A Telemetric Multispace Formulation of Riemannian Geometry, General Relativity, and Cosmology: Implications for Relativistic Cosmology and the True Reality of Time

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This thesis reveals an extended world-picture of Riemannian geometry as a telemetric multispace model of real space on the cosmological scale: certain new aspects of General Relativity are presented in terms of a fundamental membrane-transition picture of the deeper reality of time. We refer to this as “telemetric multispace formulation of General Relativity”, a world-system with heavy emphasis on Riemannian geometry “per se” (in light of a particular set of extensive, purely geometric techniques), without all the usual historical-artificial restrictions imposed on it. This seminal model gives the purely geometric realization of instantaneous long-range action in the whole space-time of General Relativity whose sub-structure is extended to include an intrinsic, degenerate gravitational-rotational zero-space hosting zero-particles. The mathematical basis of modern cosmology is the four-dimensional pseudo-Riemannian space which is the curved space-time of General Relativity. The additional restrictions pre-imposed on space-time due to so-called “physical reasons” are, regularly: the signature conditions, the prohibition of super-luminal velocities, and the strictly uni-directional flow of time. We here study the peculiar conditions by which the observable time 1) is stopped; 2) flows from the future to the past. Our world and the world wherein time flows oppositely to us are considered as spaces such that they are “mirror images” of each other. The space wherein time stops (the present) is the “mirror” reflecting the future and the past. Then we consider the interaction between a sphere of incompressible liquid (the Schwarzschild bubble) and the de Sitter bubble filled with physical vacuum: this is an example of the interaction between the future and the past through the state of the present.

1 Riemannian geometry as a mathematical model of the real world

A brief historical background is at hand, followed by a critical mathematical reappraisal. As known, the mathematical basis of modern cosmology is the four-dimensional pseudo-Riemannian space — the curved space-time of General Relativity. It belongs to the whole spectrum of Riemannian spaces obtained by Bernhard Riemann as a generalization of Carl Gauss’ work on curved surfaces. Riemannian spaces possess any number dimension \( n \). The numerical value of \( n \) is determined by a maximal number of independent basis vectors (general basis, in the collective sense) of the Riemannian space \( V_n \) [1]. The basis of the \( V_n \) is introduced at every point of the flat space \( E_n \) which is tangent to the \( V_n \) at this point. If the basis vectors are linearly dependent, the dimension of the \( V_n \) is less than that of the space wherein the basis vectors are independent of each other. There exist two types of basis vectors possessing: 1) the positive square of the length (a real vector); 2) the negative square of the length (an imaginary vector). As familiar, if all the basis vectors of the space are real or imaginary, it is known as the Riemannian space. If some of the basis vectors are real while other ones are imaginary, the space is known as the pseudo-Riemannian space. Flat Riemannian spaces, where all the basis vectors possess unit or imaginary unit lengths, are known as the Euclidean spaces \( E_n \). For example, the \( E_3 \) is the ordinary flat three-dimensional space where the unitary system of Cartesian coordinates can be introduced. Flat Riemannian spaces where some basis vectors are real and other ones are imaginary, are known as the pseudo-Euclidean spaces. The four-dimensional pseudo-Euclidean space \( E_4 \) which possesses one imaginary basis vector along with three real ones, is known as the Minkowski space (German Minkowski introduced time as the fourth coordinate \( x^4 = ct \), where \( t \) is the coordinate time while \( c \) is the light velocity). The pseudo-Euclidean space \( E_4 \) is of course the basic space (space-time) of Special Relativity. The pseudo-Riemannian (curved) four-dimensional space \( V_4 \) with the same set of the basis vectors is the basic space (space-time) of General Relativity. The idea of applying the four-dimensional pseudo-Riemannian space to the description of the real world was suggested Marcel Grossman, a close mathematician friend of Albert Einstein. Einstein agreed with him, because the metric properties of Riemannian spaces are simplest in comparison to the properties of other metric spaces. The point is that Riemannian metrics are invariant relative to transformations of coordinates. It implies that the square of the elementary
infinitesimal vector $dx^a$ conserves its length:

$$ds^2 = g_{ab}dx^adx^b, \quad \alpha, \beta = 0, 1, 2, 3,$$

(1)

where the contraction by indices $\alpha, \beta$ denotes the summation.

Metrics of Riemannian spaces are symmetric ($g_{ab} = g_{ba}$) and non-degenerate ($g = \det \|g_{ab}\| \neq 0$), while the elementary four-dimensional interval is invariant relative to any reference system ($ds^2 = \text{const}$). The invariance of the $ds^2$ is a very important argument on behalf of Riemannian geometry as the mathematical basis of General Relativity.

The metric coefficients are of course the cosines of the angles between the basis vectors in the locally flat tangent space. This is because $ds^2$ is the scalar product of $dx^a$ with itself. The dimension of the flat tangent space and the correlation between the imaginary and real basis vectors are the same as in the corresponding Riemannian space. A system of basis vectors $e_a$ can be introduced at any point of the locally tangent space. The $e_a$ are tangent to the coordinate lines $x^a$.

The fundamental metric tensor can be expressed through the basis vectors $e_a$ as [2]:

$$g_{ab} = e_a e_b \cos \left( x^a, x^b \right),$$

(2)

where $e_a$ is the length of the $e_a$. Assume here the temporal basis vector $e_0$ to be real, while, correspondingly, the spatial basis vectors $e_i (i = 1, 2, 3)$ are imaginary.

We recall that the interval $ds^2$ can be positive, negative, or null. The value $ds$ is used as the parameter along trajectories of particles (world-lines of particles). These lines can be: 1) real by $ds^2 > 0$, 2) imaginary by $ds^2 < 0$, 3) zero by $ds^2 = 0$. The value $ds$ is used as the global parameter along worldlines. Real mass-bearing particles (the rest-mass $m_0 \neq 0$, the relativistic mass $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ (where $v = \frac{dx}{\sqrt{g_{00}}}$ is real) move along the non-isotropic lines ($ds \neq 0$) at sub-luminal velocities $V < c$; imaginary mass-bearing particles or hypothetical tachyons (the rest-mass $m_0 = 0$, the relativistic mass $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ is imaginary) move along non-isotropic lines ($ds \neq 0$) at super-luminal velocities $V > c$; massless particles (the rest-mass $m_0 = 0$, the relativistic mass $m = 0$) move along isotropic lines ($ds = 0$) at light velocity $V = c$. Thus, for example, photons are actual light-like particles.

The description of the world is to be linked with the real reference frame of a real observer who actually defines both geometrical and mechanical properties of the space of reference he inhabits. The reference frame is a reference body where coordinate nets are spanned and clocks are installed at the every point of the reference’s body. The profound problem of the introduction of physically observable quantities in the whole inhomogeneous, anisotropic curved space of General Relativity is to determine which components of the every four-dimensional quantity are the physically observable quantities. This problem was solved decisively and comprehensively by A. Zelmanov [2]. He introduced chronometric invariants (chr.-inv.) as physically observable geometric quantities in General Relativity. These fundamental quantities are linked to the reference body which can, in general, gravitate, rotate, and deform. The three-dimensional observable space (the reference space) can be both curved and flat. The reference body is considered as a set of real coordinate systems, to which the observer compares all results of his measurements. Therefore the physically observable quantities are constructed as the result of fundamentally (in a unified, simultaneous geometrical-mechanical fashion) projecting four-dimensional quantities on the lines of time and on the three-dimensional space.

The chr.-inv. form of the four-dimensional interval $ds^2$ is [2]

$$ds^2 = c^2 dt^2 - dr^2, \quad dt = \left( 1 - \frac{w}{c^2} \right) dt - \frac{v dx^i}{c},$$

(3)

$$ds^2 = h_{\alpha \beta} dx^\alpha dx^\beta, \quad h_{\alpha \beta} = -g_{\alpha \beta} + \frac{v_{\alpha \beta}}{ct}, \quad i, k = 1, 2, 3,$$

where $dr$ is the interval of the observable space, $d\sigma^2$ is the observable spatial interval, $w = c^2 (1 - \sqrt{\frac{m}{\gamma_0}})$ is the three-dimensional gravitational potential, $v_{\alpha \beta} = -g_{\alpha \beta}$ is the linear velocity of the space rotation, $h_{\alpha \beta}$ is the three-dimensional fundamental metric tensor. The expression (3) may be rewritten in the form

$$ds^2 = c^2 dt^2 \left( 1 - \frac{V^2}{c^2} \right), \quad V^i = \frac{dx^i}{dt}, \quad V^2 = h_{\alpha \beta} V^\alpha V^\beta,$$

(4)

where $V^i$ is the observable three-dimensional velocity.

It follows from (4) that $ds$ is: 1) real if $V < c$, 2) imaginary if $V > c$, 3) zero if $V = c$. The condition $ds = 0$ has the form

$$cd\tau = \pm dr,$$

(5)

which is of course the equation of the elementary light cone. The term elementary means that this cone can be introduced only at every point of the space-time, but not into the whole space. The elements of the cone are trajectories of massless particles moving along null geodesic lines.

As follows from (4,5), photons are at rest within the space-time ($ds = 0$) itself, but they move at light velocity ($V = c$) along three-dimensional trajectories ($cd\tau = dr$) within the three-dimensional observable space. The light cone is known as the “light barrier” which “prohibits” motions at super-luminal velocities. Really, this restriction means that mass-bearing particles, both real ones and tachyons, cannot move at light velocity. The zero-particles penetrating the light cone are considered in detail in [3]. These particles are essentially thinner structures than light, because their relativistic masses $m$ are zeroes. Zero-particles possess non-zero gravitational-rotational masses $M = \frac{m}{1 + \omega u/c}$, where $u = \frac{dx}{dt}$. Zero-particles transfer instantly ($d\tau = 0$) along three-dimensional null trajectories ($d\sigma = 0$). The light cone is therefore transparent for zero-particles and non-transparent for mass-bearing real particles.
and tachyons. As such, we may call it a “membrane”. Thus the apparatus of General Relativity allows the existence of long-range action as truly instantaneous-transfer zero-particles. Moreover, this fundamental transfer unifies the worlds of both real particles and tachyons. As for the other new aspects of General Relativity, we shall introduce them in the next sections.

2 The past and the future as the mirror reflections each other

Most contemporary scientists presuppose that time flows only in a single direction — from the past to the future. The mathematical apparatus of General Relativity does not prohibit the reverse flow of time, i.e. from the future to the past. Nevertheless, the reverse flow of time is not introduced in contemporary physics and cosmology, partly because modern scientists refer to Hans Reichenbach’s “arrow of time”, which is directed always to the future. However, upon further analysis, Reichenbach, speaking about a unidirectional flow of time, implied a rather limited world-process of evolution (transfer mechanism of energy). He wrote: “Super-time has not a direction, but only an order. Super-time itself, however, contains local sections, each of whom has a direction, while the directions change from one section to another” [4]. Contemporary scientists consider the light cone of Minkowski space as a mathematical illustration of the time arrow: the lower half of the cone means the past, while the upper half — the future. The past automatically turns into the future at the point \( t = 0 \), meaning the present. But such an automatic transfer is due to the fact that the Minkowski space of Special Relativity is de facto empty. Besides, it does not at all include both gravitation and rotation (in addition to deformation and the whole curvature), therefore the ideal, uniformly flowing time of Special Relativity does not (and can not) depend on gravitation and rotation. In other words, this transfer does not require fundamental transformations of matter. In fact, in this picture, photons flow continuously from the lower half of the cone to the upper one. However the “real space” perceived by us as the “present” is ultimately penetrated by gravitation. Besides, the objects of the said space, ranging from electrons to galaxies and their clusters, do rotate around their centers of gravitational attraction. The problem is therefore to describe, in the framework of General Relativity, the fundamental interaction between the future and the past as a proper energetic transfer through the present state. Such description of the future-past transfer is a more exact approximation, than in the self-limited Minkowski space, because the observable time \( \tau \) essentially depends on both gravitation and rotation: see (3.5). The expressions \( d\tau = 0 \), \( d\sigma = 0 \) describe the membrane, which is situated between the past and the future. These expressions can be rewritten in the form [3]:

\[
w + v_i u^i = c^2, \quad h_{ik} dx^i dx^k = 0, \quad u^i = \frac{dx^i}{dt}.
\]

As the metric form \( d\sigma^2 \) is positively determined, the condition \( d\sigma^2 = 0 \) means that it is degenerated: \( h = \det |h_{ik}| = 0 \). The determinants of the matrices \( g = \det |g_{ik}| \) and \( h \) are linked by the relation \( \sqrt{g} = \sqrt{|g_{ik}|} \) [2], therefore the four-dimensional matrix \( |g_{ik}| \) is degenerated: \( g = \det |g_{ik}| = 0 \). The condition of the membrane transition can be written in the form [3]:

\[
w + v_i u^i = c^2, \quad d\mu^2 = g_{ik} dx^i dx^k = \left(1 - \frac{w^2}{c^2}\right) c^2 dt^2,
\]

where the first expression characterizes the condition of the stopped time, the second expression describes the geometry of the hyper-surface, where events of the present are realized.

