Mass of a Charged Particle with Complex Structure in Zeropoint Field

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A charged particle immersed in the fluctuating zeropoint field may be visualized as an oscillator and such an oscillating particle is considered to possess an extended structure with center of mass and center of charge separated by radius of rotation in a complex vector space. Considering stochastic electrodynamics with spin, the zeropoint energy absorbed by the particle due to its internal motion has been derived. One may initially assume a massless charged particle with complex structure and after interaction with zeropoint field, the absorbed energy of the particle may correspond to the particle mass. This gives an idea that an elementary particle may acquire mass from the interaction of zeropoint field. When the particle moves as a whole, there appears to be a small energy correction of the order of fine structure constant and it may be attributed to the mass correction due to particle motion in the zeropoint field.

1 Introduction

The Dirac electron executes rapid oscillations superimposed on its normal average translational motion and this oscillatory motion is known as zitterbewegung and it was first shown by Schrödinger. In the zitterbewegung motion, the electron appears vibrating rapidly with a very high frequency equal to $2mc^2\hbar^{-1}$ and with internal velocity equal to the velocity of light. These oscillations are confined to a region of the order of Compton wavelength of the particle. It has been shown by several authors over decades that the center of charge and center of mass of charged particle are not one and the same but they are separated by a distance of the order of Compton wavelength of the particle. The approach of extended particle structure was developed by Wyssenhoff and Raabe [1], Barut and Zhanghi [2], Salesi and Recami [3] and others. The list of references connected with the validity of the extended or internal structure of charged particle are too many and some of them are mentioned in the reference [4]. Thus the structure of an elementary charged particle is not definitely a point particle with charge and mass or a spherical rigid body with charge distribution. The structure of electron may be visualized as the point charge in a circular motion with spin angular momentum. The frequency of rotation is equal to the zitterbewegung frequency and the radius of rotation is equal to half the average Compton wavelength. The circular motion is observed from the rest frame positioned at the centre of rotation which is the centre of mass point. Thus the centre of mass point and the centre of charge point are separated by the radius of rotation. The electron spin generated from the circular motion of zitterbewegung was advocated by several researchers. Holten [5] discussed the classical and quantum electrodynamics of spinning particles. In the Holten theory, the spinning particle emerges as a modification of relativistic time dilation by a spin dependent term and the zitterbewegung appears as a circular motion and the angular momentum of such circular motion represents the spin. In the Hestene model of Dirac electron [6], the spin was considered as a dynamical property of the electron motion. In the approach of geometric algebra, using multivector valued Lagrangian, the angular momentum of this internal rotation represents particle spin and it has been explicitly shown as a bivector quantity representing the orientation of the plane of rotation [7, 8]. In quantum theories, the internal oscillations of the particle are attributed due to vacuum fluctuations. However, in stochastic electrodynamics, the internal oscillatory motion of the particle is attributed to the presence of zeropoint field throughout space [9]. The mass of the particle is seen as the energy of oscillations confined to a region of space of dimensions of the order of Compton wavelength [10].

The classical concept of space is an infinite void and featureless. However, it has been replaced by the vacuum field or the zeropoint random electromagnetic field when the quantum oscillator energy was found to contain certain zeropoint energy and with the substitution of the quantum oscillator energy into the Planck's radiation formula yields the energy density of zeropoint field at absolute zero temperature [11]. In a classical approach to the radiation problem, Einstein and Stern obtained blackbody radiation spectrum and suggested that a dipole oscillator possessed zeropoint energy. In 1916, Nernst proposed that the universe might actually contain ubiquitous zeropoint field without any presence of external electromagnetic sources [12, 13]. Thus the origin of zeropoint field is presumed to be purely a quantum mechanical effect and considered to be uniformly present throughout space in the form of stochastic fluctuating electromagnetic field. The zeropoint radiation is found to be homogeneous and isotropic in space. The spectral density of zeropoint radiation is proportional to ω^3 and it is therefore Lorentz invariant. The electromagnetic zeropoint field consists of fluctuating radiation that can be expressed as a superposition of polarised plane waves. Because of the random impulses from fluctuating zeropoint field, a free particle cannot remain at rest but oscillates about its equilibrium position.

