls. In the case of the perfect absorber, Planck considered that all of the energy was contained in the radiation field. Yet, Planck still needed to allow his oscillators the opportunity to have some momentary interaction with radiative energy. Otherwise, no photons could be produced or absorbed. Similarly, the perfect reflector must be allowed to have some momentary interaction with conductive energy. Johnson can no more deny the presence of thermal conduction than he can deny the presence of thermal emission and absorption. Silver, an near perfect reflector in the infrared, still has access to conductive paths of heat transfer.

Johnson tries to dismiss this thought experiment [20] and with good reason. It constitutes strong evidence that Kirchhoff's Law could never have been correct. In fact, let us revisit this setting, as it also helps to dispel Planck's ill-conceived claims relative to the carbon particle acting as a catalyst.

First, note that the radiation contained within the inner cavity depends on its history prior to the cavity being closed [20]. It will contain whatever radiation was present within the outer perfectly absorbing cavity at that time. That is, it will be defined by the temperature of the outer cavity at closure (i.e. 4 K). The radiation within the inner cavity persists as Planck claims [4, § 51], but in a state which was well-defined by history, not just any arbitrary state.

If we place a carbon particle in the perfectly reflecting cavity and if this particle does not act to transform heat from the wall into the radiative field, but can only act as a catalyst, as Planck claimed [4, § 51] (relative to the existing radiation which initially corresponded to 4 K radiation [20]), the interior of the cavity could never become black. That is because the interior of the second cavity lacks sufficient energy in its 4 K photons to adopt the proper blackbody intensity for the new higher temperature of its walls, when the both cavities have been brought to room temperature [20]. The carbon particle, can never act to shift the Wien's peak to higher frequencies because Planck denies that it can contain any significant heat on its own [4, § 51]. The cavity, in this instance, could not contain black radiation at room temperature, without violation of the First Law of Thermodynamics. That is the central problem in Planck's notion that the carbon particle was merely a catalyst. In the example provided, Planck would stand in violation of the First Law, if he persisted in insisting that the carbon particle was not transforming the energy content of the walls and if he maintained his insistence that the cavity became filled with blackbody radiation.

Should the carbon particle be characterized with a temperature, but interaction with the walls still prevented, then it could convert the radiation within the cavity to the proper Planckian distribution for the higher temperature. But this radiation will always remain gray, as the temperature of the carbon particle cannot be allowed to fall and since it has no access to other sources of heat. Once again, Planck is restricted by the First Law. The relative distributions of frequencies might become correct, but their intensity will always be too low.

It is only when the carbon particle is allowed to transform the thermal energy contained within the wall of the perfectly reflecting cavity that we obtain the correct answer and that the interior of the second cavity can become black [15]. That is why the carbon particle was never a catalyst. Planck ignores its ability to transform thermal energy contained within the walls. He was only concerned with the radiation field and this was a crucial error.

3 Robitaille and Crothers' 2015 paper

This section begins, once again, by claiming that there was a *volte-face* relative to my position on Stewart's mechanism. As noted previously, such claims are unwarranted. I have never supported *"Stewart's mechanism"* as providing a valid means of extending Kirchhoff's claims to all cavities made from arbitrary materials. Neither has Steve Crothers.

While defending Max Planck, Johnson has failed to recognize that there can be a substantial difference between 1) what Planck claims to have done, 2) what he actually did, and 3) what nature permits. For instance, when Planck denied the absorptivity of the surface layer and inserted only reflectivity, he made claims which were *demonstrably false* in the laboratory, relative to the nature of a blackbody surface. He inappropriately applied polarized light and Brewster's Law to secure his proof, when such an approach was disallowed based on the very definition of heat radiation. Finally, he concluded that his unnumbered equation at the end of section § 36 [4],

$$\frac{\mathbf{K}_\nu}{\mathbf{K}'_\nu} \cdot \frac{q^2}{q'^2} = \frac{1-\rho'}{1-\rho},$$