The conditions (7) describe the zero-space, where, from a viewpoint of a real observer, zero-particles extend instantly \( (d\sigma = 0) \) along three-dimensional null lines \( (d\tau = 0) \) [3]. The instant transfer of zero-particles means the long-range-action. We conclude that the future-past transfer is realized instantaneously, i.e. it is the long-range-action. Note, the coordinate length \( d\mu = c (1 - \frac{w^2}{c^2}) cd\tau \) depends, in part, on the gravitational potential \( w \), wherein \( d\mu = 0 \) by the collapse condition: \( w = c^2 \). Thus the metric on the hyper-surface is, in general, not a Riemannian one, because its interval \( d\mu^2 = 0 \) is not invariant (yet it is invariant by the collapse, as in this case \( d\mu^2 = 0 \)). The region of space-time, which is located between the spaces of the past and the future, is perceptible by a real observer as the present. It is the hyper-surface where all events are realized at the same moment of observable time \( \tau_0 = \text{const} \), i.e. such events are synchronized. The momentary interaction (the long-range-action) is transferred by particles of a special kind — zero-particles. They possess zero rest-mass \( m_0 \), zero relativistic mass \( m \), and non-zero gravitational-rotational mass \( M \). This quantity is determined in the generalized space-time where the condition \( g = 0 \) is satisfied. The mass \( M \) in the generalized space has the form [3]

\[
M = \frac{mc^2}{c^2 - (w + v_i u^i)}.
\]

Thus the elements of the elementary curved light cone (the so-called “light barrier”) are indeed penetrable for zero-particles. As follows from (5), trajectories of photons belong to both the space and time, because they extend along null four-dimensional trajectories \( ds = 0 \). The three-dimensional body of the real observer can thus move at pre-light velocity in the three-dimensional space, but it is always rigidly attached to the moment of time, which is perceptible as the present.

A brief philosophical digression: transfers both in the past and in the future are possible, so far, only mentally. The typical human mind does remember the past (not always clearly) and does predict the future (not always exactly). It is possible to say that the past and the future are virtual, because only the human consciousness moves in these virtual spaces, but the
physical body is strictly in the present (“reality”). Studying the past of the Earth and remembering our own past, we see a recurrence of some events, both planetary and individual. We know what happened with the Earth in the past due to mainly the tales of our ancestors, if not historians. Events (three-dimensional points, as well as threads extended in time) are ordered in a determined sequence in time. Comparing similar events from different intervals of time, we can say that the past and the future are similar, being mirror reflections of one other. The object of the three-dimensional space and its mirror reflection differ only by the notions of “left” and “right” possessing the opposite sense for every one of them. The intervals of both coordinate time and observable time are linked by the formula [3]

\[ \frac{dt}{d\tau} = \frac{v_i V^i}{c^2 - \sqrt{g_{00}}} + 1. \]  

The expression (8) was studied in [3] by the condition \( \sqrt{g_{00}} > 0 \). It means that we did not consider in [3] the reverse of time while simultaneously taking into account the state of collapse \( g_{00} = 0 \). As follows from (8), the coordinate time \( t \): 1) is stopped \((dt = 0)\) if \( v_i V^i = \mp c^2 \); 2) possesses direct flow \((dt > 0)\) if \( v_i V^i > \mp c^2 \); 3) possesses reverse flow \((dt < 0)\) if \( v_i V^i < \mp c^2 \). Thus the spaces with direct and the reverse flows of coordinate time \( t \) are divided by a fundamental surface of rotation, where the vectors \( v_i \) and \( V^i \) are linked by the relation, see (2,3):

\[ v_i V^i = \mp c^2 |e_i||V^i| \cos (v_i, V^i) = \mp c^2 |e_i||V^i| \cos (e_i, V^i), \]

where \( e_i \) is the spatial basis vector in the tangent Minkowski space. It is evident that this relation is realized for two cases:

1) the vectors \( v_i \) and \( V^i \) are co-directed, \(|v_i| = |V^i| = c\);
2) the vectors \( v_i \) and \( V^i \) are anti-directed, \(|v_i| = |V^i| = c\).

Since the vector \( v_i \) means the linear velocity of space rotation, we conclude that the very surface dividing the spaces with direct and reverse flow of coordinate time rotates at light velocity. The rotation is either left or right.

A real observer measures that the time \( \tau \) coincides completely with the coordinate time \( t \) only in the case wherein the reference space does not rotate \((v_i = 0)\) nor gravitate \((w = 0)\): see (3). If \( w \neq 0 \) or \( v_i \neq 0 \), the \( \tau \), in contrast to \( t \), depends essentially on gravitation and rotation. Because we live in the real world, where gravitation and rotation do exist, we will further consider the observable time.

The observable Universe, which is a part of the Infinite Whole, can belong to one of the aforementioned spaces (either possessing positive or negative flow of coordinate time). Let the flow of coordinate time in the region, where the observer is situated, be positive: \( dt > 0 \). The observable time is divided by the consciousness of a real observer into the “past”, the “present” and the “future”: time flows from the past to the future through the present. The problem stated in the beginning of this paper is to study the future-past transfer from the point of view of a real observer, who is located in the world of positive flow of coordinate time \( dt > 0 \). This problem is essentially simplified in the case where the reference space does not rotate. Then the expression (8) can be rewritten in the form

\[ d\tau = \pm \sqrt{g_{00}} dt = \pm \left(1 - \frac{w}{c^2}\right) dt. \]  

Taking into account the collapse condition \( \sqrt{g_{00}} \), we shall study the direction of observable time flow in the gravitational field. It follows from (9) that the observable time \( \tau \): 1) possesses positive direction if \( \sqrt{g_{00}} > 0 \), 2) possesses negative direction if \( \sqrt{g_{00}} < 0 \), 3) stops if \( \sqrt{g_{00}} = 0 \). Because the condition \( g_{00} = 0 \) is the collapse condition, the surface of the collapsar is the mirror separating the spaces with both positive and negative flow of the observable time. The observable time is perceptible by human consciousness as flowing from the past to the future, therefore we call the space of such direct flow of time the “space of the past”. Then the space of reverse flow of observable time is necessarily the “space of the future”. The present space is situated between these spaces. The concrete spaces reflecting from the surface of the collapsar, as from the mirror, will be studied in detail in the next section.

3 The interaction between the Schwarzschild and de Sitter bubbles as instantaneous transfer

All objects in the Universe consist of the same fluid substance being at different stages of cosmic evolution. Many cosmic bodies (planets, stars, . . .) are spheroids, namely spinning, deforming spheres. Probably the physical body of the Universe has the same form. The problem is to introduce the space-time (gravitational field) created by a liquid incompressible sphere. A similar model was introduced earlier by the German astronomer Karl Schwarzschild [5]. He solved the field equations (Einstein equations) for the sphere by the assumption that the solution must be everywhere regular. In other words, Schwarzschild ruled out the existence of singularities. Meanwhile the problem of singularities is very actual for astrophysics and cosmology. The more general, allowing singularities, solution of the Einstein equations for the sphere filled by ideal incompressible liquid was obtained in [6]. The substance filling the sphere is described by the energy-impulse tensor

\[ T^{\alpha\beta} = \left(\rho + \frac{p}{c^2}\right)b^\alpha b^\beta - \frac{p}{c^2}g^{\alpha\beta}, \]

where \( \rho = \text{const} \) is the density of substance, \( p \) is the pressure, \( b^\alpha = \frac{dx^\alpha}{dt} \) is the four-dimensional unit velocity vector: \( g_{\alpha\beta}b^\alpha b^\beta = 1 \).

The solution allowing singulary states of the space-time has the form [6]
\[ ds^2 = \frac{1}{2} \left( 3 \sqrt{1 - \frac{x^2 a^2}{3}} - \sqrt{1 - \frac{x^2 r^2}{3}} \right)^2 c^2 dt^2 - \frac{dr^2}{1 - \frac{x^2 r^2}{3}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \]  

(11)

where \( x = \frac{r \rho_r}{c^2 G} \) is the Einstein gravitational constant, \( G \) is the Newton gravitational constant, \( a \) is its radius, \( r, \theta, \varphi \) are the spherical coordinates.

The gravitational field described by (11) has two singularities [6]:

1) it collapses if
\[ 3 \sqrt{1 - \frac{x^2 a^2}{3}} = \sqrt{1 - \frac{x^2 r^2}{3}}; \]

2) it breaks the space if
\[ \frac{x^2 r^2}{3} = 1. \]

The radius of the collapsar \( r_c \) and the radius of the breaking space \( r_{br} \) have the forms, respectively:

\[ r_c = \sqrt{9a^2 - \frac{24}{\rho_0}} = \sqrt{9a^2 - 8r_{br}^2}, \]

(12)

where the breaking radius \( r_{br} = \frac{a}{\sqrt{9a^2 - 8r_{br}^2}} \) cm.

It follows from (12) that the incompressible liquid sphere collapses if \( a > \frac{8}{9} r_{br} = 0.94 r_{br} \). (Because by \( a = \frac{8}{9} r_{br} \) the collapsing object transforms into the point \( r_c = 0 \), we do not consider this case non-sense in the physical meaning). If \( \rho = 10^{-29} \text{ g/cm}^3 \) (the assumed value of the density of matter in the observable Universe), then the sphere collapses by \( a > 1.2 \times 10^{28} \text{ cm} \) and breaks the surrounding space by \( a = 1.3 \times 10^{28} \text{ cm} \). If the density of matter inside the sphere is \( \rho = 10^{14} \text{ g/cm}^3 \) (as inside the atomic nucleus), then \( a > 3.8 \times 10^6 \text{ cm} \) and \( r_{br} = 4 \times 10^6 \text{ cm} \). The density of matter inside a typical neutron star is regularly assumed to be the same as the nuclear density, while its radius is about a dozen kilometers. With these, larger-sized neutron stars may be non-observable, because they are gravitational collapsars. Estimate now the minimal value of the mass of the neutron star by the assumption that it collapses. If \( a = 3.8 \times 10^6 \text{ cm} \), then the mass \( M = \frac{4 \pi a^3 \rho}{3} = 23 \times 10^{33} \text{ g} = 11.5M_\odot \), where \( M_\odot \) is the mass of the Sun. Assuming \( \rho = 1 \text{ g/cm}^3 \) (the density of hydrodynamical fluid), we find \( r_{br} = 4 \times 10^{11} \text{ cm} \). It means, such a fluid sphere collapses if its radius is \( a > 4 \times 10^{11} \text{ cm} \).

A sphere of incompressible liquid with a constant volume and a constant density, which is situated in the state of weightlessness, is a kind of condensed matter. The planets, rotating around the Sun, as well as the stars, rotating around the center of the Galaxy, are in the state of weightlessness [6]. Assume that stationary stars consist of condensed matter. For example, consider the Sun as an actual sphere of condensed matter. The density of the Sun is \( \rho_\odot = 1.4 \text{ g/cm}^3 \), and its radius is \( a = 7 \times 10^8 \text{ cm} \). We find \( r_{br} = 3.4 \times 10^{13} \text{ cm} \). It follows from (12) that the collapse of the Sun is impossible in this state of matter, because \( r_c \) has an imaginary value. It is interesting to note that the surface of breaking of the Sun is at the distance \( r_{br} = 2.3 \text{ AU} \), where the Astronomical Unit (AU) is the average distance between the the Sun and the Earth: 1 AU = 1.49×10^{13} \text{ cm}. So we obtain that the surface of breaking (curvature discontinuity), created by the Sun, is actually situated inside the asteroid strip region, very close to the orbit of the maximal concentration of asteroids: 2.5 AU from the Sun [6]. (As known, the asteroid strip’s distance from the Sun is within the limit of 2.1 to 4.3 AU).