The Planck's idea of zeropoint radiation field was revisited by Marshall and explicitly showed that the equivalence between classical and quantum oscillators in the ground state [14]. This has inspired interesting modifications to classical electrodynamics and the developed subject is called stochastic electrodynamics. Stochastic electrodynamics deals with the movement of charged particles in the classical electromagnetic fluctuating zeropint field. The presence of classical, isotropic, homogeneous and Lorentz invariant zeropoint field in the universe is an important constituent of stochastic electrodynamics. The stochastic electrodynamics approach was used to explain classically several important fundamental results and problems of quantum mechanics [15–20]. Boyer [15] showed that for a harmonic oscillator, the fluctuations produced by zeropint field are exactly in agreement with the quantum theory and as a consequence the Heisenberg minimum uncertainty relation is satisfied for the oscillator immersed in the zeropoint field. Stochastic electrodynamics was used to explain the long standing problems of quantum mechanics, namely the stability of an atom, Van der Walls force between molecules [16], Casimir force [17], etc. All these studies reveal the fact that the conventional concept of space has been changed by the emergence of zeropoint field. A detailed account of stochastic electrodynamics as a real classical electromagnetic field and a phenomenological stochastic approach to the fundamental aspects of quantum mechanics was given by de La Pena *et al.,* [13, 21]. In the stochastic electrodynamics, if the upper cut-off frequency to the spectrum of zeropoint field is not imposed, the energy of the oscillator would be divergent. Despite of its success in explaining several quantum phenomena, the results obtained in the stochastic electrodynamics have certain drawbacks [20]; it neglects Lorentz force due to zeropoint magnetic field, it fails in the case of nonlinear forces, explanation of sharp spectral lines is not possible, diffraction of electrons cannot be explained and further the Schrödinger equation can be derived in particular cases only.

A charged point particle immersed in the fluctuating electromagnetic zeropoint field is considered as an oscillator. In the stochastic electrodynamics approach, the equation of motion of the charged particle in the zeropoint field is known as Brafford-Marshall equation [13] which is simply the Abraham-Lorentz [22] equation of motion of a charged particle of mass m and charge e and it is given by

$$m\ddot{\mathbf{x}} - \Gamma_a m\ddot{\mathbf{v}} + m\omega_0^2\mathbf{x} = e\mathbf{E}_z(\mathbf{x}, t), \tag{1}$$

where $\Gamma_a = 2e^2/3mc^3$, ω_0 is the frequency of oscillations of the particle, $\mathbf{v}$ is the velocity of the particle, c is the velocity of light, $\mathbf{E}_z(\mathbf{x}, t)$ is the external electric zeropoint field and an over dot denotes differentiation with respect to time. In the above equation, the force term contains three parts; the binding force $m\omega_0^2\mathbf{x}$, damping force $\Gamma_a m\ddot{\mathbf{v}}$ and external electric zeropoint field force $e\mathbf{E}_z(\mathbf{x}, t)$. In the case of point particles, the strength of these forces follows the relation

$$m\omega_0^2\mathbf{x} < \Gamma_a m\ddot{\mathbf{v}} < e\mathbf{E}_z(\mathbf{x}, t). \tag{2}$$

The energy absorbed by the particle oscillator in the zeropoint field was given by several authors by introducing certain approximations. There are two main approaches found in the literature; one is due to Boyer [6] and the other is due to Rueda [19]. In addition to these main approaches, recently Cavalleri *et al.,* [20] introduced stochastic electrodynamics with spin and explained several interesting phenomena for example, stability of elliptical orbits in an atom, the origin of special relativity and the explanation for diffraction of electrons. It has been shown that the drawbacks of stochastic electrodynamics can be removed with the introduction of spin into the problem. The particle has a natural cut-off frequency equal to the spin frequency which is the maximum frequency radiated by the electron in the zitterbewegung interpretation. This eliminates the problem of divergence in stochastic electrodynamics. These recent advancements in the field of stochastic electrodynamics fully support the assumption that the stochastic electromagnetic field represents the zeropoint field and renew the interest in studying the fundamental aspects of quantum systems and in particular the charged particle oscillator in zeropoint fields.