could be satisfied by all values of ρ and ρ'. Yet, when $\rho = 1$, this expression became undefined. As such, both Crothers and I maintain that Planck's "proof" [4] of Kirchhoff's Law remains fundamentally flawed and invalid. Planck has, therefore, been deprived of any justification in claiming universality. In his initial paper [3] and in the latter portion of his text [4], Planck correctly derived an expression for the blackbody function. But Planck can never state, based on § 35-37, that interiors of all cavities contain black radiation. This remains a serious crack in the armor of modern physics and Johnson's letter has not helped to rectify the problem.

3.1 The meaning of Planck's term "surface"

Within his classic text, Planck described how he has deviated from Kirchhoff's definition of a blackbody. For Johnson, Planck's new definition was permitted, whereas, in truth, it constituted a rejection of nature itself. As we have highlighted [25, p. 124], Planck stated within a footnote *"In defining a blackbody Kirchhoff also assumes that the absorption of incident rays takes place in a layer "infinitely thin". We do not include this in our definition"* [4, § 10]. This was not footnote material, as it constitutes a critical redefinition of the blackbody. In opposition to Kirchhoff, Planck decided to write: *"The creation of a heat ray is generally denoted by the word emission. According to the principle of the conservation of energy, emission always takes place at the expense of other forms of energy (heat, chemical or electric energy, etc.) and hence it follows that only material particles, not geometrical volumes or surfaces, can emit heat rays. It is true that for the sake of brevity we frequently speak of the surface of a body as radiating heat to the surroundings, but this form of expression does not imply that the surface actually emits heat rays. Strictly speaking, the surface of a body never emits rays, but rather it allows part of the rays coming from the interior to pass through. The other part is reflected inward and according as the fraction transmitted is larger or smaller the surface seems to emit more or less intense radiations"* [4, § 2].

Was Kirchhoff actually correct? Does the absorption of incident rays take place in a layer *"infinitely thin"* [4, § 10]? Or, did Planck more closely approximate nature: *"Strictly speaking, the surface of a body never emits rays, but rather it allows part of the rays coming from the interior to pass through"* [4, § 2]. Of course, if a surface, strictly speaking, cannot emit rays, it also cannot absorb rays.

The answer to this problem has been provided in the laboratory. If one considers the hexagonal planar structure of graphite and the reality that soot (or lampblack) has always played an important role relative to the creation of blackbodies (see references within [16, 17]), then the answer is readily apparent. For soot shares, in large measure, the hexagonal planar structure of graphite, although more breaks exist in the lattice. The surface of graphite or soot, is well represented by graphene [56, 57], as this alone constitutes the outer layer of a sheet of graphite.

Mak et al [58] speak of the absorption of graphene, *"Indeed, it was the strong absorption of single-layer graphene (with its absorbance of ∼2.3%, ... that permitted the initial discovery of exfoliated monolayers by visual inspection under an optical microscope"*. The authors are referring to the work of Novoselov and Geim [56] (Nobel Prize, Physics, 2010). Moreover, even a single layer of graphene has been shown to be an absolutely phenomenal emitter, when driven by current [59].

Consequently, Planck's position that, *"Strictly speaking, the surface of a body never emits rays, but rather it allows* *part of the rays coming from the interior to pass through"* [4, § 2], simply cannot be upheld. Laboratory evidence is firm on this point: some of the rays will begin to be absorbed even by the first mono-layer of atoms and less than 50 hexagonal planes of atoms should result in near complete absorption.