Let’s now study the simultaneous mechanical and geometrical properties of the metric (11). As shown in [2], the three-dimensional observable space (the reference space) is characterized by the three mechanical characteristics and one geometrical. The mechanical characteristics are: the vector of the gravitational inertial force \( F_i \), the tensor of the angular velocity of rotation \( A_{ik} \), and the tensor of the rate of deformation \( D_{ik} \): 

\[ F_i = \frac{c^2}{c^2 - \rho} \left( \frac{\partial \omega}{\partial x^i} - \frac{\partial \omega}{\partial t} \right), \quad \frac{1}{2} \frac{\partial \omega}{\partial x^i} = \frac{1}{2} \frac{\partial \omega}{\partial x^i} = \frac{1}{c^2} (F_{ik} - F_{ik} v_k), \]

where \( \frac{\partial \omega}{\partial x^i} = \frac{1}{c^2} \frac{\partial \omega}{\partial x^i} \) is the chr.-inv. operator of differentiation along the temporal coordinate.

We find that the reference space of the metric (11) does not rotate (\( A_{ik} = 0 \)) and deform (\( D_{ik} = 0 \)), but it gravitates. The gravitational inertial force \( F_i \) has the only non-zero component [6]

\[ F_i = -\frac{xpc^2}{3} \left( \frac{1}{\sqrt{1 - \frac{x^2 a^2}{3}}} - \frac{1}{\sqrt{1 - \frac{x^2 r^2}{3}}} \right) \right) \sqrt{\frac{1 - x^2 r^2}{3}} \]

(13)

\[ F_1 < 0. \]

Thus the quantity \( F_1 \) is the non-Newtonian force of attraction. Then \( F_1 \to \infty \) both by the collapse and the breaking of space [6].

The pressure of the ideal liquid \( p \) is determined from the conservation law [6]. It has the form

\[ p = \rho c^2 \left( \frac{1}{\sqrt{1 - \frac{x^2 a^2}{3}}} - \frac{1}{\sqrt{1 - \frac{x^2 r^2}{3}}} \right) > 0. \]

(14)
It follows from (14) that \( p \to \infty \) by the collapse and \( p = -\frac{\kappa^2}{3} \) at the surface of break.

The geometric characteristic of the reference space is the chr.-inv. three-dimensional tensor of curvature \( C_{ijkl} \) [2] possessing all the algebraic properties of the Riemann-Christoffel four-dimensional tensor of curvature \( R_{a'b'y'y} \). The \( C_{ijkl} \) has the form [2]:

\[
C_{ijkl} = \frac{1}{4} (H_{ijkl} + H_{ikjl} - H_{ijlk} + H_{iljk}),
\]

where \( H_{ijkl} \) is the chr.-inv. close analog of the Schouten tensor

\[
H_{ijkl} = \frac{\partial \Delta^j}{\partial x^i} - \frac{\partial \Delta^i}{\partial x^j} + \Delta^a_{ik} \Delta^j_{lm} - \Delta^a_{ij} \Delta^k_{lm},
\]

are the chr.-inv. Curvatures of the second and first kind, respectively, \( \frac{\partial}{\partial \sigma} = \frac{\partial}{\partial x} + \frac{i}{\sqrt{g}} \frac{\partial}{\partial \xi} \) is the chr.-inv. operator of differentiation along spatial coordinates [2].

The tensors \( H_{ijkl} \) and \( C_{ijkl} \) are linked by the relation [2]

\[
H_{ijkl} = C_{ijkl} + \frac{1}{c^2} \left( 2 A_{ik} D_{lj} + A_{ik} D_{jl} + A_{ij} D_{kl} \right).
\]

It is evident, therefore, that \( C_{ijkl} = H_{ijkl} \) if \( A_{ik} = 0 \) or \( D_{ik} = 0 \). Calculating the Christoffel symbols of the second kind, we obtain for the non-zero components:

\[
\Delta^1_{11} = \frac{\kappa \rho}{3} - \frac{\kappa \rho^2}{3},
\]

\[
\Delta^1_{22} = \frac{\Delta^1_{33}}{\sin^2 \theta} = -r \left( 1 - \frac{\kappa \rho^2}{3} \right),
\]

\[
\Delta^2_2 = \Delta^3_3 = \frac{1}{r}, \ \Delta^1_3 = -sin \theta cos \theta,
\]

\[
\Delta^3_2 = cot \theta.
\]

Substituting (19) into (16) and lowering the upper indices, we find the non-zero components \( C_{ijkl} \) for the space-time described by the metric (11)

\[
C_{1212} = \frac{C_{1133}}{\sin^2 \theta} = \frac{\kappa \rho^2}{3} - \frac{1}{1 - \frac{\kappa \rho^2}{3}},
\]

\[
C_{2323} = \frac{\kappa \rho^2}{3} \sin^2 \theta.
\]

The components \( C_{ik} = \Gamma_{ik}^{lm} C_{mlak} \) and the three-dimensional scalar \( C = R^a C_{ik} \) have the form [7]

\[
C_{11} = \frac{2 \kappa \rho}{3} - \frac{1}{1 - \frac{\kappa \rho^2}{3}}, \ \ C_{22} = \frac{C_{33}}{\sin^2 \theta} = \frac{2 \kappa \rho^2}{3},
\]

\[
C = 2 \kappa \rho > 0 .
\]

The three-dimensional reference space satisfies the condition

\[
C_{ijkl} = q (h_{ik} h_{jl} - h_{ik} h_{jl}), \ \ q = \frac{\kappa \rho}{3} = const, \quad (22)
\]

therefore it is the space of constant positive curvature, where \( q \) is the Gaussian curvature of the three-dimensional reference space. It follows from (12) that the radius of curvature is \( r = \sqrt{\frac{1}{3q}} \). It is necessary to note that the Gaussian curvature and, consequently, the radius of space breaking depend on the density of incompressible liquid.

Thus we have found that the three-dimensional reference space of the space-time (11) is the space of constant positive curvature. Study now the geometric properties of the four-dimensional space (11). As is well-known, the geometric properties of every curved (Riemannian) space are described by the Riemann tensor

\[
R_{a'b'y'y} = \frac{1}{2} \left( \partial_{a'b'} g_{y'y} + \partial_{a'y'} g_{b'y} - \partial_{b'y'} g_{a'y} - \partial_{y'y} g_{a'b'} \right) + \frac{q^2}{r^4} \left( \Gamma_{a'b'a'} \Gamma_{y'y'y} - \Gamma_{a'b'y} \Gamma_{y'y'} \right),
\]

where \( \Gamma_{a'b'a'} \) are the Christoffel symbols of the first kind

\[
\Gamma_{a'b'a'} = \frac{1}{2} \left( \partial_{a'b'} g_{a'b'} + \partial_{b'a'} g_{b'a'} - \partial_{a'a'} g_{b'b'} \right).
\]

Calculating the values \( \Gamma_{a'b'a'} \) for the metric (11) we obtain

\[
\Gamma_{01,0} = -\Gamma_{00,1} = \frac{3}{12} \sqrt{1 - \frac{\kappa \rho^2}{3}} \sqrt{1 - \frac{\kappa \rho^2}{3}},
\]

\[
\Gamma_{11,1} = -\frac{\kappa \rho}{3} \left( 1 - \frac{\kappa \rho^2}{3} \right),
\]

\[
\Gamma_{22,1} = -\Gamma_{12,2} = r,
\]

\[
\Gamma_{33,1} = -\Gamma_{13,3} = r \sin^2 \theta,
\]

\[
\Gamma_{33,2} = -\Gamma_{23,3} = r^2 \sin^2 \theta.
\]

Calculating the components of Riemann tensor (23) for the metric (11) we find

\[
R_{0101} = -\frac{3}{4r^2} \left[ 1 - \frac{a^2}{r^2} - \frac{r^2}{r_{br}^2} \right],
\]

\[
R_{0202} = -\frac{r^2}{4r^2} \left[ 3 - \frac{a^2}{r^2} - \frac{r^2}{r_{br}^2} \right],
\]

\[
R_{1212} = -\frac{r^2}{r_{br}^2} \left[ 1 - \frac{r^2}{r_{br}^2} \right], \ \ R_{2323} = -\frac{r^4}{r_{br}^2} \sin^2 \theta,
\]

\[
R_{0003} = R_{0202} \sin^2 \theta, \ \ R_{1313} = R_{1212} \sin^2 \theta,
\]

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where \( r_{br}^2 = \frac{1}{a} = \frac{1}{\sqrt{q}}. \)

The space-time is therefore not a constant-curvature space, because the components \( R_{00k} \) of the Riemann tensor do not satisfy the condition

\[
R_{ijkl} = K(g_{ij}g_{kl} - g_{ik}g_{jl}), \quad K = \text{const}, \tag{27}
\]

which is a necessary and sufficient condition that the space-time possesses constant curvature. Note that the spatial components \( R_{ijkl} \) satisfy (27), while the mixed components \( R_{ij0k} \) are zeroes. It means, due the structure of the components \( R_{00k} \), the space-time (11) does not possess constant curvature.

So forth, study the geometric properties of the space-time (11) in terms of Zelmanov’s theory of physically observable quantities. Zelmanov selected three groups of all independent curvature components \( R_{ijkl} \) — the projections on time, the projections on space, and the mixed projections [2]:

\[
X^{ik} = -c^2 g_{0k} R_{0ij} \quad Y^{ik} = c^2 g_{0k} R_{0ij} \quad Z^{ik} = c^2 R^{klj}. \tag{29}
\]

Here we have only interest in the components \( X^{ik} \). Calculating these components, we obtain

\[
X_{11} = \frac{c^2}{\rho_{br}^2} \left( \frac{1}{3} \sqrt{1 - \frac{a^2}{r_{br}^2}} - \frac{1 - \frac{a^2}{r_{br}^2}}{1 - \frac{a^2}{r_{br}^2}} \right) > 0, \tag{28}
\]

\[
X_{22} = \frac{X_{33}}{\sin^2 \theta} = \frac{c^2 r^2}{r_{br}^2} \left( \frac{1}{3} \sqrt{1 - \frac{a^2}{r_{br}^2}} - \frac{1 - \frac{a^2}{r_{br}^2}}{1 - \frac{a^2}{r_{br}^2}} \right) > 0. \tag{29}
\]

All components \( X^{ik} \to \infty \) in the state of collapse. Besides, if the breaking of space takes place, the \( X_{11} \to \infty \) and \( X_{22} = X_{33} \) are zeroes. Comparing (13) and (28), we find that the gravitational inertial force \( F_1 \) and the radial projection of the Riemann tensor on time \( X_{11} \) are linked by the relation

\[
F_1 = -r X_{11}. \tag{30}
\]

It means that the sign of the \( r \)-directed force is opposite to the sign of the temporal projection of the Riemannian tensor (the “curvature of the time”) in this direction: the negative non-Newtonian force of attraction is due to the positive curvature of time.

The partial case of the collapse of the incompressible liquid sphere \( r_c = r_{br} \) is studied in detail in [7]. As follows from (12), in this case the surface of the sphere is simultaneously both the surface of the collapsar and the surface of the breaking of the space. Remember that \( a = \frac{1}{\sqrt{q}} \) is also the radius of curvature of the sphere of condensed matter, where \( q \) is the Gaussian curvature of the reference space. Assuming \( a = r_{br} = \sqrt{\frac{3}{\lambda q}} \) and substituting this expression in (11), we obtain the de Sitter metric

\[
ds^2 = \frac{1}{4} \left( 1 - \frac{r_c^2}{a^2} \right)^2 dt^2 - \frac{dr^2}{1 - \frac{r_c^2}{a^2}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \tag{30}
\]

The space-time described by the metric (30) satisfies the Einstein equations

\[
R_{ij} - \frac{1}{2} g_{ij} R = \lambda g_{ij}. \tag{31}
\]

where the cosmological constant \( \lambda = \frac{\alpha^2}{\alpha} \).

