Matter in a Space of a Fractional Dimension. A Cosmological System of Spaces and Evolution of the Universe

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In this article, we propose a model of evolution of the Universe from topological spaces as a sequence generating one space from another. While the Universe is modelled in the form of a fraction-dimensional space, where time is the manifestation of the fractional dimension of the space.

Introduction

The origin of the Universe is a key topic in modern physics. Expansion of the Universe still demands an explanation.

Forty years ago, the physicist A. D. Sakharov has introduced a hypothesis: objects of the three-dimensional space are compositions of objects of a two-dimensional space and a one-dimensional space.

Based on this hypothesis, we propose a new cosmological model. In the framework of this cosmological model, the three-dimensional space is generated by its sub-spaces of lower dimensions. So forth this cosmological model and the process generating the spaces are described in detail.

The cosmological model of the Universe

The topological approaches described in Alexandrov's *Combinatorial Topology* [1] are used here to introduce the new cosmological model of the Universe. So we have:

R^{-1} — a space, which dimension is -1 that means a lack of space;

R^0 — a space of zero dimension means a space of energy, which is similar to energy of a quark;

R^1 — a space, which dimension is 1, means a space of electric energy;

R^2 — a space, which dimension is 2, means a space of magnetic energy;

R^3 — a space, which dimension is 3, means a space of gravitational energy (the "weight space");

R^1, R^2 and R^3 — Euclidean spaces. Dimension of such a space is the number of freedom degrees of a material point located therein.

The process generating the aforementioned topological spaces is as follows.

The space R^0. As a result of inflation [2] as symmetrization, an R^{-1} space generates an R^0 space. The space R^0 is not Euclidean. Each object located in a space contains a part of the total energy of the space. As such one, the space R^0 contains two groups of symmetric objects. The additive energy of objects located in the space is zero. Interaction between objects of one group is proportional to their distance from each other. Objects of the space R^0 uniquely define this space itself. Hence, the space R^0 is a space of quark-like energy. Distances between the objects is determined by the difference in energies of these objects. Time is a factor of evolution of the space. This evolution factor (time) is manifested in the redistribution of energy between the objects, and in the change in the objects' number in this space (i.e. transition from one state of the space into another state of the space). When interaction between the objects of the space reached symmetry, time disappears. In this case, the space R^0 arrives at a singular state. As is known, a space is identical to a specific type of energy. Quark-like energy is identical to the space R^0. So, quark-like energy and generates the space R^0.

The space R^1. Due to symmetrization of the singularity of the space R^0, synthesis of two objects which are attributed to two different groups of the space R^0 generates an object of a higher-dimensional space R^1. This is a space of electric energy (see above). Thus the space of electric energy is generated. Objects of the space R^1 are charges. The numerical value of such a charge is equal to the modulus of the energy difference of two objects attributed to the space R^0. Interaction between two charges is proportional to the multiplying result of their numerical values. Time in the space R^1 is determined by transformation of energy of the space R^0 into electric energy. The space R^1 evolves from the space R^0 to the singularity state. Singularity of a space is another space in which time is absent. So, after the entire energy of the space R^0 is transformed into electric energy, time disappears. Energy of each single charge is unlimitedly and continuously distributed along the space R^1 according to the interaction.

The space R^2. Due to symmetrization of the singularity state of the space R^1, charges in the space are separated from each other by the sign of difference of the objects attributed to the groups of the space R^0. The groups of charges differ by their signs. Synthesis of two charges bearing different signs generates an object of a higher-dimensional space R^2 (a space of magnetic energy, see above). Thus the space of magnetic energy is generated.

Consider the generation process by the example of a single photon. The photon is a result of synthesis of two charges bearing different signs (the space R^1), which are equal in their absolute values. The photon energy is continuously and unlimitedly distributed along the space R^2. Interaction between two photons is inversely proportional to the distance between them, and is proportional to the product of their energies. A single photon is an object of the magnetic energy space R^2. Objects in the space R^2 have the rotational degree of freedom (the spin).