It is a fact that, even a single layer of graphene, the only structure which can be associated with the surface of a graphite blackbody, has powerful absorbance. Max Planck cannot be permitted to neglect this layer, when discussing blackbodies. Planck knew Kirchhoff's definition and chose to ignore it, even though he recognized that he could make the entire radiation within a cavity black, by introducing even the smallest of carbon particles [4, § 51]. Planck's error was in not allowing any absorption or emission at all, not in allowing that a single layer did not have 100% absorption. In this respect, Planck's statement was imprudent at the time and laboratory experiments have now demonstrated that, indeed, it was false. Johnson cannot correct this situation. Yet, as we shall see below, this was a critical step towards Planck's faulty derivation of Kirchhoff's Law. As for Kirchhoff, given his period in history, it is clear that his definition remains valid. For a single layer of atoms is as "infinitely thin" as nature can allow and 50 layers of atoms about as "infinitely thin" as a man could conceive in Kirchhoff's days. He could have no concept of the dimensions of atoms in 1860 [1, 2].

In continuing his letter, Johnson then attempts to justify Planck's insistence that the term *"bounding surface"* referred to a geometrical surface dividing two media, and that *"the material effects of emission and absorption take place within the adjoining media"* [36]. In this respect, we return to the question of what Planck has said and what he can be permitted to say.

In § 35 of his textbook [4], Planck outlined the notation relative to primed and unprimed superscripts: *"Let the specific intensity of radiation of frequency v polarised in an arbitrary plane be $\mathbf{K}_\nu$ in the first substance ..., and $\mathbf{K}'_\nu$ in the second, and, in general let all quantities referring to the second substance be indicated by the addition of an accent"* [4]. Planck continued in § 43, *"The most adequate method of acquiring more detailed information as to the origin and the paths of the different rays of which the radiations I_1, I_2, I_3, ... I_n consist, is to pursue the opposite course and to inquire into the future fate of that pencil, which travels exactly in the opposite direction to the pencil I and which therefore comes from the first medium in the cone $d\Omega$ and falls on the surface element $d\sigma$ of the second medium"* [4, § 43]. Here, Planck clearly assigned to the surface element $d\sigma$, properties of the second medium.

Johnson argues that Planck's bounding surface did not have to absorb any light, citing Planck's claim, *"Thus only material particles can absorb heat rays, not elements of surfaces, although sometimes for the sake of brevity the expression absorbing surfaces is used"* [4, § 12]. But what Johnson fails to understand is that, should he argue along these lines, he would be brought to accept yet another truth from Robitaille and Crothers which I now state: Only material particles can reflect light! Thus, Planck cannot be allowed an imaginary surface which reflects light, while at the same time denying that this same surface can absorb or emit light.

The truth being that when Planck placed two materials together, the bounding element, $d\sigma$, must be characterized on one side by the reflectivity and aborptivity of the first material and on the other side, by the reflectivity and absorptivity of the second material. That is because, the elements in either of the materials are not properly characterized only by reflectivity. This is precisely why Crothers and I object to Planck's use of a bounding surface which does not *fully represent* the materials which it unites.

Planck is welcome to claim that he can place a hypothetical bounding surface between two materials which considers only transmission and reflection. As for Crothers and I, we continue to object. The bounding surface which Planck envisioned was completely detached from reality. The issue is not that Planck cannot place the geometric surface between two layers. That is self-evident. The issue is that Planck cannot detach this geometric bounding layer from the material properties of those substances which he claims it characterizes. It is impossible to extract only the reflectivity of a particle, assign it to a geometric bounding surface, and at the very same time, ignore the absorptivity of this same particle. Contrary to Johnson, our statement that *"Planck neglected the fact that real materials can possess finite and differing absorptivities"* [25, p. 127] is entirely appropriate and valid. Planck's own textbook provides additional insight: *"Whenever absorption takes place, the heat ray passing through the medium under consideration is weakened by a certain fraction of its intensity for every element of path traversed"* [4, § 12]. By necessity, the element contained within the bounding section is one of the elements in the path traversed. Planck cannot ignore its absorption, because its properties can only be related to the medium to which it is linked.