The term \( \lambda g_{ij} \) can be expressed in the form [7]

\[
\lambda g_{ij} = x T_{ij}. \tag{32}
\]

Thus the \( \lambda \)-field generating the de Sitter space (30) is equivalent to the substance described by the energy-impulse tensor

\[
\mathbf{T}_{ij} = \frac{\lambda}{\alpha} g_{ij}. \tag{33}
\]

Calculating the physically observable components of the energy-impulse tensor (33) [2], we find

\[
\rho_0 = \frac{T_{00}}{g_{00}} = \frac{\lambda}{\alpha}, \quad J_0^i = \frac{\alpha T_0^i}{\sqrt{g_{00}}} = 0, \tag{34}
\]

\[
U_0^ik = c^2 T_{0ik} = -\frac{\alpha c^2}{\alpha}, \tag{35}
\]

where \( \rho_0, J_0^i \) and \( U_0^ik \) are the chr.-inv. density of matter, the (vector) density of impulse, and the tensor of stress, respectively.

As seen, the expression (10) transforms into (33) if the condition is

\[
p = -\rho_0 c^2 = -\frac{\alpha c^2}{\alpha}, \tag{36}
\]

i.e. it describes matter in the state of inflation. Thus the energy-impulse tensor (33) describes substance with positive constant density \( \rho_0 = \frac{\alpha}{2} \) and negative constant pressure \( p_0 = -\rho_0 c^2 \). The flow of energy is given as \( q_0 = J_0^c c^2 = 0 \). This substance is called physical vacuum. We conclude that the collapsing sphere of ideal incompressible liquid transforms into a de Sitter vacuum bubble by the special case of collapse, when the radius of the sphere \( a \) equals the breaking radius \( r_{br} \).

\[
a = r_{br} = \sqrt{\frac{3}{2q}} = r_c. \tag{36}
\]

where the radius of the collapsar \( r_c \) coincides with the radius of the sphere and the breaking radius.
The physical vacuum is an actual substance, possessing positive density and negative pressure. Because the bubble is stationary, the negative pressure, which inflates the bubble, must be balanced by attraction, thereby compressing it. To solve the problem of stability of inflation collapsar, it is necessary to find this compressing factor. Study the physical and geometrical characteristics of the de Sitter bubble and compare them with the corresponding characteristics of the liquid bubble. This comparison allows us to consider the process of transformation of the gravitational collapsar (“black hole”) into the inflational collapsar (“white hole”).

The physical and geometrical properties of the de Sitter bubble, described by the metric (30), are studied in detail in [7]. The local reference space does not rotate and deform. The gravitational inertial force has the form

\[ F_1 = \frac{c^2 r}{a^2 - r^2} > 0, \quad F^1 = \frac{c^2 r}{a^2} > 0, \]  

(37)
i.e. is the force of repulsion. As seen, the formula (13) transforms into (37) by the condition (36). Thus the gravitational inertial force of attraction (13), acting inside the liquid bubble, transforms into a force of repulsion, acting inside the vacuum bubble. Using the collapse condition (36), rewrite (37) in the form

\[ F^1 = \frac{xpoc^2}{3} = -\frac{xpr}{3} > 0. \]  

(38)

It is easy to see that both the positive density and the negative pressure both inflate the vacuum bubble. As known, the generally accepted viewpoint consists in that the stability of the de Sitter vacuum bubble is due to the action of two opposite factors: 1) compression due to the positive density; 2) inflation due to the negative pressure. As follows from (38), the positive density and negative pressure effects are identical, consequently it is necessary to find the factor, which causes the compression of the bubble.

Studying the physical and geometrical characteristics of the Schwarzschild liquid bubble, we have found that the force of attraction (13) is balanced by the value \(-rX_{11}\), which possesses the dimension of acceleration: see (29). The quantity \(X_{11} > 0\) is the observable projection of the Riemann tensor component \(R_{0101}\) on time — the “curvature of time in the radial direction”. Thus the non-Newtonian force of attraction, which is proportional to the radial distance \(r\), is balanced by the action of the “positive curvature of the time” (the term \(rX_{11}\)).

Consider the problem of the stability of the vacuum bubble. Calculating the Riemann tensor (23) for the metric (30), we find

\[ R_{0101} = \frac{1}{4a^2}, \quad R_{0202} = \frac{R_{0003}}{\sin^2 \theta} = -\frac{r^2(a^2 - r^2)}{4a^2}, \]

\[ R_{1212} = \frac{R_{1113}}{\sin^2 \theta} = -\frac{r^2}{a^2 - r^2}, \quad R_{2323} = -\frac{r^4 \sin^2 \theta}{a^2}. \]  

(39)

It is easy to see that the components (26) transform into (39) by the condition \(a = r_{bh}\). The components (39) satisfy the condition (27), where the four-dimensional constant curvature is negative: \(K = -\frac{1}{a^2}\).

The quantities \(C_{ijkl}, C_{ik}\) and \(C\) (20–21) of the reference space (30) then take the form

\[ C_{1212} = \frac{R_{1113}}{\sin^2 \theta} = \frac{r^2}{a^2 - r^2}, \quad C_{2323} = \frac{r^4 \sin^2 \theta}{a^2}, \]

\[ C_{11} = \frac{2}{a^2 - r^2}, \quad C_{22} = \frac{R_{0003}}{\sin^2 \theta} = \frac{2r^2}{a^2}, \]

\[ C = \frac{6}{a^2} > 0. \]  

(40)

The components \(C_{ijkl}\) (40) satisfy the condition (22), where the three-dimensional Gaussian curvature is \(q = \frac{1}{4a}\), consequently the reference space of the vacuum bubble is a three-dimensional sphere of the real radius \(a = \frac{1}{4q}\). We have shown above that the de Sitter space (30) possesses negative four-dimensional Gaussian curvature \(K = -\frac{1}{a^2} = -q\), consequently it is a four-dimensional sphere with the imaginary radius \(\Re = iq\).

Comparing the obtained results with the analogical ones for the liquid sphere (11), we find that both reference spaces possess positive constant curvature, but the four-dimensional de Sitter space possesses constant negative curvature. Calculating the physically observable components of the Riemann-Christoffel tensor \(\mathcal{X}_k\) (28) for the de Sitter vacuum bubble, we find

\[ X_{11} = -\frac{c^2}{a^2 - r^2} > 0, \quad X_{22} = \frac{R_{0003}}{\sin^2 \theta} = -\frac{c^2 r^2}{a^2} < 0. \]  

(41)

We conclude therefore that the sign of curvature of the de Sitter vacuum bubble coincides with the signs of the \(R_{ijkl}\) projections onto time (the “negative curvature of time”).

Comparing the component \(X_{11}\) (41) with the expression of the gravitational inertial force (37), we find that these quantities satisfy the condition (29), i.e. the signs of the \(F_1\) and \(X_{11}\) are opposite. We conclude that the non-Newtonian force of attraction inside the liquid sphere (11) is due to the positive curvature of time, the force of repulsion inside the vacuum bubble (30) is due to the negative curvature of time.

These results are connected with the geometric structure of the physically observable curvature components \(X_{ik}\). Generally speaking, they depend on the deformation, rotation, and gravitation of the reference space [2]. If locally the space does not deform and rotate, the components \(X_{ik}\) take the form

\[ X_{ik} = \frac{1}{2} \left( \nabla_i F_k + \nabla_k F_i \right) - \frac{1}{c^2} F_i F_k, \]  

(42)

where \(\nabla_i\) is the ch.-inv. operator of covariant differentiation [2].
We have thus shown that the collapsing liquid bubble (11) transforms instantly into the vacuum bubble (30) by the special case of collapse: \( a = r_\nu \). The surface \( r = a \) in this case is simultaneously: 1) the breaking surface; 2) the surface of the inflation collapser.

Calculating the elementary observable interval of time for the metrics (11) and (30), we find, respectively:

1) the Schwarzschild liquid bubble

\[
\begin{align*}
\text{d}t_1 &= \pm \frac{1}{2} \left( 3 \sqrt{1 - \frac{xpr^2}{3}} - \sqrt{1 - \frac{xpr^2}{3}} \right) \text{dt};
\end{align*}
\]

2) the de Sitter vacuum bubble

\[
\begin{align*}
\text{d}t_\nu &= \pm \frac{1}{2} \left( \sqrt{1 - \frac{r^2}{a^2}} \right) \text{dt}.
\end{align*}
\]

Assuming in (43) \( a = \sqrt{\frac{1}{\nu}} = r_\nu \), we obtain

\[
\begin{align*}
\text{d}t_\nu &= \pm \frac{1}{2} \left( \sqrt{1 - \frac{r^2}{a^2}} \right) \text{dt}.
\end{align*}
\]

We have obtained as a result that the observable time \( \tau \) inside these bubbles flows in the opposite direction. We consider usually the observable time as flowing in the positive direction — from the past to the future. In order to determine one of the two signs in the formulae (43–44), it is necessary to ask, which of the two bubbles is more applicable as the model of the observed Universe: the Schwarzschild liquid bubble or the de Sitter vacuum bubble? This question will be studied in detail in the next section.

4 The de Sitter bubble as a proposed cosmological model

Consider the Schwarzschild and de Sitter bubbles as the two possible cosmological models. The choice of such a model must be in accordance with astronomical data. The most important criterion for the choice is the observed red-shift. In other words, the model, which allows the red-shift, can be chosen as the cosmological model. The effect of the spectral line displacement is calculated exactly for every gravitational field configuration.

As known, the world-lines of light-like particles (null geodesic lines) are described by the equations of the parallel transfer of the isotropic (null) four-dimensional wave vector \( K^a \)

\[
\begin{align*}
\frac{dK^a}{d\sigma} + \Gamma^a_{\mu\nu} K^\mu \frac{dx^\mu}{d\sigma} = 0, \\
K_a K^a = 0,
\end{align*}
\]

where \( \omega \) is the cyclic frequency, \( \Gamma^a_{\mu\nu} \) is the Christoffel symbols of the second kind, \( \sigma \) is the parameter of differentiation, \( \frac{dx}{d\sigma} \) is the isotropic (null) vector of the 4-velocity, which is tangent to the world-lines (\( g_{\alpha\beta} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} = 0 \)).

These equations have the form in terms of the physically observable quantities (viz. the theory of chronometric invariants) [9]

\[
\begin{align*}
1 \frac{d\omega}{\omega} - \frac{1}{c^2} D_{ik} \frac{dx^i}{d\tau} \frac{dx^k}{d\tau} - \frac{1}{c^2} F_{ik} \frac{dx^i}{d\tau} = 0, \\
\frac{d}{d\tau} \left( \omega \frac{dx^i}{d\tau} \right) + 2 \mu (D^i_{\nu} + A^i_{\nu}) \frac{dx^\nu}{d\tau} - \omega F^i + \omega A^i_{\nu} \frac{dx^\nu}{d\tau} = 0,
\end{align*}
\]

\[
\begin{align*}
h_{ik} \frac{dx^i}{d\tau} \frac{dx^k}{d\tau} = c^2.
\end{align*}
\]

The system of equations (47–49) is the chr.-inv. form of the parallel transfer equations of the four-dimensional wave vector \( K^a = \frac{dx}{d\tau} \), where the equations (47–48) are linked by the relation (49). Solving the system for every metric, we find the frequency of the photon and the associated spatial trajectory in the given space-time.

If the reference space does not rotate and deform, the equations (47–48) take the form

\[
\begin{align*}
1 \frac{d\omega}{\omega} - \frac{1}{c^2} F_{ik} \frac{dx^i}{d\tau} = 0, \\
1 \frac{d}{d\tau} \left( \omega \frac{dx^i}{d\tau} \right) + F^i + \Delta^i_{\nu} \frac{dx^\nu}{d\tau} = 0,
\end{align*}
\]

Substituting into (50) the expressions for gravitational inertial force \( F_i \) (13) and (40), we obtain the equations describing the behaviour of the cyclic frequency inside both the condensed matter and physical vacuum bubbles, respectively:

1) the Schwarzschild bubble

\[
\begin{align*}
\frac{d\omega}{\omega} - \frac{\kappa pc^2}{3} \left( 3 \sqrt{1 - \frac{\kappa pr^2}{3}} - \sqrt{1 - \frac{\kappa pr^2}{3}} \right) \frac{dr}{\tau};
\end{align*}
\]

2) the de Sitter bubble

\[
\begin{align*}
\frac{d\omega}{\omega} = \frac{r}{a^2 - r^2} \frac{dr}{\tau}.
\end{align*}
\]

Integrating (52–53), we obtain, respectively:

1) the Schwarzschild bubble

\[
\begin{align*}
\omega = \frac{P}{3 \sqrt{1 - \frac{\kappa pr^2}{3}}} , P = \text{const};
\end{align*}
\]

2) the de Sitter bubble

\[
\begin{align*}
\omega = \frac{Q}{\sqrt{1 - \frac{r^2}{a^2}}} , Q = \text{const},
\end{align*}
\]
where \( P \) and \( Q \) are integration constants.