In Boyer's extensive studies, the harmonic oscillator was developed under the dipole approximation and the charged particle was considered as a point particle without any internal structure. The point particle limit is endowed with two assumptions; i) when the particle size tends to zero, $\omega_c\tau \ll 1$, where ω_c is the cut-off frequency and τ is the characteristic time and ii) when the radiation damping term is very small compared to the external force, $\Gamma_a\omega_c \ll 1$. In Boyer's process of finding the zeropoint energy associated with the charged particle, an integral under narrow line width approximation was solved and finally the zeropoint energy per mode of the oscillator was obtained [16]. This energy has been shown to be equal to the zeropoint energy of the quantum oscillator.

In Rueda's approach, the classical particle was considered as a homogeneously charged rigid sphere and to find the energy absorbed by the particle, the radiation damping and binding terms were neglected when compared to the force term in the Lorentz Abraham equation of motion. The integration was performed over a range 0 to τ, where τ is the characteristic time taken by the electromagnetic wave to traverse a distance equal to the diameter of the particle. The main difference from Boyer's approach is that Rueda assumed $\omega_c\tau \gg 1$ and this condition means the cut-off wavelength is much smaller than the particle size. Further, Rueda introduced a convergence factor $\eta(\omega)$ in the zeropoint energy of the particle oscillator. The average zeropint energy of the oscillator is given by [19]

$$\langle E_0\rangle = \frac{\Gamma_a\hbar\omega_c^2}{\pi}\,\eta(\omega_c). \tag{3}$$

In the later studies, Haitch, Rueda and Puthoff [23] studied an accelerated charged particle under the influence of zeropint field and obtained a relation for inertial mass of a charged particle which is similar to (3). Recently, Haitch *et al.* [9] suggested that the radiation damping constant in the zeropoint field as Γ_z which is not necessarily equal to the damping constant Γ_a of Larmor formula for power radiated by an accelerated charged particle. If we set $\eta(\omega)\Gamma_z\omega_c \sim 1$, the ground state energy of the particle oscillator in the zeropoint field is written as $(\hbar\omega_c)/\pi$. In the case the cut-off frequency is simlar to the resonant frequency of the particle oscillator in the electromagnetic zeropoint field, the ground state energy is equal to the zitterbewegung energy of the Dirac electron. Here, the frequency ω_c is not generally equal to the frequency of oscillation of the particle and it differs by a fraction of fine structure constant. However, the reason for assuming Γ_a as Γ_z is obscure. It may be understood that the energy in (3) corresponds only to a mass correction but not to the mass of the charged particle.

In the stochastic electrodynamics with spin, the particle is considered to possess an extended internal structure and the particle spin is sensitive to the zeropoint frequency that is equal to the frequency of gyration. The particle gyration motion explains the spin properties and refers to a circular motion at the speed of light [20]. The velocity of the particle is not the real velocity of gyrating particle, but centre of mass point around which the particle revolves. The special relativity is not present at the particle level and arises mainly because of the helical motion of the particle when observed from an arbitrary inertial frame of reference [24]. The centre of circular motion responds only to the force parallel to the spin direction. The equation of motion of centre of mass point can be expressed by (1) provided the external force is parallel to the spin direction.

Clifford algebra or Geometric algebra has been considered to be a superior mathematical tool to express many of the physical concepts and proved to provide simpler and straightforward description to the mathematical and physical problems. The geometric algebra was rediscovered by Hestenes [25] in 1960's and it is being used by a growing number of physicists today. In Geometric algebra, a complex vector is defined as a sum of a vector and a bivector. In the complex vector algebra, the oscillations of a charged particle immersed in zeropoint field have been studied recently by the author [26]. The oscillations of the particle in the zeropoint field may be considered as complex rotations in complex vector space. The local particle harmonic oscillator is analysed in the complex vector formalism considering the algebra of complex null vectors. It has been shown that the average zeropoint energy of the particle is proportional to particle bivector spin and the mass of the particle may be interpreted as a local spatial complex rotation in the rest frame.

