The Dirac-Electron Vacuum Wave

William C. Daywitt

National Institute for Standards and Technology (retired), Boulder, Colorado. E-mail: wcdaywitt@me.com

This paper argues that the Dirac equation can be interpreted as an interaction between the electron core and the Planck vacuum state, where the positive and negative solutions represent respectively the dynamics of the electron core and a vacuum wave propagating within the vacuum state. Results show that the nonrelativistic positive solution reduces to the Schrödinger wave equation.

1 Introduction

In its rest frame the massive electron core $(-e_*, m)$ exerts the two-term coupling force [1, Sec.7-8]

$$F(r) = \frac{e_*^2}{r^2} - \frac{mc^2}{r} = \frac{(-e_*)(-e_*)}{r^2} - \frac{mm_*G}{r_*r} \tag{1}$$

on the PV quasi-continuum, where e_* is the massless bare charge with its derived electron mass m, and $G\,(= e_*^2/m_*^2)$ is Newton's gravitational constant. The first $(-e_*)$ in (1) belongs to the electron and the second to the separate Planck particles making up the degenerate PV state. The two terms in (1) represent respectively the Coulomb repulsion between the electron charge and the separate PV charges, and their mutual gravitational attraction.

The particle/PV coupling force (1) vanishes at the electron Compton radius $r_c\,(= e_*^2/mc^2)$. In addition, the vanishing of $F(r_c)$ is a Lorentz invariant constant [2] that leads to the important Compton-(de Broglie) relations

$$r_c \cdot mc^2 = r_d \cdot cp = r_L \cdot E = r_* \cdot m_*c^2 = e_*^2 \; (= c\hbar) \tag{2}$$

where $r_d = r_c/\beta_0\gamma_0$ and $r_L = r_c/\gamma_0$, and $r_*\,(= e_*^2/m_*c^2)$ and m_* are the Compton radius and mass of the Planck particles within the PV. The ratio of the electron speed v to the speed of light c is β_0 and $\gamma_0 = 1/(1-\beta_0^2)^{1/2}$. The relativistic momentum and energy following from the invariance of $F(r_c) = 0$ are $p\,(= m\gamma_0 v)$ and $E\,(= m\gamma_0 c^2)$, from which $E = (m^2c^4 + c^2p^2)^{1/2}$ is the relativistically important energy-momentum relationship.

The results of the previous paragraph show that the important relativistic energy E and momentum p (or its vector counterpart $\mathbf{p}$) are determined at the basic core-PV interaction level. Furthermore, since the core is many orders-of-magnitude smaller than the electron Compton radius, it is reasonable to assume that this point-core picks up its wave-particle nature (including its Compton radius and its energy and momentum operators) from its coupling to the PV continuum.

2 Dirac equation

The Dirac equation [3, p.79]

$$i\hbar \frac{\partial}{\partial t}\begin{pmatrix}\phi\\ \chi\end{pmatrix} = \begin{pmatrix}c\vec{\sigma}\cdot\widehat{\mathbf{p}}\chi\\ c\vec{\sigma}\cdot\widehat{\mathbf{p}}\phi\end{pmatrix} + mc^2\begin{pmatrix}\phi\\ -\chi\end{pmatrix} \tag{3}$$

where $\widehat{\mathbf{p}}\;(=-i\hbar\nabla)$ is the momentum operator and $\hbar$ is the reduced Planck constant, can be expressed using (2) as

$$ir_c \frac{\partial}{c\partial t}\begin{pmatrix}\phi\\ \chi\end{pmatrix} + \begin{pmatrix}\vec{\sigma}\cdot ir_c\nabla\chi\\ \vec{\sigma}\cdot ir_c\nabla\phi\end{pmatrix} = \begin{pmatrix}\phi\\ -\chi\end{pmatrix} \tag{4}$$

where the solutions ϕ and χ for this electron-vacuum system are 2x1 Dirac spinors, and $\vec{\sigma}$ is the Pauli 2x2 vector matrix derived from the three 2x2 Pauli spin matrices σ_k $(k = 1, 2, 3)$ [3, p.12]

$$\sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \sigma_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \quad \sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{5}$$

The gradient operators $\partial/c\partial t$ and ∇ in (4) are normalized by the electron Compton radius r_c

$$\frac{\partial}{c\partial t/r_c} \qquad \text{and} \qquad \frac{\partial}{\partial x_k/r_c} \tag{6}$$

whose denominators can be looked upon as the normalized line elements cdt/r_c and dx/r_c of a spacetime [4, p.27] perturbed by the electron core $(-e_*, m)$ in (1). Following from this viewpoint is the concept that *the 2x1 spinors ϕ and χ represent, respectively, the PV response to the electron core $(-e_*, m)$ and some type of vacuum wave.* Furthermore, the vacuum wave cannot be a Planck-particle wave, since the PV is a degenerate state (where the vacuum eigenstates are fully occupied). Thus the wave must be of the nature of a percussion wave, analogous to a wave traveling on the head of a kettle drum.

3 Dirac-Schrödinger reduction

The solution χ in the two simultaneous equations of (4) is assumed in the PV theory to represent a relativistic vacuum wave propagating within the PV state. What follows derives the nonrelativistic version of that wave to add more credence and understanding the vacuum wave idea.