Cosmologists have introduced the quantity \( z \) — the relative variation of the frequency
\[
z = \frac{\omega_{em} - \omega_{obs}}{\omega_{obs}},
\]
where \( \omega_{em} \) is the frequency, emitted by the source, located at the radial distance \( r_{em} \) relative to the observer, \( \omega_{obs} \) is the observable (observed, registered) frequency of this source at the place, where the observer is located: \( r_{obs} \). The condition \( z < 0 \) means that the observable frequency is more than the emitted, consequently the observable light seems shifted more towards the blue than the emitted one (the phenomenon of blue-shift). The condition \( z > 0 \) implies a red-shift, because in this case the observable frequency is less than the emitted one.

Calculating the value \( z \) for the expressions (54–55), we obtain

1) the Schwarzschild bubble
\[
z = \sqrt{1 - \frac{\kappa \rho_{em}^2}{3}} - \sqrt{1 - \frac{\kappa \rho_{obs}^2}{3}} < 0; \quad (57)
\]
2) the de Sitter bubble
\[
z = \sqrt{a^2 - r_{obs}^2} - \sqrt{a^2 - r_{em}^2} \quad \frac{\sqrt{a^2 - r_{em}^2}}{\sqrt{a^2 - r_{obs}^2}} > 0. \quad (58)
\]

It follows from (58) that the red-shift takes place inside the de Sitter bubble, therefore namely this space-time can be considered as a cosmological model.

Let us study more exactly the behavior of the frequency of photons emitted by distant sources. Assume that the photons from the source move to the observer in the radial \( r \)-direction. Then (49) takes the form
\[
\frac{a^2}{a^2 - r^2} \left( \frac{dr}{dt} \right)^2 = c^2. \quad (59)
\]

Taking the root of (59), we obtain
\[
\frac{dr}{\sqrt{a^2 - r^2}} = \frac{c}{a} \, dt = \pm H \, dt, \quad (60)
\]
where \( H \) is the Hubble constant. Assuming \( H = 75 \) Mps/sec \( = 2.3 \times 10^{-18} \) sec\(^{-1} \), we find \( a = 1.3 \times 10^{28} \) cm.

Choose the sign + or −, respectively, if the distance between the observer and the source is taken into account: 1) from the observer to the source; 2) from the source to the observer. Integrating (60) from \( r \) (the distance from the source) until \( r = 0 \) (the location of the observer), we find
\[
\int_0^r \frac{dr}{\sqrt{a^2 - r^2}} = - \arcsin \frac{r}{a} = - H \tau, \quad (61)
\]
where \( \tau \) is the observable time, in the course that the signal from the source comes to the observer. It follows from (61) the expression for \( r \):
\[
r = a \sin H \tau, \quad (62)
\]
i.e. the photometric distance is harmonic (sinusoidal) oscillation with the amplitude \( a \) and the period \( T = \frac{\pi}{H} \). The amplitude \( a \) is the maximal distance from any observer — the so called “event horizon”. It is easy to find that the three-dimensional observable vector of the light velocity \( c^1 = \frac{dr}{d\tau} \) has the form
\[
c^1 = \frac{dr}{d\tau} = aH \cos H \tau = c \cos H \tau, \quad (63)
\]
where
\[
h_{11}c^1 = \frac{a^2}{a^2 - r^2} \left( \frac{dr}{dt} \right)^2 = c^2.
\]

This formula means that the radial component of the vector of the light velocity oscillates with a frequency \( H \) and an amplitude \( c \). This oscillation is shifted for \( \frac{\pi}{2} \) with respect to the oscillation of the radial distance \( r \) (62).

Substituting (63) into (55), we obtain
\[
\omega = \frac{Q}{\cos H \tau}, \quad 0 \leq \tau \leq \frac{\pi}{2H}. \quad (64)
\]
As seen, \( \omega \to \infty \) if \( \tau \to \frac{\pi}{2H} \), i.e. by \( r \to a \). It follows from (58) that the value of \( z \) increases infinitely by \( r \to a \). This effect takes place from the viewpoint of the real observer, because the observable time depends on the photometric distance \( r \) from the event horizon:
\[
d\tau = \frac{1}{2} \sqrt{1 - \frac{r^2}{a^2}} \, dt. \quad (65)
\]

Thus the tempo of the observable time decreases by \( r \to a \), and the observable time is stopped at the event horizon. Therefore the observable cyclic frequency of photons increases infinitely by \( r \to a \).

It was shown above, the coordinate (photometric) distance \( r \) is the sinusoidal (harmonic) oscillation (wave) with the amplitude \( a \) and the cyclic frequency \( H = \frac{c}{r} \). The quantity \( T = \frac{\pi}{2H} \) is the full period of the oscillation, the maximal value \( a \) (amplitude) is the event horizon. Taking into account only the positive values of \( r \), we are restricted only to the semi-period of the oscillation. The maximal value of \( r = a \) takes place at \( \tau = \frac{\pi}{4H} = \frac{\tau}{2} \). Introducing the used-in-contemporary cosmology value \( H = 2.3 \times 10^{-18} \) sec\(^{-1} \), we find \( T_o = \frac{\pi}{H} = 21.6 \times 10^9 \) years — the time of passing of the light signal from the event horizon to the observer. Contemporary cosmologists calculate the time of the life of the Universe as the interval of time after the Big Bang. They obtained the age of the Universe approximately \( 13.75 \times 10^9 \)
years. If we’ll introduce $H$ as the ordinary (not the cyclic) frequency $H = \frac{H}{2\pi} = \frac{1}{T}$, we find $T = 13.74 \times 10^9$ years.

As is well known, the mathematical basis of contemporary relativistic cosmology is the theory of a non-stationary (extending) universe. It is founded on Friedman’s cosmological models, which belong to a particular class of solutions to Einstein’s field equation, obtained by the imposing condition that the space of the observable Universe is homogeneous and isotropic. This class of solutions is described by the metric

$$ds^2 = c^2dt^2 - R(t)^2 \left[ \frac{dx^2 + dy^2 + dz^2}{1 + \frac{k}{2}(x^2 + y^2 + z^2)} \right], \quad k = 0, \pm 1, \quad (66)$$

where $R(t)$ is the scale factor: $\frac{1}{R} \frac{dR}{dt} = H$. In accordance with the value $k$ of the three-dimensional space: 1) is flat ($k = 0$); 2) has negative curvature ($k = -1$); 3) has positive curvature ($k = +1$). Models with $k = 0$, $-1$ are called open, and models with $k = +1$ are closed ones. Friedman’s spaces are both empty ($T_{ab} = 0$) and filled by ideal liquid described by (10).

The special reference space (68) does not rotate and gravitate, but it does deform. The tensor of the rate of deformation is described by the formula $D_{ik} = R \frac{\partial \Omega}{\partial x^i}$, which describes more near-to-the observer regions ($r \ll a$).

Cosmologists explain the observable red-shift by the Doppler effect which is due to the expansion of the space of the Universe. The generally accepted model of the non-stationary (extending) Universe is the Standard Cosmological Model. The age of the Universe is determined by means of extrapolation of the uniformly flowing time from the present to the past — the beginning of the Universe caused by the Big Bang. The age of the observable Universe, according to Friedman’s theory, is determined approximately as $13 \times 10^9$ years — the interval of the time from the Big Bang of the initial singularity (the “point” consisting of super-compact initial substance).

Now we come to the essential question: What cosmological model is more applicable for the description of the observable Universe: the stationary de Sitter space or the extending Friedman’s space? The criterium of the choice must be the results of astronomical observations. It follows from the observations of spectra of galaxies that the observable red-shift is linear for more near galaxies and it rapidly increases for the most distant objects. Most cosmologists explain this result as the accelerated expansion of space, while routinely avoiding some principal weaknesses. The correct theoretical explanation of this fact has not been obtained until now. Moreover, contemporary cosmologists do not calculate variations of frequencies as exact solutions to the general relativistic equation of motion of null geodesic lines. The observable phenomena of the red-shift is explained by the temporal variations of the scale factor $R(t)$. It is necessary to note that the exact solution(s) to the equations (47–49) can be found only for concrete metrics. In particular, the expression of the cyclic frequency $\omega$ for Friedman’s metric can be obtained only if the exact expression for $R(t)$ is known and the value of $k$ is chosen. In other words, in order to study variations of frequencies of cosmic objects, it is necessary before hand to assign the function $R(t)$, which determines the kind of deformation, and the value of $k$, which determines the geometry of the reference space.

The exact value of the frequency (55) is obtained here as the solution to equation of motion of null geodesic lines (47–49). It follows from (55, 59) that the observable frequency $\omega$ and the quantity $\xi$ increase infinitely while approaching the event horizon. If $r \ll a$, the quantity $\xi$ can be transformed as

$$z \approx \frac{r_{\text{em}}^2 - r_{\text{obs}}^2}{2a^2}. \quad (67)$$

It means that the red-shift in the spectra of near-to-the observer objects ($r \ll a$) is subject to the parabolic law. In other words, the linear red-shift cannot be explained in the de Sitter space. The gravitational inertial force of repulsion inside the de Sitter bubble causes the parabolic red-shift for near sources and the infinite increase at the maximal distance from the observer — the event horizon. Thus the red-shift in the de Sitter bubble is due to the non-Newtonian force of repulsion, which is proportional to the radial (photometric) distance $r$.

We conclude: neither the Friedman expansion, which is caused by the deformation of the reference space, nor the de Sitter force of repulsion can explain simultaneously both the linear red-shift for near sources and the sharp, non-linear increase for most distant sources. Probably, this problem can be solved in frames of a generalized metric which includes both Friedman’s expansion and the de Sitter repulsion. It is possible that the de Sitter space is applicable near the event horizon ($r \sim a$), while the Friedman extending space correctly describes more near-to-the observer regions ($r \ll a$).

5 The past, the present, and the future are three multi-space aspects of the observable time

Now, let us consider in detail the collapse mechanism of the liquid bubble into the vacuum bubble. We have obtained above the key rôle in the very process the condition (36) plays. If such a state is realized, then the interval of the observable time interior to Schwarzschild’s liquid bubble $d\tau$ transforms into the interval of the observable time inside de Sitter’s vacuum bubble $d\tau$; moreover, each of these intervals possesses the opposite sign:

$$d\tau = -d\tau.$$

It means that the observable time inside the vacuum de Sitter bubble flows in the opposite direction. We have assumed in the previous section that once the de Sitter bubble is
applicable as a cosmological model, the flow of the $\tau$ in this space is positive: the observable time flows from the past to the future. Then the observable time inside the Schwarzschild liquid bubble flows from the future to the past, and its interval has the form:

$$dt_1 = -\frac{1}{2} \left( 3 \sqrt{1 - \frac{a^2}{r_0^2}} - \sqrt{1 - \frac{r^2}{r_0^2}} \right) dt < 0. \quad (68)$$

If $a = r_0$, then (70) transforms into the expression

$$d\tau_2 = \frac{1}{2} \left( \sqrt{1 - \frac{r^2}{a^2}} \right) dt > 0, \quad (69)$$

which is the interval of the observable time inside the de Sitter bubble.