In the electromagnetic world, the particle mass originates from the electromagnetic field and it is purely electromagnetic in nature [27]. In the classical Lorentz theory of electron, the self-energy is closely connected to the electromagnetic mass of the electron. The self-energy problem in classical theory or quantum theory is essentially connected to the structure of electron and it may not be correct to assign the structure to the electron as a form factor [28]. Further the classical electromagnetic field may be only responsible for the interaction and gives the particle mass as purely electromagnetic in nature. In quantum field theories, the energy, momentum and charge of a particle appear as a consequence of field quantisation and leads to natural classification of particles depending on their spin values. In the renormalization procedure of quantum field theory with finite cut-off for the radiatively induced mass, it has been shown that mass depends on particle spin in the limit when the bare mass tends to zero [29]. However, in the quantum electrodynamics it is well known that the sum of bare mass and the mass correction equals the electron mass and the mass correction is due to the interaction of the particle with vacuum fluctuations [11]. Recently, Pollock interpreted particle mass (fermion or boson) arising from the zeropoint vacuum oscillations by introducing a matrix mass term in the Dirac equation [30]. The standard model deals with the fundamental particles through interaction of bosons, and at a deeper level one may consider the particles as field excitations. Though the vacuum fluctuations have been treated in a different manner in quantum theory and in quantum electrodynamics, the particle oscillations considered either in the vacuum field or in the classical stochastic electrodynamics with spin, are attributed to the fluctuations of the zeropoint field. The idea that the mass arises from the external electromagnetic interaction may lead to the conclusion that charge retains intrinsic masslessness [31]. It has been argued that for there to be correspondence with the particle mass, perhaps at pre-quantum level, inertial mass must originate from external electromagnetic interaction [32].

The aim of this article is to find the energy absorbed by the particle due to its intrinsic motion in the presence of zeropoint field and to discuss the possible origin of mass generation. In section 2, we have explained the modalities of the extended structure of the charged particle in the complex vector algebra. In the present extended particle structure, since we have considered the center of mass point and center of charge separated by radius of rotation in the complex plane, the equation of motion of the particle as a whole is considered as a combination of equation of motion of center of charge and the equation of motion of center of mass. These equations of motion of center of charge and center of mass are derived in section 3. Considering the equation of motion of center of charge in the zeropoint field, the energy absorbed by an extended charged particle is obtained in section 4, and the possible origin of mass generation is discussed in section 5. Finally, conclusions are presented in section 6. Throughout this article a charged particle implies a particle like electron.

2 The complex structure of a charged particle

In the extended particle structure, the centre of mass and the centre of charge positions are considered as separate. Denoting the centre of local complex rotations by the position vector $\mathbf{x}$ and the radius of rotation by the vector ξ , a complex vector connected with both the motion of the centre of mass point and internal complex rotation is expressed as [26]

$$X(t) = \mathbf{x}(t) + \mathbf{i}\,\xi(t). \tag{4}$$

In the geometric algebra, a bivector represents an oriented plane and $\mathbf{i}$ is a pseudoscalar which represents an oriented volume [33]. Differentiating (4) with respect to time gives the velocity complex vector.

$$U(t) = \mathbf{v}(t) + \mathbf{i}\,\mathbf{u}(t). \tag{5}$$

Here, the velocity of centre of mass point is $\mathbf{v}$ and the internal particle velocity is $\mathbf{u}$. A reversion operation on U gives $\bar{U} = \mathbf{v} - \mathbf{i}\,\mathbf{u}$ and the product

$$U\bar{U} = v^2 + u^2. \tag{6}$$

In the particle rest frame $\mathbf{v} = 0$ and $U\bar{U} = u^2$. Since the particle internal velocity in the particle rest frame $u = c$ the velocity of light, $|U| = u = c$. However, when the particle is observed from an arbitrary frame different from the rest frame of the particle centre of mass, as the centre of mass moves with velocity $\mathbf{v}$, the particle motion contains both translational and internal rotational motion of the particle. Then the particle internal velocity can be seen as

$$u^2 = c^2 - v^2 \tag{7}$$

or

$$u = c(1 - \beta^2)^{1/2} = c\gamma^{-1}, \tag{8}$$

where $\boldsymbol{\beta} = \mathbf{v}/c$ and the factor γ is the usual Lorentz factor. The angular frequency of rotation of the particle internal motion is equal to the ratio between the velocity c and radius of rotation ξ, $\omega_s = c/\xi$. When observed from an arbitrary frame, the angular frequency ω would be equal to the ratio between u and ξ

