An Explanation of De Broglie Matter Waves in Terms of the Electron Coupling to the Vacuum State

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This paper examines the de Broglie wave theory derived by Synge who assumed that Hamilton’s variational principle in three dimensions applies also to the four-dimensional Minkowski spacetime. Based on the Planck vacuum (PV) view of the electron coupling to the vacuum state, calculations here suggest that the Synge de Broglie waves exist and travel within the PV state.

1 Introduction
In the early part of the twentieth century when it was realized that the massless photon had particle-like properties, de Broglie figured therefor that the massive electron must have wave-like properties — and the de Broglie matter wave was born [1, p.55].

In circa 1954 Synge [2] published a study on the idea of 3-waves propagating in the 4-dimensional Minkowski spacetime. The study was based on the properties of a medium which the present author interprets as a vacuum medium. In the PV theory, the occurrence of 3-waves in a 4-dimensional spacetime is symptomatic of an invisible medium. In the PV theory, the occurrence of 3-waves propagating in the 4-dimensional Minkowski space-time. Based on the Planck vacuum view of the electron, calculations here suggest that the Synge de Broglie waves exist and travel within the PV state.

2 Compton-(de Broglie) relations
In the PV theory the interaction of the electron with the vacuum state leads to the Compton-(de Broglie) relations [3]

\[ r_e \cdot mc^2 = r_d \cdot cp = r_L \cdot E = r_n \cdot m_e c^2 = \frac{e^2}{\hbar c} \]  

(1)

where \( e \) is the massless bare charge that is related to the electronic charge via \( e = \alpha^{1/2} e_x \), and \( \alpha \) and \( m_e \) are the fine structure constant and the Planck particle mass. The radii \( r_e (= e^2 / mc^2) \) and \( r_d (= e^2 / m_e c^2) \) are the electron and Planck-particle Compton radii respectively and \( m \) is the electron mass. The magnitudes of \( r_e \) and \( m_e \) are equal to the Planck length and mass respectively [4, p. 1234]. It is because \( r_n \neq 0 \) that the PV state is a quasi-continuum.

The de Broglie radii, \( r_L \) and \( r_d \), are derived from \( r_e \) and the Lorentz invariance of the vanishing electron/PV coupling force (A1) at \( r = r_e \):

\[ \beta = \frac{v}{c} \quad \text{and} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} \]

where \( \beta = v/c \) and \( \gamma = 1/\sqrt{1 - \beta^2} \). From (3) \( \beta = r_L / r_d \) yields the relative velocity of the electron in the coupled electron/PV system. The relations in (1) also lead to the relativistic electron energy \( E = \sqrt{m^2 c^4 + v^2 c^2} \).

The relativistic scalar wave equation is [5, p. 319]

\[ \left( \frac{\hbar}{mc} \right)^2 \left( \nabla^2 - \frac{\partial^2}{c^2 \partial t^2} \right) \psi = r_e^2 \left( \nabla^2 - \frac{\partial^2}{c^2 \partial t^2} \right) \psi = \psi \]  

(4)

while its nonrelativistic counterpart reads

\[ \left( \nabla^2 - \frac{\partial^2}{\nu^2 \partial t^2} \right) \psi = 0 \]  

(5)

where \( v \ll c \); so the electron radii in (1) and (4) are relativistic parameters.

The plane-wave solution \( (\psi \propto \exp i \phi) \) to (4) in the \( z \)-direction involves the phase

\[ \phi = \frac{Et - pz}{\hbar} = \frac{Ect - cpz}{e^2} \]

(6)

where the relativistic energies \( E = m e c^2 \) and \( cp = cm ey \) from (1) are used in the final expression. The normalization of \( ct \) and \( z \) by the de Broglie radii \( r_L \) and \( r_d \) is a characteristic of the PV model of the vacuum state, and is related to the Synge primitive (plane-wave) quantization of spacetime to be discussed below.

3 De Broglie waves

The Ray/3-Wave diagram that represents the Synge de Broglie wave propagation in spacetime [2, p. 106] is shown in Fig. 1, where the electron propagates upward at a uniform velocity \( e \) along the Ray. The vertical axis is \( \zeta = i ec \) and the horizontal axes are represented by \((x,y,z)\). The need for quantizing the Synge vacuum waves (adding the parameters \( r_e, r_L, \) and \( r_d \) to Fig. 1) is explained in the following quote. “So far the [variational-principle] theory has been confined to the domain of geometrical mechanics. The de Broglie waves have no phase, no frequency, no wave-length. It is by quantization...
Fig. 1: Planewave quantization of de Broglie Waves in Spacetime. The figure consists of a single Ray and a partial picture of the corresponding 3-Waves propagating toward the upper left. The Ray and their 3-Waves are orthogonal in the 4-dimensional spacetime sense. The quantization consists of the normalization constants \( r_c \), \( r_L \), and \( r_d \).

that these things are introduced, in much the same way as they are introduced in the transition from geometrical optics to physical optics" [2, p. 105].

The parameters in the figure correspond to \( r_c = r_L \), \( r_L = r_d \); the Compton and de Broglie radii from (1). In the quantization, the Synge theory utilizes the wavelengths \( 2\pi r_c \), \( 2\pi r_L \) and \( 2\pi r_d \). Thus there is a one-to-one correspondence between the electron/PV coupling radii from (1) and the Synge wave theory regarding Fig. 1. In the PV theory the electron radii \( (r_c, r_L, r_d) \) are parameters generated by the electron/PV interaction — thus it is reasonable to conclude that the de Broglie waves travel within the vacuum state.

Appendix A: Coupling force

In its rest frame, the coupling force the electron core \( (-e_*, m) \) exerts on the PV quasi-continuum is [3]

\[
i \left( \frac{e^2}{r^2} - \frac{mc^2}{r} \right)
\]

where the radius \( r \) begins at the electron core. The spacetime coordinates are denoted by

\[
x_\mu = (x_0, x_1, x_2, x_3) = (ict, x, y, z)
\]

where \( \mu = (0, 1, 2, 3) \) and \( r = (x^2 + y^2 + z^2)^{1/2} \).

The force (A1) vanishes at the electron Compton radius \( r_c \) and leads to:

\[
i \left( \frac{e^2}{r_c^2} - \frac{mc^2}{r_c} \right) = 0
\]

in the electron rest frame; and when (A3) is Lorentz transformed it results in the two coupling forces

\[
i \left( \frac{e^2}{r_c^2} - \frac{mc^2}{r_c} \right) = 0
\]

and

\[
i \left( \frac{e^2}{r_d^2} - \frac{mc^2}{r_d} \right) = 0
\]

in the uniformly moving frame.

To complete the PV perspective, the space and time derivatives in the scaler wave equation

\[
r_c^2 \left( \nabla^2 - \frac{\partial^2}{c^2 \partial t^2} \right) \psi = \psi
\]

and the Dirac electron equation [6, p. 74]

\[
ic \hbar (\alpha \cdot \nabla + \frac{\partial}{c \partial t}) \psi = mc^2 \beta \psi
\]

or

\[
i r_c \left( \alpha \cdot \nabla + \frac{\partial}{c \partial t} \right) \psi = \beta \psi
\]

are quantized (normalized, scaled) by the same constant \( r_c \), where the 4x4 vector matrix \( \alpha \) accounts for electron spin. [The \( \beta \) in (A7) is a 4x4 matrix, not a relative velocity.]

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References