Thus the surface $a = \sqrt{\frac{2}{\rho_0}} = r_0$ is the mirror dividing two worlds — the space of the future and the space of the past. It means, this surface is the space of the present. As was shown above, the surface $a$ is singular. It means, the present is the instantaneous state between the future and the past, where the future transforms into the past by means of passing through the singularity state. The space of the future is here the vacuum de Sitter liquid bubble, where the observable time flows from the future to the present: that is, the future “goes to us”. The future, after the passage through the said singularity surface, becomes the past: the present “leaves us”. Thus the singular surface is not only a mirror (a reflecting surface). It is simultaneously a membrane: a telemetric, multispace membrane connecting the worlds of the past and the future. The future penetrates into the inflation collapse namely through this “mirror-like membrane” — the interior layer between the past and the future. This situation can be illustrated in terms of the well-known description of the interaction between a light beam and some incident surface (as the light beam falls upon the surface). This beam splits into three beams: 1) the reflected; 2) the refracted; 3) the absorbed. The light beam within the framework of General Relativity is the trajectory of photons — the world-line of the null four-dimensional length $ds = 0$, where here every individual photon is said to be the event itself. The world-lines with $ds \neq 0$ also consist of four-dimensional world-points. It is possible to say therefore that the light beam of events, falling onto the singular surface, splits into: 1) the reflected light beam (returned into the space of the future); 2) the refracted light beam (directed into the space of the past); 3) the absorbed light beam, by the said singularity surface. The first light beam describes those events, which cannot be realized (materialized) in the present (for example, using analogy with daily life, certain ideas or epochs which are far too advanced for the time). The second light beam describes those events, which could be realized in principle, but they can not actually be realized (in part, these are not readily perceived by the bulk human consciousness). Finally, those events in the likeness of the absorbed light-beam represent the world of the present, which is uniquely perceived by our consciousness (taking into account varying internal degrees of consciousness) as “reality”. The said non-realized (for a while) events can be called virtual events.

An event in General Relativity is the four-dimensional point of the space-time $V_4$ — the three-dimensional point, which is expanded into a “thread”. This thread is the four-dimensional trajectory of the event — the world-line. These lines can be: 1) non-null (trajectories of mass-bearing particles, both real and imaginary); 2) null (trajectories of light-like particles; in particular, photons). Interlacing of these threads creates the “material of the space”. Because we assume here fundamental interactions between the past, the present, and the future, we must introduce a “medium”, which realizes these interactions. We will consider in this paper only null world-lines, i.e. we will study events of the “life of photons”.

It is evident that those particles, which realize the transfer of energies between the future into the past, must penetrate the singularity surface. As known, regular photons cannot pass through the singularity surface, but this surface is penetrable for zero-particles, introduced in [3]. These particles exist in the generalized space-time $V_4$, which is determined as an immediate generalization of the Riemannian space-time $V_4$ of General Relativity (both at the differential-geometric manifold and sub-manifold levels): $\tilde{V}_4 = V_4 \cup Z$, where $Z$ is the zero-space. Zero-particles have zero rest-mass $m_0$, zero relativistic mass $m$, and non-zero gravitational-rotational mass $M$, which is described in the $\tilde{V}_4$ as

$$M = \frac{m}{1 - \frac{w + pl}{c^2}}, \quad u^i = \frac{dx^i}{dt}. \quad (70)$$

The four-dimensional metric of $\tilde{V}_4$ satisfies the condition $g = \det|g_{ij}| < 0$, i.e. it allows the versatile degeneration of the metric. The manifold $\tilde{V}_4$ is the ordinary space-time $V_4$ by $g < 0$ and in the zero-space $Z$ by $g = 0$. Zero-particles transfer instantaneously ($d\tau = 0$), from the viewpoint of a real observer, along three-dimensional lines of null observable length ($d\sigma = 0$), i.e. they are mediums for the long-range-action. Zero-particles can be considered as the more tenuous and thinner structures than the photon. The condition (5) takes for zero-particles the form $d\sigma = d\tau = 0$.

The four-dimensional null wave vector $K^\alpha$ of the $\tilde{V}_4$ can be expressed both in the corpuscular form and in the wave form

$$K^\alpha = \frac{\omega}{c} \frac{dx^\alpha}{ds}, \quad K_\sigma = \frac{\partial \psi}{\partial x^\sigma}, \quad (71)$$

where $\psi$ is the phase of the wave (the eikonal).

The physically observable characteristics of $K^\alpha$ are [3]

$$K_0 = \frac{\omega}{\sqrt{g_{00}}} = \pm \omega \frac{\partial \psi}{\partial t}, \quad K^i = \frac{\omega}{c^2} \frac{dx^i}{dt} = -\hbar \frac{\partial \psi}{\partial x^i}, \quad (72)$$

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are the chr.-inv. operators of differentiation along the temporal and spatial coordinates, respectively [2]. The signs (+) and (−) are related to the spaces possessing the direct and reverse flow of time, respectively.

The wave form of the condition $K_\alpha K^\alpha = 0$ is the well-known eikonal equation

$$g^{\alpha\beta} \frac{\partial \psi}{\partial x^\alpha} \frac{\partial \psi}{\partial x^\beta} = 0. \quad (73)$$

Expressing (73) in terms of physically observable values, we obtain

$$\frac{1}{c^2} \left( \frac{\partial \psi}{\partial t} \right)^2 - \frac{\partial ^2 \psi}{\partial x^i \partial x^i} = 0. \quad (74)$$

The cyclic frequency of zero-particles is $\omega = 0$, consequently the equation (74) takes the form of the standing wave [3]

$$\kappa ^2 \frac{\partial \psi}{\partial x^i} \frac{\partial \psi}{\partial x^i} = 0, \quad (75)$$

which can certainly be interpreted as a hologram, i.e., a standing wave of the extended space-time. Thus the present, in the sense of geometric optics, is a holographic picture perceived by our consciousness as the material (real) world.

We conclude therefore that zero-particles are the mediums of the long-range-action in the space of the present — the boundary between the spaces of the future and the past. Zero-particles can be considered as a result of the fundamental interaction between the photons themselves, moving in time in the two above-mentioned opposite directions and possessing certain cyclic frequencies of the opposite signs.

In other words, the standing wave can be interpreted as a result of the summarization of the two waves $\varphi_+ \psi_-$, directed from the past to the future and from the future to the past, respectively. Let photons, moving in the space of the past, possess positive frequencies $\omega_+$, and photons moving in the space of the future, possess negative frequencies $\omega_-$, respectively. The interaction between the $\varphi$-waves, oppositely oriented in time, generates information, which is transmitted instantaneously by means of zero-particles. This information creates a hologram (the unique “reality” of the present moment), which exists during the infinitely small interval of time as well as after it is substituted by the next hologram.

By analogy, the perception of the continuity (and solidity) of the present is due to the fact that the successive frames of a movie are substituted very quickly.

We do not consider here the whole unique process of the chain of sequential materializations: zero-particles $\rightarrow$ photons $\rightarrow$ mass-bearing particles, because this problem is very difficult and impractical to be considered in further detail. We introduce here instead the problem of observation of cosmic objects. Consider the information which comes to us from stars and galaxies in the form of light beams. Because the cosmic objects are distant from us, we register the photons later than they were first emitted. It means, the observer, registering the electromagnetic radiation of the source, studies the past state of this cosmic object. This state corresponds to the moment of radiation of the electromagnetic signal. The information about the present state of the object can be obtained by means of registration of zero-particles, emitted by the source at the moment of observation. But the observer does not perceive it, because he does not use corresponding intermediary instruments. Contemporary astronomers use instruments, which can register only different ranges of electromagnetic radiation transferring at the light velocity.

6 Newtonian and non-Newtonian forces in the Universe

We have studied until now only non-Newtonian forces:

1) the force of attraction (13), created by the homogeneous liquid sphere (11);
2) the force of repulsion (37), created by the vacuum bubble (30);
3) the values of these forces are proportional to the radial coordinate $r$;
4) both forces are connected to the observable components of the Riemann tensor by the correlation (29).

The metrics (11) and (30) describe the gravitational fields created by the continuous bodies (bubbles). It is necessary to note that the force of attraction (13) transforms into the force of repulsion (37) as a result of the collapse of the liquid bubble, and both forces are non-Newtonian. The force of attraction (13) is created by the liquid sphere, which was initially introduced by Schwarzschild for the description of the Sun. On the other hand, the Sun as an attracting body is described by the well-known Schwarzschild metric of a single mass (mass-point) in emptiness ($R_{\alpha\beta} = 0$) [8]. This metric has the form

$$ds^2 = \left( 1 - \frac{r_g}{r} \right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 (d\varphi^2 + \sin^2 \theta d\varphi^2), \quad (76)$$

where $r_g$ is the gravitational (Hilbert) radius and $M$ is the mass of the gravitating mass-point.

The space-time (76) collapses by the condition $r = r_g$, and the surface $r = r_g$ is called the Schwarzschild surface. Besides this, the space experiences breakage by the same condition. Thus the mass-point stops the time and breaks the space by $r = r_g = r_{fr}$.

The metric (76) is applied for the description of the gravitational field of the Sun and the motion of the planets of the Solar System. It allows the post-Newtonian approximation, consequently it must include Newtonian gravitation. Let us study in detail the physical and geometrical characteristics
of the gravitational field of the mass-point in order to compare the obtained results with the analogous results for the metric (11), which describes the continuous body — a liquid sphere. This approach allows us to determine the problem of the connection between the local geometry of space-time and the character of attractive forces therein.

We have obtained for the metric (11) that the radial non-Newtonian force of attraction (13) is linked to the radial projection of the “curvature of time” (28) by the correlation (29). As follows from (29), the force of attraction is due to the positive curvature of time. Let us study the connection between the observable components of the Riemann tensor and the gravitational inertial force for the space-time (76).

The reference space described by (76) does not rotate and deform, but it gravitates. Calculating the gravitational inertial force $F_1$ by the formula $F_1 = \frac{r^2}{\alpha^2} \frac{\partial}{\partial \alpha}$, we obtain

$$F_1 = -\frac{c^2r_g}{2\alpha^2} \frac{1}{1 - \frac{r_g}{r}}, \quad F_1^1 = -\frac{c^2r_g}{2\alpha^2}. \quad (77)$$

Substituting into the expression for $F_1$ the value $r_g = \frac{2GM}{c^2}$, we rewrite (77) in the form

$$F_1 = -\frac{GM}{r^2} \frac{1}{1 - \frac{2GM}{c^2}}, \quad F_1^1 = -\frac{GM}{r^2}. \quad (78)$$

We see that the component $F_1^1$ is the ordinary Newtonian force of attraction. Calculating the observable components of the Riemann tensor $X_{11}$ by the formula (42), we find

$$X_{11} = -\frac{c^2r_g}{r^2} \frac{1}{1 - \frac{r_g}{r}} < 0. \quad (79)$$

It follows from (78–79) the relation between the force of attraction and the “curvature of time” in the radial direction:

$$F_1 = \frac{r}{2} X_{11}. \quad (80)$$

The signs of $F_1$ and $X_{11}$ coincide in contrast to the analogous relation (29), which is satisfied for both the de Sitter and Schwarzschild bubbles. It means that the Newtonian force of attraction is due to the “negative curvature of time”. The point is that the Non-Newtonian and Newtonian gravitational forces of attraction are originated by different sources. As shown earlier, the non-Newtonian force of attraction is connected to the continuous body (the liquid sphere). The Newtonian force is connected usually to the mass, which is concentrated inside a small volume, so called a “mass-point” [8]. Meanwhile, it is evident that continuous bodies possess the said Newtonian force, because they attract bodies with smaller masses. Therefore, it is necessary to state correctly the criterium, which will determine what kind of cosmic bodies must be described as “continuous bodies” and what kind — as “mass-points”.

The gravitational field of the mass-point is described by the Schwarzschild metric (76), which includes Newtonian gravitation (as well as the post-Newtonian approximation). The motion of cosmic bodies, which move around the attracting center (mass-point), is usually studied in either the framework of Newtonian gravitation or that of the post-Newtonian theory of gravitation. In the second case, the motion of cosmic objects is calculated in the Schwarzschild mass-point field by the condition $r_g \ll r$. This condition means that the Hilbert radius is very small in comparison to the distance between the attracting center and the object moving around the center. This approach is applicable both to the Sun and to the planets, asteroids, etc. On the other hand, continuous bodies also possess gravitational attraction. In part, the gravitational inertial force of attraction in the reference space of the homogeneous liquid sphere is described by (13). The question now arises: what are the conditions, by which the Newtonian force of attraction is the partial case of the non-Newtonian force (13)?