$$\omega = \frac{u}{\xi} = \omega_s \gamma^{-1}. \tag{9}$$

Thus the angular frequency of rotation decreases when observed from an arbitrary frame and the decrease depends on the velocity of the centre of mass. Considering the helical motion of the particle, this method of calculation for time dilation was first shown in a simple manner by Cavelleri [24]. The above analysis shows that the basic reason for the relativistic effects that we observe is due to the internal rotation which is a consequence of fluctuating zeropoint field and elucidates a deeper understanding of relativity at particle level in addition to the constancy of velocity of light postulate. The difference between ω and ω_s corresponds to the particle velocity. In other words, when the particle moves with velocity $\mathbf{v}$, an important consequence is that the particle itself induces certain modification in the field to take place at a lower frequency ω_B. Thus the motion of a free particle is conveniently visualized as a superposition of frequencies ω_0 and ω_B such that the particle motion as observed from an arbitrary frame appears to be a modulated wave containing internal high frequency ω_0 and an envelope frequency ω_B. The ratio between the envelop frequency and the internal frequency is then expressed as

$$\frac{\omega_B}{\omega_0} = \frac{v}{c}. \tag{10}$$

This result is simply a consequence of superposition of internal complex rotations on translational motion of the particle. The relativistic momentum of the center of mass point can be expressed as $\mathbf{p} = \gamma m\mathbf{v}$ and in the complex vector formalism momentum complex vector is given by [26]

$$P = \mathbf{p} + \mathbf{i}\,\pi, \tag{11}$$

where $\pi = m\mathbf{u}$. The total energy of the particle is now expressed as

$$E^2 = P\bar{P}c^2 = (\mathbf{p} + \mathbf{i}\,\pi)(\mathbf{p} - \mathbf{i}\,\pi) = p^2c^2 + m^2c^4. \tag{12}$$

However, in the presence of external electromagnetic field we normally replace the momentum by $\mathbf{p} - e\mathbf{A}/c$ in the minimal coupling prescription. Now, using $\mathbf{p} \to \mathbf{p} - e\mathbf{A}/c$ in (12) and equating the scalar parts, the total energy of the particle becomes

$$E^2 = p^2c^2 - 2ec\mathbf{p}.\mathbf{A} + e^2A^2 + m^2c^4. \tag{13}$$

Here, $\mathbf{A}$ represents the zeropoint electromagnetic field vector potential. In the rest frame of the particle, i.e., when the velocity $\mathbf{v} = 0$, the above expression reduces to

$$E_0 \sim mc^2 + \frac{e^2A^2}{2mc^2}, \tag{14}$$

where the higher order terms are neglected. Thus, under the influence of zeropoint field, the term $e^2A^2/2mc^2$ in the above equation gives a correction to mass. Expanding the vector potential in terms of its creation and annihilation operators and averaging in the standard form, it can be shown that the correction term [23]

$$\frac{e^2}{2mc^2}\langle A^2\rangle = \frac{\alpha}{2\pi}\frac{(\hbar\omega_c)^2}{mc^2}, \tag{15}$$

where ω_c is the cut-off frequency and α is the fine structure constant. When the cut-off frequency is equal to the frequency of oscillation of the particle, $\omega_c = \omega_0$ and using Einstein-de Broglie formula $\hbar\omega_0 = mc^2$, the mass correction can be expressed in the following form

$$\langle \Delta E_0\rangle = \delta mc^2 = \frac{\alpha}{2\pi}\, mc^2. \tag{16}$$

Thus in the presence of zeropoint field, the vector potential term in (15) gives the mass correction and it was obtained by Schwinger in quantum electrodynamics. The particle mass which arises due to local complex rotations in the zeropoint field is regarded as the so called bare mass and when the particle is observed from an arbitrary frame, the particle mass has some mass correction due to the presence of external zeropoint field.