Johnson then attempts to counter our statement: *"Third, the simplest means of nullifying the proof leading to Planck's Eq. 42, is to use a perfect reflector as the second medium. In that case, a refractive wave could never enter the second medium and Planck's proof fails"* [25, p. 127]. In order to counter this argument, Johnson tries to make the bounding surface perfectly reflecting, but unfortunately, he is not allowed to adopt such an approach, as Planck's proof intrinsically depends on the transmissivity of this bounding surface. Johnson cannot make it a perfect reflector, as in doing so, he optically isolates the two media. Furthermore, Johnson has failed to notice what has been mentioned above; namely, if $\rho = 1$, then $(1 - \rho) = 0$ and Planck's equation, at the bottom of Planck's § 36 (see § 3 herein) becomes undefined. Planck needs this equation to be valid in order to obtain his Eq. 41, $q^2\mathbf{K}_\nu = q'^2\mathbf{K}'_\nu$. But after he obtains Eq. 40, $\rho=\rho'$, Planck must return to the equation he lists at the end of § 36 and this expression is not always true.

It also remains the case that Planck's entire proof of Kirchhoff's Law collapses, as Crothers and I correctly highlighted in our joint paper [25, p. 127], when we replaced the second medium with a perfect reflector.

At first, it was difficult for me to even understand why Johnson would have wanted to replace the geometric bounding surface with a perfectly reflective surface. The answer rests in his use of this quote from Max Planck: *"Since the equilibrium is nowise disturbed, if we think of the surface separating the two media as being replaced for an instant by an area entirely impermeable to heat radiation, the laws of the last paragraphs must hold for each of the two substances separately"* [4, § 35]. However, in that case, Planck was referring to the treatment he had just outlined when addressing a single medium. Note that Planck writes in § 32, *"that the total state of radiation of the medium is the same on the surface as in the interior.* Then in § 33, Planck writes, *"While the radiation that starts from a surface element and is directed towards the interior of the medium is in every respect equal to that emanating from an equally large parallel element of area in the interior, it nevertheless has a different history. That is to say, since the surface of the medium was assumed to be impermeable to heat, it is produced only by reflection at the surface of radiation coming from the interior"* [4, § 35]. Planck had assumed that the surface in this case was impermeable to heat because this was the only way he could treat the isolated medium near its surface.

However, when Planck moved to two media, he no longer used a boundary impermeable to heat, but assumed that the surface of each medium was *"smooth"* [4, § 36]. In § 9, Planck had defined a smooth surface as one which can partially reflect and transmit the incoming radiation [4, § 9]. Planck required transmission for his later proof of Kirchhoff's law in § 35 and § 36. This is an essential element, which Johnson failed to consider in stating that a perfectly reflecting boundary enabled $\rho = \rho'$. In that case, as mentioned above, the equation at the bottom of Planck's § 36 would become undefined. It is for this reason that Johnson cannot support Planck's position, by making the bounding surface a perfect reflector.

Robitaille and Crothers remain correct. Planck improperly treated absorption and reflection in his derivation. Furthermore, the use of a perfect reflector for the second medium was all that was needed to shatter Planck's proof of Kirchhoff's Law, as we have previously noted [25].

3.2 Absorption and transmission

This section of Johnson's letter begins by quoting from the paper by Robitaille and Crothers: *"With his words, Planck redefined the meaning of a blackbody. The step, once again, was vital to his derivation of Kirchhoff's Law, as he relied on transmissive arguments to arrive at its proof. Yet, blackbody radiation relates to opaque objects and this is the first indication that the proofs of Kirchhoff's Law must not be centered on arguments which rely upon transmission. Planck ignored that real surface elements must possess absorption, in apparent contrast with Kirchhoff and without any experimental justification"* [25, p. 124].