It follows from (77–78) that the gravitational inertial force coincides with the Newtonian force of attraction if $r_g \ll r$. Because the Newtonian theory of gravitation is constructed in the flat three-dimensional (Euclidean) space, we can assume that the homogeneous gravitating mass $M$ has the form

$$M = \rho V, \quad V = \frac{4\pi a^3 \rho}{3}, \quad (81)$$

where $V$ is the volume of the mass, $a$ is its radius, $\rho = \text{const}$ is the density of mass. This assumption is admissible also for any homogeneous sphere. Using (81), we can rewrite the expression (13) in the form

$$F_1 = -\frac{c^2r_g}{a^3} \left(\frac{r}{3 - \frac{r_g}{a} - \sqrt{1 - \frac{r_g}{a} \frac{r_g}{a^2} \frac{a^2}{a^2}} \sqrt{1 - \frac{r_g}{a} \frac{a^2}{a^2}} - \frac{r_g}{a^2}}\right). \quad (82)$$

Let $r_g \ll r \leq a$. Expressing the value $\sqrt{1 - \frac{r_g}{a}}$ into series, neglecting the members of the second kind and assuming $\sqrt{1 - \frac{r_g}{a}} \approx 1 - \frac{r_g}{a}$, we obtain, after transformations, the expression for the $F_1$ in the form

$$F_1 \approx -\frac{c^2r_g r}{2a^3} = -\frac{GMr}{a^2}. \quad (83)$$

If $r = a$, then (83) transforms into the expression for the Newtonian force of attraction, created by the sphere of radius $a$

$$F_1 = -\frac{GM}{a^2}. \quad (84)$$

The expression (84) coincides completely with (78) by $r_g \ll r = a$. Thus the Newtonian gravitational force is the
partial case of the non-Newtonian force of gravitation (82) by the condition \( r_\varrho \ll r = a \). But this fact does not mean that we must use the Newtonian theory of gravitation for the description of the gravitational field of the single body, whose Hilbert radius is small in comparison with its radius. The point is that the application of the relativistic mass-point metric (76) allows us to calculate the well-known effects (e.g. the perihelion motion of Mercury, the gravitational shift of light beams, the gravitational shift of spectral lines). It is possible that many other effects, unknown until now, will be explained by means of this metric.

We have studied until now only the case \( r_\varrho \ll r = a \). This condition corresponds to a single body, whose Hilbert radius \( r_\varrho \) is negligible in comparison with its geometrical radius \( a \). Consider now the case \( r_\varrho \ll r \), where the radial coordinate \( r \) can possess any values. Then the value \( \frac{2Gm}{c^2 a} = \frac{M}{a} \) is not infinitely small for \( r \gg a \). It follows from (11) that the condition \( \frac{2Gm}{c^2 a} = 1 \) is the condition of space breaking, consequently the quantity \( r_{br} = \sqrt{\frac{2Gm}{c^2 a}} \) is the breaking radius. Using the expressions for the \( r_\varrho \) and \( r_{br} \), we can rewrite (13) in the form:

\[ F_1 = -\frac{2GM}{c^2 a^3} \left( \frac{r}{\sqrt{1 - \frac{2GM}{c^2 a r}}, \sqrt{1 - \frac{r^2}{r_{br}^2}}, \sqrt{1 - \frac{r^2}{r_{br}^2}}} \right) \tag{85} \]

The formula (85) describes the gravitational inertial force of the liquid sphere, whose Hilbert radius is small in comparison with the radius of the sphere \( \left( \frac{a}{a} \ll 1 \right) \) and the sphere of space breaking \( r = r_{br} \) is outside the liquid sphere \( (r_{br} > a) \). It follows from (85) that the force \( F_1 \to \infty \) by \( r \to r_{br} \). It is evident that the force (85) is the non-Newtonian force of attraction, manifesting a curvature discontinuity in the environment.

The condition of space breaking was initially studied in [6]. The Sun was introduced as a liquid homogeneous sphere. It was shown that the Sun would break the surrounding space, with the breaking radius \( r_{br} = 3.43 \times 10^{13} \text{ cm} = 2.3 \text{ AU} \) (1 AU = 1.49 \times 10^{13} \text{ cm}), where 1 AU is the distance between the Sun and the Earth. Thus the breaking (curvature discontinuity) of the Sun’s space is located inside the asteroid strip, i.e. outside the gravitating body (the Sun). The Hilbert radius of the Sun is \( r_\varrho = 2.9 \times 10^3 \text{ cm} \), the proper radius being \( a = 6.95 \times 10^{10} \text{ cm} \). It is easy to calculate \( \frac{a}{r_\varrho} = 4.2 \times 10^{-6} \ll 1 \), and \( \frac{2Gm}{c^2 a} = 4.9 \times 10^2 \). It is possible that this non-Newtonian force creates the additional effect on the motion of the bodies in the Solar System. In partial, those bodies, which recede from the Sun in the radial direction, must possess additional negative (directed to the Sun) acceleration.

Analogous calculations were realized for all the planets of the Solar System [6]. It is important to note that the breaking spheres of the Earth, Mars, and Jupiter intersect with the asteroid strip near the hypothetical planet Phaeton, according to the Titus-Bode law at \( r = 2.8 \text{ AU} \). It is possible that the breaking of the Solar System space by the Sun and the mentioned planets plays an important role in the very formation of the Solar System itself. It means that not only the Sun, but also other planets of the Solar System exert an effect on the motion of different objects, including artificial satellites, moving in the orthogonal direction with respect to the orbits of planets. The additional non-Newtonian force of attraction is proportional to the radial distance \( r \), and the Newtonian force decreases as \( \frac{1}{r^2} \). It means that the more distant the body moves away from the center of attraction, the more appreciable the effect of the non-Newtonian part of the force is. It is possible that the Pioneer anomaly can be explained by the existence of non-Newtonian forces: this effect is registered near the boundary of the Solar System, because Newtonian attraction here decreases (with radial distance), and non-Newtonian attraction increases.

Thus the gravitational field of a single mass, whose Hilbert radius is considerably smaller than its radius, can be described by the Schwarzschild mass-point metric (76) by way of performing calculations of the orbital motions of the test bodies. The analogical field must be described by the metric of a continuous body (such as the simplest metric of the homogeneous liquid sphere), i.e. if we consider the radial motion of the moving test body.

Consider now a cosmic body whose Hilbert radius is comparable with its proper radius: \( r_\varrho \sim a \). A model of the observable Universe whose whole radius matches the Hilbert radius was first suggested by Stanyukovich [10]. He studied some geometric properties of the liquid body in the state of gravitational collapse, but he did not introduce the concrete metric. Stanyukovich assumed that the space of the Universe was a collapsar, whose Hilbert radius \( r_\varrho \) was equal to the distance up to horizon of events \( a \). According to this concept, the mass of the Universe could be calculated by the formula \( M = \frac{3\pi}{2G} \). Assuming \( a = 1.3 \times 10^{20} \text{ cm} \) (the maximal observed distance), we should find \( M = 8.78 \times 10^{35} \text{ g} \). This value coincides approximately with estimates obtained by way of other sorts of reasoning.

The average value of the density of the liquid substance is \( \rho = \frac{M}{V} \). Calculating the value of the density of the mass-point collapsar \( M = \frac{ac^2}{2G} \) by the assumption \( V = \frac{4\pi ac^3}{3} \), we obtain

\[ \rho = \frac{3c^2}{8\pi Ga^2} = \frac{3H^2}{8\pi G} = 9.5 \times 10^{-30} \frac{g}{\text{cm}^3}, \quad H = \frac{c}{a}. \tag{86} \]

This value corresponds to the range of values obtained from observational data. Moreover, it corresponds to the theoretical value of the critical density \( \rho_c \), by the condition \( H = 2.3 \times 10^{-18} \text{ sec}^{-1} \).

It is necessary to note that the critical density is determined in standard cosmology as the density of the Friedman model (66), whose three-dimensional space is flat: \( k = 0 \). It is evident that this space-time is not a collapsar, because the observable time \( \tau \) coincides with the coordinate time \( t: dt = d\tau \).
consequently \( g_{00} = 1 \). (Recall that the collapse condition is \( \sqrt{g_{00}} = 0 \).) Calculating the volume of the gravitational collapsar by the formula \( V = \frac{4\pi M^3}{3} \), we have assumed in fact that the space inside the collapsar is flat. Let us study this problem in detail below.

Recall once again that Stanyukovich considered the Universe as the result of the collapse of the space-time (76), created in emptiness by the mass-point, because he actually used the Hilbert radius \( r_g \) [10]. We have introduced in this paper the collapse of a specific continuous body — a homogenous liquid sphere (liquid bubble). It follows from (12) that the radius of the liquid sphere (11) in the collapse condition \( r_c \) equals its proper radius \( a \) and the breaking radius \( r_{br} \), if

\[
r_c = a = r_{br} = \sqrt{\frac{3}{8\rho}}.
\]

Substituting into (87) \( \kappa = \frac{8\pi G}{c^2} \) and \( \rho = \frac{3M}{4\pi a^3} \), we find, after elementary transformations,

\[
r_c = a = r_{br} = r_g = \frac{2GM}{c^2},
\]

where \( M \) is the mass of both the liquid and vacuum bubbles, because the liquid bubble in the state of collapse is precisely the vacuum bubble.

We have interpreted above that the liquid and vacuum bubbles are the spaces of the future and the past, respectively. This is partly how we geometrize the reality of time in terms of its flows (successive states) and in a cosmological framework. Then the space of the present must: 1) belong to these states simultaneously; 2) be situated between the future and past spaces. Of special interest, the singular surface \( r = a \) (the event horizon) satisfies both conditions. Firstly, the event horizon belongs to the gravitational and inflation collapsars; secondly, it is between the future and the past, since the observable time at the surface of the collapsar is stopped.

Since the event horizon is the characteristic surface of both the gravitational and inflation collapsars, it is simultaneously the surface of both the “white” and “black” holes. The collapsing liquid bubble transforms instantaneously into the de Sitter vacuum bubble — the inflation collapsar. Besides, the space inside the inflation collapsar (the “white hole”) is simultaneously also the space inside the gravitational collapsar (the “black hole”). The white and black holes possess the generic surface \( r = a \), which is simultaneously: 1) the radius of the liquid sphere and its breaking radius; 2) the event horizon itself and the radius of curvature of the vacuum bubble; 3) the Hilbert radius of the whole mass-point, which equals both the masses of the liquid and vacuum bubbles. The transformation of the liquid into the vacuum is accompanied by the inversion of the observable time: the flow of time changes the direction by way of transformation. Let us consider the causes of this transformation in detail. The question is: where, in the reality of time, is the mass \( M \)? The answer is: the liquid and vacuum bubbles are reflections of one other, where the mirror is the singular surface, therefore the mass is in the very present state of time, i.e. at the singular surface. Thus the materialization of the present (“reality”) is the transfer of time flows through the said singularity.

Let us return for a moment to the “black-and-white” model of the Universe. This object is the result of some transformations: 1) the liquid substance transforms instantly into the physical vacuum in the state of inflation; 2) the “curvature of time” changes its sign; 3) the Non-Newtonian force of attraction transforms into the force of repulsion. In fact, the liquid sphere overturns itself in time. This overturning is similar to the transfer of a time flow from one side of the Möbius strip onto the other side where the respective time on each of these sides flows in the opposite direction (compared to the other). We know that the Möbius strip is a two-dimensional one-sided surface which can be included (embedded) in three-dimensional Euclidean space \( E_3 \) (otherwise, it is generally non-orientable).

It is possible to say, therefore, that the observable time has three dimensions: the past, the present, the future. Time is perceived by human consciousness as one-dimensional and directed from the past to the future. Meanwhile, similar events are repeated for different epochs, demonstrating that the past and the future are mirror images of one other, where the mirror is the present. But these events are not identical. It is possible to say that the spaces of the past and the future are created from “different cosmic substances”; which depends on the time of creation of each space. Thus the past, present, and future are the three dimensions of the temporal volume, and these dimensions are different in principle. The past contains the consequence of holograms — physically realized (materialized) events. Besides, it also contains non-realized events. The future is virtual, because it contains only non-materialized events. Some events will be physically realized, others will be virtual. Such materialized events create the hologram (standing-wave picture) of the events, which is perceived by human consciousness as the (present) “reality”.

As such, our Universe transforms the space of the future through the singular surface (the present) into the space of the past, consequently the following materialization is none other than time transfer through the pertinent singularity — the event horizon. This singular surface is the place of interaction of two opposite forces — attraction and repulsion. The energy of physical vacuum creates the force of attraction, appearing as the “scattering of galaxies”. It can be called “radiant energy”. The energy of compression, which is due to the force of attraction, can be called “dark energy”. These two types of energy are divided and connected at the same time by said singular surface, which transforms the future into the past. When the course of the future reaches an end, the radiant energy will not develop, and the observable Universe will be compressed into the state of initial singularity. The cos-
mos will exist the way it does at present until it transforms all
the virtual realities of the future time (as it flows from the fu-
ture to the past). When this mechanism is exhausted, the ob-
servable Universe will compress itself into a Schwarzschild
black hole, namely the initial singularity. It is possible that
the mass of the singularity itself is the hypothetical “hidden
mass”, which exerts a definite effect on the motion of stars
and galaxies.