3 Equation of motion of the particle with complex structure

It should be noted that, (1) contains the so called runaway and causal problems. In the Landau approximation, the damping term is written as a derivative of external force. In this case, the runaway and causal problems are eliminated and the exact equation of motion of a charged particle was recently given by Rohlrich [34] and Yaghjian [35]. In the equation of motion of the charged particle, centre of mass appears as if the total charge is at that point. In other words, there is no distinction between centre of mass and centre of charge points. In the case of extended particle structure, it has been clarified in the previous sections that the external zeropoint field must be responsible for the internal complex rotations and at the same time for the deviations in the path of the particle when it is moving with certain velocity. The external zeropoint field is then expressed as a function of complex vector X, $\mathbf{E}_z = \mathbf{E}_z(X,t)$ and expanding it gives

$$\mathbf{E}_z(X,t) = \mathbf{E}_z(\mathbf{x},t) + \mathbf{i}\,\xi \left|\frac{\partial \mathbf{E}_z(\mathbf{x},t)}{\partial \mathbf{x}}\right|_{\mathbf{x}\to 0} + O(\xi^2). \quad (17)$$

The second term on right hand side of the above equation is independent of x and it is a function of ξ only. Neglecting higher order terms in (17) and representing the second term on right by $\mathbf{i}\,\mathbf{E}_z(\xi,t)$, the external zeropoint field $\mathbf{E}_z(X,t)$ can be decomposed into a vector and a bivector parts

$$\mathbf{E}_z(X,t) = \mathbf{E}_z(\mathbf{x},t) + \mathbf{i}\,\mathbf{E}_z(\xi,t). \quad (18)$$

The random fluctuations produce kicks in all directions and leads to random fluctuations of the centre of mass point and at the same time random fluctuations also produce internal complex oscillations or rotations. Thus the force acting on the charged particle can be decomposed into two terms, the force acting on the centre of mass and the force acting on the centre of charge. For the field acting on the centre of mass, the particle mass and charge appear as if they are at the centre of mass point and we treat the equation of motion of the particle in the point particle limit. However, for the field acting on the centre of charge, the effective mass seen by the zeropoint field is the mass due to the potential $U_z \sim e^2/2R \sim m_z c^2$. The magnitude of R is of the order of Compton wavelength. Then the effective mass m_z in the zeropoint field is approximately equal to the electromagnetic mass which is proportional to the electromagnetic potential due to charge e at the center of mass position. Replacing the position vector $\mathbf{x}$ by the complex vector X and $\mathbf{E}_z(x,t)$ by the complex field vector $\mathbf{E}_z(X,t)$ in (1) and separating vector and bivector parts gives the equations of motion of the centre of mass and the centre of charge respectively. The equation of motion of center of mass is the Abraham-Lorentz equation of motion of a charged point particle in the external electromagnetic zeropoint field given by (1) and the motion of the centre of mass of the particle is observed from an arbitrary frame of reference. In the rest frame of the particle, the equation of motion represents the equation of motion of center of charge

$$m_z\ddot{\xi} - \Gamma_z m_z \ddot{\mathbf{u}} + m_z\omega_0^2\xi = e\mathbf{E}_z(\xi,t). \quad (19)$$

The terms $\Gamma_z m_z \ddot{\mathbf{u}}$ and $m_z\omega_0^2\xi$ are radiation damping and binding terms respectively. The damping constant in the above equation is defined as $\Gamma_z = (2e^2)/(3m_z c^3)$.

4 Average zeropoint energy associated with the particle in its rest frame

The zeropoint field and particle interaction takes place at resonance and the particle oscillates at resonant frequency ω_0. In other words, the particle oscillator absorbs energy from the zeropoint field at a single frequency which is the characteristic frequency of oscillation. Since, both radiation damping and binding terms are much smaller than the force term in (19) one can neglect these terms and integrating with respect to time t gives the internal velocity of rotation of the particle

$$\mathbf{u}(t) = \frac{e}{m_z}\int_0^{\tau} \mathbf{E}_z(\xi,t)dt. \quad (20)$$

Here, the upper limit of integration is chosen as the characteristic time τ required by the electromagnetic wave to traverse a distance equal to the size of the particle. The electric field vector $\mathbf{E}_z(\xi,t)$ is expressed in the same form as that of Rueda [19],