Strangely, Johnson then concludes from this quotation that *"the apparent problem arises from the fact that Planck's surface is a geometrical one, whilst Robitaille and Crothers are obviously referring to a surface layer in which, they maintain, all absorption must take place because transmission is not permitted through a black body"* [36]. But we never stated that *all* of the absorption must take place from the surface layer. We stated that *"real surface elements must possess absorption"* [25, p. 124]. The surface need not have 100% absorption, as only a slight absorption is sufficient to invalidate Planck's proof. It is obvious, from our treatment of the first section of Planck's proof, that we do in fact allow transmission to take place within the medium and for elements within the blackbody to absorb, exactly like Max Planck [25, § 4.2]. We caution, however, that blackbodies are opaque objects and that Planck's proof cannot rely exclusively on transmission and reflection. Our point remains valid, as well demonstrated by the experimental realities outlined relative to graphene in § 3.1 above.

Again quoting from our paper, Johnson then attempts to argue that Planck was correct in inferring that "... *while in the case of bodies with vanishingly small absorbing power only a layer of infinite thickness may be regarded as black"* [4, § 10]. Once again, it is difficult to understand how Johnson can come to Planck's defense in this case. An opaque object which has a low absorptivity, also has a high reflectivity *by definition*. If not, it would not be opaque. As such, most photons which approach an opaque surface with low absorptivity are reflected away from the body. For Planck's argument to work, one would have to discount the surface reflection from an opaque object with a low emissivity which is counter to all laboratory experience. This highlights that Planck's new definition of a blackbody is completely outside the laws of nature. Planck cannot argue that he can neglect surface reflection, simply to salvage his derivation of Kirchhoff's Law.

Our point remains valid *"Blackbodies are opaque objects without transmission, by definition"* [25, p. 125]. Still, we have, in fact, allowed Planck to have some mathematical latitude and some level of transmission within the object, as presented in our § 4.2 [25]. But we cannot allow Planck to completely negate the presence of the reflection which is known to occur at the surface of an opaque object of low emissivity. Johnson and Planck shall not redefine nature.

3.3 Reflection

Relative to neglecting the reflection which occurs within a medium, we never stated that such an approach was invalid, merely that it was suboptimal. In fact, in § 4.2 of our paper, we specifically outline the effects of neglecting the reflection taking place within the medium [25].

Johnson, however, is under the impression that reflection is strictly a surface phenomenon and cannot take place within the medium. At the end of this section, Johnson emphasizes the point when he states *"Note that Planck is still talking about the interior of the medium where reflection is not applicable because there is no surface; therefore Robitaille and Crothers' objection cannot be maintained"* [36, § 3.3]. Johnson is confused on this point.

Planck himself explicitly commented on scattering within media: *"The propagation of the radiation in the medium assumed to be homogeneous, isotropic, and at rest takes place in straight lines and with the same velocity in all directions, diffraction phenomena being entirely excluded. Yet, in general, each ray suffers during its propagation a certain weakening, because a certain fraction of its energy is continuously deviated from its original direction and scattered in all directions. This phenomenon of "scattering", ... takes place, generally speaking, in all media differing from absolute vacuum ... "* [4, § 8]. Later in the same section, Planck noted that, beyond diffraction, scattering also depends on reflection [4, § 8]. Hence, contrary to Johnson's claims, Planck understood that reflection is not strictly a surface phenomenon.

Crothers and I have properly considered internal reflection [25, § 4.2]. We have demonstrated that, when internal reflection is considered, powerful new insight is gained. Rather than simply obtaining Kirchhoff's formulation, $\mathbf{K}_\nu = \epsilon_\nu/\alpha_\nu$, which is potentially undefined, we can actually extract $\epsilon_\nu = (1-\rho_\nu)\mathbf{K}_\nu$, which is never undefined [25, § 4.2]. The insight provided by this treatment is important, contrary to what Johnson implies when insisting, without justification and in opposition to Planck's own statements, that reflection is only a surface phenomenon.