Let us now calculate the values $r_{hr}$ and $r_g$ for the Earth, the
Galaxy, and the observable Universe: see Table 1. Besides,
let us include into Table 1 the relative values $r_{hr}/a$ and $r_g/a$
for the mentioned objects. It follows from the Table that the
physical-geometric properties of the Universe differ in princi-
ple from the analogous properties of other objects (the Earth,
the Sun, the Milky Way). In reality, only the Universe is si-
multaneously both a white hole and a black hole, because its
Hilbert radius $r_g$ equals the radius of the inflation collapsar
$a$. These values coincide completely with the radius of space
breaking in the curvature of time. It is possible to say that
the forces of attraction and repulsion in the cosmos are in the
state of equilibrium. It is evident that the observable Universe
must be described as a stretched meta-body filled with matter
(physical vacuum in the given case).

The other objects (the Earth, the Sun, the Milky Way) con
tain black holes, whose Hilbert radii $r_g$ are very small
in comparison to their radiuses $a$. In addition, these objects
break the surrounding space, and the respective spheres of
spatial discontinuities are located out of the bodies (sources),
far away from them. Since the Hilbert radius $r_g \ll a$ depends
only on the mass of the body, we will consider these bodies
as mass-points, for example, by studying test bodies motion
in their gravitational fields. But if we want to study this case
in detail, we must consider the sources as stretched bodies
filled with matter. This approach applied in [6] to the Solar
System allows us to study the coupling between them. It is
easily obtained from the formula (12), that the Earth, the Sun,
and the Galaxy cannot be “white holes”, since the value $r_{hr}$
is imaginary. Therefore, these objects include Hilbert “black
holes” inside their spaces, but the respective space breakings
are outside of them.

7 Conclusion

The seminal process of time-transfer transformation of the
future into the past has been considered in this paper. The
future and past spaces are introduced geometrically as two
telemetric spheres (bubbles), filled with ideal substances —
liquid vacuum and physical vacuum respectively. These bub-
bles are mirror reflections of each other, where the mirror is
the singular surface. It means that the transfer of time from
the future to the past is realized through the singular state —
the very space of the present. The singular surface is simulta-
nously the surface of both the gravitational and inflation
collapsar, which can be called the dual “black-white hole”.

Thus, the present is the result of the collapse of the future
space, where the singular surface (the present) is the event
horizon. The collapsar is in the state of equilibrium, because
the two oppositely directed forces equalize each other. They
are 1) the gravitational force of attraction; 2) the force of re-
repulsion, which can be called the “force of anti-gravitation”.
The present is stable, until these forces neutralize one other.
If the force of attraction is greater than the force of repulsion,
the event horizon approaches the observer in space-time: the
space of the observable Universe “compresses”. If the force
repulsion is greater than the force of attraction, the event hori-
zon recedes from the observer: the space of the Universe ob-
servable “expands”.

We have obtained that observable time flows in the oppo-
site directions inside the liquid and vacuum bubbles. As was
shown in [3], spaces with the opposite directions of time are
mirror reflections of each other. In essence, the very term the
“mirror space” is linked immediately to the “arrow of time”.
The widely accepted opinion is that the “arrow of time” can
be directed only from the past to the future. The mathe-
matical apparatus of General Relativity does not prohibit the re-
verse flow of time, i.e. from the future to the past. Never-
theless the reverse flow of time is not introduced in contem-
porary physics and cosmology, because modern scientists re-
fer to Hans Reichenbach’s “arrow of time”, which is directed
always to the future [4]. However, Reichenbach stipulating
unidirectional time also implied a world process of evolution
(transfer of energy). In particular, in the geometric frame-
work of General Relativity, time can be stopped (as light can
also be frozen) or be directed to the past or the future. Setting
free cosmology from the unidirectional time concept gives us
a definite advantage as to introduce the potentielly revolu-
tionary Mirror Universe into General Relativity.

It is therefore more correct to introduce time as an ul-
timate kind of energy, although formally time is one of the
coordinates of the four-dimensional Riemannian manifold —
the space-time of General Relativity. But the three spatial
coordinates are measured by lines, while time is measured
by clocks, consequently space and time are two aspects of the
indivisible manifold — the space-time. Clearly speaking,
space-time can be considered as material (space), which is
filled with time (time-energy). Time-filled spaces exists only
in pseudo-Riemannian spaces, because the principal differ-
ence between coordinates exists, namely in spaces where the
basis vectors possess both real and imaginary lengths.

It is necessary to mention “rulers” of a special kind, which
are used in contemporary astronomy and cosmology, namely
light rays. Because light transfers at the finite velocity $c$, ob-
servation of electromagnetic radiation ensuing from cosmic
objects allows us to study only the past states of these ob-
jects. It is evident that the present states of these cosmic ob-
jects could be studied by means of instruments, which could
register a long-range action. The unfortunate negation of a
long-range action allows us to consider only the past states
of the Universe. In reality, our telescopes perceive only those light rays from stars and galaxies, emitted in the past. But if we’d only virtually reflect on the very boundaries of the observable Universe, that the present exists simultaneously in the whole space of the Universe, we might be able to build a space-time apparatus capable of registering the momentary (present) action of cosmic objects. (For example, such apparatuses have been constructed and tested by Nikolai Kozyrev).

It is well-known that the consensual opinion exists that General Relativity prohibits a long-range action due to the “light barrier”. This opinion is fundamentally incorrect: only the typical human consciousness produces this imaginary barrier. In fact, the mathematical apparatus of General Relativity allows the existence of zero-particles possessed of instantaneous transfer. The rejection of the notion of the “light barrier” allows us to construct, in principle, instruments for the registration of zero-particles.

All the innovative techniques in this paper are substantially based on Riemannian geometry only. The usual imaginary prohibitions (e.g., the speed of light barrier) by way of consensus in the field of General Relativity retard the development of General Relativity and science as a whole on the furthest horizon, which is a way to negate General Relativity as a whole. Clearly, those typical conditions restricting Special Relativity (as in the usual particle physics) do not ultimately exist in General Relativity as a whole by way of the vastness and versatility of the underlying Riemannian geometry (in our extensive case as shown in [3], the basic Riemannian geometry of General Relativity is extended at the sub-manifold level by the presence of degenerate, generally rotating zero-spaces and zero-particles). Meanwhile, in principle, the fundamental elements of Riemannian geometry allow for the existence of both the long-range action and the reverse flow of time: the long-range action is realized by null-particles, while the reverse flow of time is due to gravitation and rotation. It is necessary to note that these results are obtained by the condition that gravitation and rotation are rather strong. Meanwhile, most specialists in General Relativity consider gravitation and rotation as weak factors. For example, the gravitational potential \(w\) and the linear velocity \(v\) from the expression of \(d\tau\) (3) are taken into account by the usual problem of the synchronization of clocks as merely small corrections. Moreover, contemporary cosmologists assume that the reality of time of the Universe is the same in the whole space (being limited usually by the Hubble volume), since the observable time in the Friedman cosmological model flows uniformly: \(d\tau = dt\). But, as shown here, even using very simple non-rotating model of the gravitating Universe (the de Sitter bubble) as a start, we have seen that gravitation causes the accelerated extension of the space of the Universe near the event horizon.

All that has been said above is similar to the observation of a thunderstorm: we first see a lightning flash, only then the thunderpeal is registered by our ears. This is because light and sound travel at different speeds. A blind observer will, however, perceive only the thunderpeal. Moreover, having not a visual connection to the source of this sound (which is the lightning flash), he will be unable to determine the distance to the lightning. (A normal, sighted observer merely multiplies the sound speed in the air by the duration between the observed lightning and the heard thunderpeal, thus calculating the distance to the lightning.) Most astronomers may now be compared to the previous blind researcher of the thunderstorm: the instruments they use in their astronomical observations register only electromagnetic radiations of different sorts (visible light, radio-waves, x-rays, etc.), while all these radiations travel at the speed of light (in vacuum) or even slower than light (if travelling in a medium); their current instruments are not able to register real cosmic signals which are faster than light. In other words, those astronomers merely focus on the registration of the “short-range action” (transferred by photons, in particular). They do not take the possibility of the “long-range action” (instantaneous geometric interactions) into account. The key role in this primitive approach is played by the psychological wall erected against superluminal (and instantaneous) interactions. There is an easily popular bias that this prohibition is due to Einstein, whose prior postulate of the Special Theory of Relativity stipulated that signals travelling faster than light was practically impossible. This is, however, not true in the bigger picture. Einstein claimed this postulate in his early “positivistic” publication prior to General Relativity, in the framework of his theory of observable phenomena registered by means of signals of light: superluminal (and instant) signals were naturally out-of-access for such an observer. However, the geometric (if not hypergeometric) structure of the four-dimensional pseudo-Riemannian space (which is the basic space-time of Einstein’s General Theory of Relativity, being geometrically more complete, vast, and versatile in comparison to the Special Theory of Relativity) allows more diverse paths along which particles (signals) of different kinds may travel. For instance, particles bearing non-zero rest-mass/energy inhabit the sub-light speed region of the space-time (located “inside” the light cone); meanwhile, particles bearing imaginary masses and energies inhabit the superluminal space-time region (located “outside” the light cone); subsequently, there exist light-like particles bearing zero rest-mass (they are always in motion), while their relativistic masses and energies (“kinetic” masses and energies of motion) are non-zero, as they travel along space-time trajectories located along the light cone. There are also the so-called “zero-particles”: they are the ultimate case of light-like particles, and travel along the fully degenerate light-like trajectories which seem to have zero length and duration to an external observer; as a result zero-particles seem to be travelled instantaneously, thus transferring long-range action such as that in the case of the geometric non-quantum teleportation as shown in [3].
A real observing human whose body is made of regular substance such as atoms and molecules cannot travel at the speed of light. At the same time, he perceives light by his physical organs and the other (artificial) instruments of observation: there is not a barrier dividing him and light. In analogy to this case, instruments registering zero-particles (which seem to be travelling instantaneously) may be invented. All that the innovative engineers need to do is set themselves free of the psychological prohibition and limitation in traveling at the light speed, as to be professionally equipped with the full extent of the General Theory of Relativity which has already theoretically predicted zero-particles carrying the long-range action (geometric non-quantum teleportation).

Again, there are unfortunately many popular biases about Einstein’s General Theory of Relativity. Most of them originated in the non-technically equipped reporters of pop-science, or the pop-science authors themselves whose knowledge in this field is limited with those “first-grade” rudimentary textbooks on the theory of Relativity. Such books present Einstein’s theory rather very shallowly, paying attention to mostly the native examples based on Einstein’s early postulates revolving around his theory of exchanging light signals. The greater true meaning of Einstein’s theory — the deeper picture of space-time geometry as the basis of all the physical world — is regularly out-of-scope in such books due to the psychological threshold of the need to master Riemannian geometry and tensor calculus at a certain great level of mathematical and physical depth (which is not a trivial task for a beginner and indeed most would-be specialists, with the exception of very few gifted and versatile ones). As a result, we have such a popular bias (not based on geometry) as the above-mentioned aforementioned myth about the unsurpassable nature of the light speed limit, and also the myth about the irreversibility of the arrow of time (which naturally depends on the physical conditions of observation in different space-time regions). There is also another myth saying that the General Theory of Relativity can result in only small corrections to Classical Mechanics and Electrodynamics (this is not true on cosmological scales where the effects of General Relativity greatly rule), and many other biases concerning Einstein’s theory.

Setting ourselves free from these popular, primitive, anti-progressive biases, and following the deeper versatile trajectory (geometry) of the theory of space-time-matter established by Albert Einstein, no doubt certain researchers could arrive at new instruments of observation based on the geometric resurgence of the long-range action (in parallel with certain gravitational and gauge field instantons of the Plebski type). These new developments, based on completely different principles than the usual electromagnetic interactions, could lead to certain cosmic engines allowing for (geometric) non-quantum teleportation, as well as other new exotic technologies in order to carry the human species to an unprecedented Golden Age in the cosmos.

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