$$\mathbf{E}_z(\xi,t) = \sum_{\lambda=1}^{2}\int d^3k\epsilon(\mathbf{k},\lambda)\frac{H(\omega)}{2}\times \left[ae^{i(\mathbf{k}.\xi-\omega t)} + a^*e^{i(\mathbf{k}.\xi-\omega t)}\right], \quad (21)$$

where $a = \exp(-i\theta(\mathbf{k},\lambda))$, $a^* = \exp(i\theta(\mathbf{k},\lambda))$ and $\epsilon(\mathbf{k},\lambda)$ is the polarization vector and the normalization constant is set equal to unity. The phase angle $\theta(\mathbf{k},\lambda)$ is a set of random variables uniformly distributed between 0 and 2π and are mutually independent for each choice of wave vector $\mathbf{k}$ and λ. The stochastic nature of the field lies in these phase angles and a statistical average of these phase angles gives an effective value of the field. For point particles, because the size is zero, we find the spectral divergence of zeropoint field. However, for particles with extended structure, one can discern a natural cut-off wavelength associated with the particle size. The convergence factor gives an upper bound to the energy available from the electromagnetic zeropoint field and it is associated with the characteristic function $H(\omega)$ of the zeropoint field. The function $H(\omega)$ is given by $2\pi^2 H^2(\omega) = \eta(\omega)\hbar\omega$. In (21), integrating the electric field vector with respect to time gives

$$I = \sum_{\lambda=1}^{2} \int_0^\infty d^3k \, \frac{H(\omega)}{2} \left[\epsilon(\mathbf{k},\lambda)\, a e^{i\mathbf{k}.\xi} \left(\frac{e^{-i\omega\tau} - 1}{-i\,\omega} \right) + \epsilon(\mathbf{k},\lambda)\, a e^{i\mathbf{k}.\xi} \left(\frac{e^{-i\omega\tau} - 1}{-i\,\omega} \right) \right].$$

The charge current in the rest frame of the particle is the charge times the internal velocity of the particle. The interaction energy of the charged particle with the zeropoint field is expressed as the charge current times the vector potential of the zeropoint field. However, one can express the vector potential as the integral of the zeropoint electric field vector. Then the average zeropoint energy acquired by the particle is expressed as

$$\langle E_0 \rangle = \frac{e^2}{m} \langle I I^* \rangle. \tag{22}$$

The averages of random phase and the polarization vector are expressed as follows

$\langle a a^* \rangle = \delta(\lambda - \lambda')\, \delta^3(k - k')\,;\ \langle a a \rangle = 0\,;\ \langle a^* a^* \rangle = 0$

$\langle \epsilon(\mathbf{k},\lambda) \epsilon^*(\mathbf{k},\lambda) \rangle = \delta_{ij} - \frac{k_1}{k^2}$

$\sum_{\lambda=1}^{2} \int d^3k \, \langle \epsilon(\mathbf{k},\lambda) \epsilon^*(\mathbf{k},\lambda) \rangle = \frac{8\pi}{3} \int \omega^2 \, d\omega\,.$

Using these stochastic averages, replacing the convergence factor by $\eta(\omega_0)$ and setting the upperlimit of integration to the frequency of oscillations in (22) gives

$$\langle E_0 \rangle = \frac{4e^2\hbar}{3\pi m_z c^3}\, \eta(\omega_0) \int_0^{\omega_0} \omega (1 - \cos\omega\tau)\, d\omega. \tag{23}$$

For an extended particle structure $\omega_0 \tau = 2\pi$ and the above equation after integration reduces to

$$\langle E_0 \rangle = \eta(\omega_0)\, \frac{\Gamma_z \hbar \omega_0^2}{\pi}. \tag{24}$$

This result is similar to the result obtained by Reuda [19] and Puthoff [36]. However, the difference is that the damping constant is now replaced by Γ_z and cut-off frequency ω_c is replaced by the resonant internal frequency of oscillation of the particle. In (24), both the values for m_z and $\eta(\omega_0)$ are not known exactly and must be approximated. Instead, one can approximate $\eta(\omega_0)\Gamma_z\omega_0 \sim 1$ for the particle with extended structure. Then the average zeropoint energy acquired by the particle in its rest frame is

$$\langle E_0 \rangle = \frac{\hbar\omega_0}{\pi}. \tag{25}$$

This energy is similar to the zitterbewegung energy of Dirac electron in quantum mechanics.