3.4 Polarization and equality of reflection

In the final section of his letter, Johnson attempts to justify Planck's use of polarized light and his assertion that the reflectivities of a pair of media at the bounding surface must be equal. He begins by quoting from our paper: *"In § 5 Planck admitted that homogeneous isotropic media emit only natural or normal, i.e. unpolarized, radiation: "Since the medium was assumed to be isotropic the emitted rays are unpolarized". This statement alone, was sufficient to counter all of the arguments which Planck later utilized to arrive at Kirchhoff's Law [Eq. 42]. That is because the important sections of Planck's derivation, namely § 35–37 make use of plane-polarized light. These steps were detached from experimental reality, relative to heat radiation [Planck, § 35] ... ".*

At this point, Johnson recalls that we have allowed Planck to resolve heat radiation into two equal orthogonal components, each plane-polarized. He objects to our statement that *"such rays could never exist in the context of heat radiation"* [25, p. 129]. Apparently, Johnson has failed to grasp that even though Planck can resolve heat radiation into two components, he is not allowed to apply only one component in his derivation. He must always consider *both* components, even if he can resolve them into two orthogonal planes.

Johnson apparently does not understand why Planck wanted to treat only one component, in part, because he seems unaware of Brewster's Law. Planck, in his derivation of Kirchhoff's Law, invoked plane-polarized radiation, such that he could set $\rho = \rho' = 0$. He could only obtain this expression, when dealing with a single plane polarized beam of light. That is because, if he sent such a beam at the proper angle and with the proper polarization towards his bounding surface, there would be no reflection, according to Brewster's Law.

However, Planck was not right in stating that there could be no reflection in the context of heat radiation. He could not obtain the plane-polarized beam of light, which he required, because the other component of the radiation, which was inappropriately ignored in his derivation, was also present. Moreover, Planck did not even test reflectivity by his argument from Brewster's law, as the latter is dependent upon the presence of a reflected ray as well as a transmitted ray. Thus, Planck could not conclude that the reflectivities of both materials were 0. The absence of a reflected ray does not imply that reflectivity is zero, as the polariscope attests. Just because Planck can resolve light into two components does not mean that he can ignore one of these components. *This is one of the most significant flaws in Planck's derivation of Kirchhoff's Law.*

Johnson then tries to defend Planck's most dramatic claim. Planck states [4, § 37]: *"Now in the special case when the rays are polarized at right angles to the plane of incidence and strike the bounding surface at the angle of polarization, $\rho = 0$, and $\rho' = 0$. The expression on the right side of the last equation then becomes 1; hence it must always be 1 and we have the general relations:*

$$\rho = \rho' \qquad (40)$$

and

$$q^2\boldsymbol{K}_\nu = q'^2\boldsymbol{K}'_\nu \quad (41)".$$

As I have just outlined, Planck cannot refer to this special case, because he does not have access to light polarized in a single plane. He must *always simultaneously treat both components*. Secondly, Planck is incorrect in asserting that the right side of the expression at the end of his § 36 [4] (also shown in § 3 of this letter), *"must always be 1"*, because it becomes undefined when $\rho = 1$. Planck was making an elementary error in mathematics. We maintain that *"The result was stunning."* [25, p. 129]. We also maintain that *"Max Planck had determined that the reflectivities of all arbitrary media were equal"* [25, p. 129].

Johnson then tries to defend Planck one last time, by insisting that what the latter *"had in fact demonstrated is that the reflectivities on each side of a geometrical surface bounding two different media are equal. Clearly if a different pair of media are chosen, the value of the reflectivity of the bounding surface may be different as well"* [36, § 3.4]. He then quoted from *The Theory of Heat Radiation*, *"Since, in general, the properties of a surface depend on both of the bodies which are in contact, this condition shows that the property of blackness as applied to a body depends not only on the nature of the body but also on that of the contiguous medium. A body which is black relatively to air need not be so relatively to glass, and vice versa"* [4, § 10].