The Dirac-to-Schrödinger reduction [3, p.79] of (4) begins with the elimination of its mass related, high-frequency, components by assuming

$$\begin{pmatrix}\phi\\ \chi\end{pmatrix} = \begin{pmatrix}\phi_0\\ \chi_0\end{pmatrix} e^{-i(mc^2t/\hbar)} = \begin{pmatrix}\phi_0\\ \chi_0\end{pmatrix} e^{-i(ct/r_c)} \tag{7}$$

where ϕ_0 and χ_0 are slowly varying functions of time compared to the exponential. This result implies that the frequency $\omega_c = c/r_c \gg \omega_0$ for any ω_0 associated with ϕ_0 or χ_0. Inserting (7) into (4) gives

$$ir_c \frac{\partial}{c\partial t}\begin{pmatrix}\phi_0\\ \chi_0\end{pmatrix} + \begin{pmatrix}\vec{\sigma}\cdot ir_c\nabla\chi_0\\ \vec{\sigma}\cdot ir_c\nabla\phi_0\end{pmatrix} = \begin{pmatrix}0\\ -2\chi_0\end{pmatrix} \tag{8}$$

where the 0 on the right is a 2x1 null spinor. This zero spinor indicates that the mass energy of the free electron core is being ignored, while the effective negative mass-energy of the vacuum wave has doubled ($-2\chi_0$). In effect, mass energy for the core-vacuum system has been conserved by shifting the mass energy from the free relativistic core to the vacuum wave.

The lower of the two simultaneous equations in (8) can be reduced from three to two terms by the assumption

$$\left| ir_c \frac{\partial\chi_0}{c\partial t}\right| \ll |-2\chi_0| \tag{9}$$

if the kinetic energy of the vacuum wave is significantly less than its effective mass energy. Inserting (9) into (8) then yields

$$ir_c \frac{\partial}{c\partial t}\begin{pmatrix}\phi_0\\ 0\end{pmatrix} + \begin{pmatrix}\vec{\sigma}\cdot ir_c\nabla\chi_0\\ \vec{\sigma}\cdot ir_c\nabla\phi_0\end{pmatrix} = \begin{pmatrix}0\\ -2\chi_0\end{pmatrix} \tag{10}$$

as the nonrelativistic version of (4). The mass energy of the free core, and the kinetic energy of the vacuum wave (associated with the lower-left null spinor), are discarded in this nonrelativistic approximation to (4).

Separating the two equations in (10) produces

$$ir_c \frac{\partial\phi_0}{c\partial t} + \vec{\sigma}\cdot ir_c\nabla\chi_0 = 0 \tag{11}$$

and

$$\vec{\sigma}\cdot ir_c\nabla\phi_0 = -2\chi_0 \tag{12}$$

where the second term in (11) and the first term in (12) represent the connection between the free-space core dynamics (ϕ_0) and the vacuum wave (χ_0). Inserting (12) into (11) then leads to [3, p.80]

$$ir_c \frac{\partial\phi_0}{c\partial t} - \frac{(\vec{\sigma}\cdot ir_c\nabla)^2}{2}\,\phi_0 = 0\,. \tag{13}$$

Finally, using the Pauli-matrix identity [3, p.12]

$$(\vec{\sigma}\cdot\nabla)^2 = I\,(\nabla)^2 \tag{14}$$

in (13) yields the free-core Schrödinger equation

$$ir_c \frac{\partial\phi_0}{c\partial t} = \frac{(ir_c\nabla)^2}{2}\,\phi_0 \quad \text{or} \quad i\hbar\frac{\partial\phi_0}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\phi_0 \tag{15}$$

where the two spin components in ϕ_0 are ignored in this nonrelativistic approximation; so ϕ_0 becomes a simple scaler wavefunction rather than a 2x1 spinor.

4 Conclusions and comments

Although the spin components are missing from the standard version of the Schrödinger equation [5, p.20], the solutions to (11) and (12) indicate that those components are still meaningful.

Using $r_c\,(= e_*^2/mc^2 = \hbar/mc)$ from (2) in (11) and (12) yields

$$ir_c \frac{\partial\phi_0}{c\partial t} = \vec{\sigma}\cdot(\widehat{\mathbf{p}}/mc)\,\chi_0 \tag{16}$$

and

$$\chi_0 = \vec{\sigma}\cdot(\widehat{\mathbf{p}}/2)\,\phi_0 \tag{17}$$

where $\widehat{\mathbf{p}}\,(= -i\hbar\nabla)$ is the vector momentum operator.

Equations (16) and (17) from the perturbed spacetime can be understood as follows: the free-space energy from ϕ_0 in the first term of (16) drives the vacuum energy associated with the second term; this χ_0 energy of the second term in (17) then feeds back into the ϕ_0 term in (16), leading to a circular simultaneity between the two equations that represent the coupled nonrelativistic behavior of the core-PV system. Furthermore, the fact that there is no kinetic-energy term in (17) suggests that the localized energy in the PV travels as a percussion wave through that vacuum state. This scenario represents the PV view of the Dirac electron equation (4): that is, the dynamics of the free-space electron core $(-e_*, m)$ lead to a vacuum wave propagating within the PV state, in step with the free electron core.

Dedication

This paper is dedicated to the memory Dr. Petr Beckmann [6], Professor Emeritus Electrical Engineering, the University of Colorado, Boulder, Colorado.

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