5 Equation of motion of the particle with complex structure

In the above procedure, initially we have considered the charged particle without any mass. Such particle interacting with zeropoint field acquires mass due to particle resonant oscillations and gains energy from the electromagnetic zeropoint field. This average zeropoint energy of the particle appears as the mass of the particle. In the complex vector formalism of internal harmonic oscillator in zeropoint field, it has been shown by the author that the average energy $\langle E_0 \rangle$ is related to the mass through particle spin and represents the mass generated from the local complex rotations produced by the interaction of zeropoint field with the particle. The relation between average zeropoint energy and particle spin is given by the expression [26]

$$\langle E_0 \rangle - \omega_0 \langle \mathbf{s} \rangle = 0. \tag{26}$$

Let us denote $\omega_s = 2\omega_0$ and write the angular velocity bivector as $\Omega_s = -\mathbf{i}\,\sigma_s \omega_s$, where σ_s is a unit vector along the direction of spin. The average value of spin is obtained by taking the average over a half cycle, $\langle \mathbf{s} \rangle = \frac{2}{\pi}\mathbf{s}$. Substituting this average value of spin and $\langle E_0 \rangle$ from (25) in (26) gives the relation between particle mass and spin

$$mc^2 = \sigma_s \Omega_s . S, \tag{27}$$

where the relation $\hbar\omega_0 = mc^2$ is used and the bivector spin $S = \mathbf{i}\,\sigma_s \hbar/2$. The unit vector σ_s acting on an idempotent $\mathcal{J}_+ = (1 + \sigma_S)/2$ gives an eigenvalue +1. This statement is represented by an equation $\sigma_s \mathcal{J}_+ = +1\mathcal{J}_+$. When (27) is multiplied from right by an idempotent $\mathcal{J}_+$ on both sides the unit vector is absorbed by the idempotent and equating the scalar parts gives

$$mc^2 = \Omega_s . S. \tag{28}$$

Thus the mass of the particle turns out to be the local internal rotational energy given by the term $\Omega_s.S$. Since, the magnitude of spin and velocity of light are constants, the value of particle mass depends on the frequency of spin rotation and the different particles may have different frequencies of spin rotation. The above analysis shows that the internal complex rotation is responsible for the existence of particle mass. Then, one may initially consider a massless charged particle and it may acquire mass from zeropoint field through a local complex rotation.

When the particle is observed from an arbitrary frame of reference, the center of mass point moves with velocity $\mathbf{v}$. The equation of motion of centre of mass point is given by (1) and solving it by assuming the radiation damping and binding terms as small when compared to the force term, one can obtain the zeropoint energy absorbed by the point particle and it is given by (3). The cut-off frequency ω_c is the limiting frequency in the integration. When we assume the cut-off frequency $\omega_c = \omega_0$ [37, 38] and after introducing the convergence factor $\eta(\omega_0) \sim 3/4$ in (3), the average energy representing the mass correction of the particle in the zeropoint field can be expressed as

$$\delta m = \frac{\alpha}{2\pi} mc^2. \tag{29}$$

This mass correction is too small and found to be similar to the expression found in quantum electrodynamics to the first order in the fine structure constant α.

6 Conclusions

In the stochastic electrodynamics with spin, it has been shown that the average zeropoint energy absorbed by the particle due to its internal motion gives the particle mass. When the particle center of mass point moves with certain velocity, we find the average energy absorbed by the particle gives the mass correction. In deriving both particle mass and mass correction, a convergence factor has been introduced for an extended particle. To understand the mechanism of mass generation of an elementary particle, one may initially assume a massless charged particle with complex structure and such a particle can be visualized as an oscillator in the fluctuating zeropoint field. Then the average energy absorbed by the oscillator refers to the particle mass. Finally, we conceive the idea that an elementary particle acquires mass from the interaction of ubiquitous zeropoint field.

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