Both Crothers and I understand what Max Planck claimed. However, we are properly concerned with what he has actually done. Planck's statement that *"the properties of a surface depend on both of the bodies which are in contact"* [4, § 10] can never be verified in the context of opaque media, precisely because his bounding surface is an abstraction. Snell's law, for instance, also relies on the interface of two media, but a bounding surface, or the changes at the surfaces, need not be introduced to obtain the proper answer. The indices of refraction of the two media alone are sufficient to treat the problem.

Planck's statements relative to the bounding surface were subject to two fundamental objections. First, they are justified by nothing; second, they constitute *"une hypothèse gratuite"* (see table presenting arguments against 19th century proofs of Kirchhoff's Law in [47, p. 16]). Planck may wish to claim that *"A body which is black relatively to air need not be so relatively to glass, and vice versa"* [4, § 10], but he had absolutely no justification for such a statement.

Rather, what Planck did possess are two isotropic media. Each of these is characterized by the absorptivity and reflectivity for each of its constitutive elements. Within his bounding surface, Planck could only introduce the reflectivity of elements contained in the media in question. When he introduced this reflectivity into his bounding surface, he had to additionally introduce some absorptivity, since this also characterized the media. Planck was not free to ignore the absorptivity. But he did so, as absorptivity in the bounding surface would prevent him from making use of Brewster's Law.

In any case, Planck could not invent a new reflectivity, which now existed only when he places the two media in contact with one another. After all, the reflectivities of the bounding surface must somehow be related to the materials under study. Furthermore, all that Planck could ever know about these materials are the reflectivities which can be measured. Neither he, nor Johnson, are allowed to hypothesize on what can never be measured in opaque media.

Planck recognized that he could not state that reflectivities of all materials are identical. As such, he postulated, *without any experimental evidence*, that his proof actually refers to something else [4, § 10]. Crothers and I dispute such claims. Planck's derivation must be taken on what the setting and the mathematics demonstrate. If we ignored Planck's mathematical errors and experimental oversights, we could much more convincingly argue that he had demonstrated that the reflectivities of all arbitrary materials were equal, using the same proof. Planck could measure nothing more than the reflectivities of each medium. Thus, he remains in violation of known optics, despite his attempts to introduce a new meaning to the reflectivity of a surface. Furthermore, Planck is forbidden from writing Eq. 40, $\rho = \rho'$, precisely because he has violated nature's rule that heat radiation is never polarized. It also remains the case that the unnumbered equation, which Planck presents at the end of his § 36 [4] (see § 3 herein), is undefined when $\rho = 1$.

4 Johnson's summary and conclusions

In opening this section of his letter, Johnson claims that, *"Stewart [33] had shown that the radiation in a cavity made from perfectly absorbing material at thermal equilibrium must be black, of an intensity appropriate to the equilibrium temperature. According to Robitaille, Kirchhoff [1] extended this finding to cavities made of arbitrary materials"*. Once again, Johnson has missed the mark.

Stewart considered plates in his experiments and Johnson is distorting what Stewart has done. It was with plates that Stewart demonstrated the Law of Equivalence (in modern notation: $\epsilon = \alpha$, or $\epsilon + \rho = \alpha + \rho$). Kirchhoff's extension to all arbitrary cavities [1, 2], went well beyond Stewart's legitimate law and has never been demonstrated to be true in the laboratory.

Johnson's claim that I have now withdrawn my objections to *"Stewart's mechanism"*, in my paper with Crothers [25], is without basis. *"Stewart's mechanism"* has numerous problems, including potential violations of the First Law of Thermodynamics, depending on the circumstances considered. It suffers from the reality that cavities made of low emissivity materials can prefer to increase the temperature of the walls, rather than emit a photon. Johnson's letter does nothing to counter this argument and that is why *"Stewart's mechanism"* cannot be realized in practice, as recognized by Crothers and myself.

Finally, Johnson admits: *"Robitaille is obviously correct to point out that black body cavities are never made from reflective materials."*. However, he then attempts to excuse the observation, in noting that, *". . . this fact appears to be more a question of practicality and the need to ensure that the walls are not perfectly reflective at any wavelength so that proper measurements of temperature can be made. It does not seem to amount to a demonstration that Kirchhoff's Law necessarily fails, as Robitaille claims."*. Again, the arguments are ill-conceived. The fact that an experiment, required to establish a law of physics, still remains impractical after 150 years, well indicates that the law was never valid.

Some have argued, for instance, that when cavities are constructed from low emissivity materials, their dimensions need to be increased. This helps to augment their absorbance when sampling return losses. But Kirchhoff's law is explicit. The dimensions of the walls are irrelevant and, at a given temperature, must remain unrelated to the emissivity of the material, provided that the diffraction limit is avoided. The diffraction limit is not set by the emissivity. Furthermore, Johnson's arguments, relative to the ability to properly measure a temperature remain unfounded. Temperature sensors in the walls of cavities can easily report such information.

5 Conclusions

Throughout his letter, Johnson demonstrates that he has not carefully considered what Stewart, Kirchhoff, and Planck have written. He attributes to them positions which they never adopted. Then, he misinterprets the positions they did take. He repeatedly makes elementary errors relative to the understanding of cavity radiation. His statements on properly measuring the temperature of a cavity are but one example. He argues for *"Stewart's mechanism"*, in building up the radiation within a cavity, while not recognizing that the introduction of specular reflection within such objects can easily lead to the formation of standing waves. He also fails to understand that a cavity can simply increase the temperature of its walls and not emit a single photon.

He rejects my experimental work on MRI cavities, as unrelated to the problem of thermal emission and notes that processes, like fluorescence, have been excluded by Max Planck. Yet, the sampling of a cavity with a network analyzer does not involve such processes. In this respect, he also fails to note that Kirchhoff had mistakenly included such processes, in his initial work [1]. This was the only work of Kirchhoff which Johnson cited.

Furthermore, he fails to recognize that microwave cavities are utilized aboard modern satellites, wherein such objects are claimed to be black. Johnson also improperly and unknowingly expresses Stewart's Law as $\epsilon_\nu = \alpha_\nu + \rho_\nu$ in a thought experiment, thereby reaching conclusions which were clearly false. Then, he ignores the very existence of thermal conduction, when he attempts to invalidate my thought experiment with two cavities. He misrepresents my statements and those of Stephen Crothers, when he tries to state that we denied that the interior of a medium can have absorbance. He failed to understand the difference between resolving a heat ray into its two plane-polarized components and making use of a single plane-polarized ray, in order to infer something about heat radiation, which is never polarized. He hypothesized that replacing Planck's geometrical bounding surface with a perfect reflector could be used to validate Planck's claims, when clearly, it leads to an undefined mathematical expression and an invalid setting.

For all these reasons, Johnson cannot state that he has, in any way, nullified my objections to Kirchhoff's Law. Still, he must not be faulted for trying to defend Kirchhoff and Planck. As I stated in the introduction, it is the first obligation of a scientist to defend established science. Moreover, the study of cavity radiation is not at all simple. In this regard, Johnson's efforts are noteworthy and he is to be given credit for the time he has invested in reviewing these many papers.

Through the exchange prompted by his letter, Johnson has been indirectly responsible for bringing to the forefront many aspects of cavity radiation. Progress is often achieved, only when old ideas are first rejected, even if the process of discovery is not smooth. The process of correction, in itself, leads to scientific advancement. Hence, through such an exchange, readers can better come to understand why Kirhchoff's Law of thermal emission was never valid. Consequently, Planck's claims for universality must be rejected.

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Dedication

This work is dedicated to my youngest sister, Mireille and her husband, John.

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*Johnson and I had indeed exchanged correspondence following our meeting at a conference in 2014 [60].

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