The space R^2 evolves from the singularity state of the space R^1 to its own state of singularity. After converting electrical energy into magnetic energy, the space R^2 arrives at its own singularity state: time disappears in the space.

The space R^3. As a result of symmetrization of the singularity state of R^2, objects of the space are separated by the rotational degree of freedom (the spin). Synthesis of two objects, which are located in the space R^2 and bear oppositely directed spins, generates an object of a higher-dimensional space R^3 (a space of gravitational energy, see above). Thus the space of gravitational energy, R^3, is generated. Objects of the space R^3 are composed of objects of the spaces which dimensions are 1 and 2. The mass of objects of R^3 is continuously and unlimitedly distributed along the space.

Evolution of the space R^3

At present, the process converting magnetic energy of the space R^2 into gravitational energy of the space R^3 is in progress. We suggest to refer magnetic energy of the space R^2 as dark energy (the vacuum-like substance according to Gliner [3,4]). In these terms, mass (gravitational energy) is represented by matter and dark matter. Dark matter is a result of conversion of the magnetic energy into the gravitational energy. In the process of evolution of the space R^3, the shared part of the gravitational energy increases. This leads to slowing the clock down in this space. The process of passage of light in a space is analogous to the process of registration of time by a clock. The speed of light in this case is the conversion factor of the length in a duration of time. This coefficient is a constant of the space R^3. The process of passage of light in the space R^3 is the process of motion of a photon in the space R^2. In the space R^3, there are regions of absorption and emission of the photon. The photon's trajectory in the space R^2 is mapped into the region of its registration in the space R^3 in the relation "one-to-many". Thus the photon is tunneling in the space R^3. With the increase in the mass fraction in space, the redshift effect arises: a clock slow down with the process of passage of light. Density of the gravitational energy of the space R^3 depends on the speed of light. The energy density of a space, reduced to time duration, is a constant value [5]: $d_t c_t^{r-1} = const$, where d_t is the density of matter at a given point of the space; c_t is the speed of light (the speed of time) at the given point of the space; r is the dimension of the space at the given point.

Matter in a space of a fractional dimension

Consider how we percept the space of our world. At present, the space is three-dimensional: three spatial coordinates with triangulation of three dimensions are required. The fourth coordinate is time. In this case, the qualitative difference between the coordinate of time and the coordinates of space is emphasized. It is suggested that there exists an infinite set of three-dimensional spaces. However, under certain conditions (such as that the light speed in vacuum is constant), the time coordinate can be expressed in terms of linear length and vice versa. This allows us to assume that the time coordinate and the space coordinates have the same nature. In this case, the question about the infinite set of three-dimensional spaces does not vanish. On the basis of the above, we consider the problem of generation of spaces in the framework of the theory of topology sets.

Consider metric spaces R^n. In accordance with [1], an empty set has a dimension of $n = -1$. A set R^0 containing only one point X_t has a dimension of $n = 0$. To go to a higher dimensional space, it is necessary to perform a continuous mapping of one point $X_t \in R^0$ into a continuous set of points $X \subseteq R^1$. Here are two ways to display the sequence: 1) in the form of the ε-displacement (see §1.1 of Chapter 6 in [1]), where the continuity sequence of the subsequent point from the previous one is observed; and 2) the transfer method, where this condition is not satisfied. Introducing the notion of a sequence maps, we thereby define the time factor. Here the time factor determines the process generating a space with a higher dimension from a space of a lower dimension. Using only the shift method to generate a space gives a set that has a beginning, i.e. the starting point of reference. To exclude the starting point of reference, it is necessary to use, at least once, the transfer method. To generate all points of the set R^1, an infinite set of steps (an infinite amount of time) is required.

Time is a quantitative characteristic of the displayed space. Introducing the time factor is equivalent to introducing a characteristic of the density of the mapping flow — the speed of time. By the speed of time, we understand the ratio of the number of displayed points of a higher dimension space to the number of points of a space of a smaller (than that generated these points) dimension. This determines the multiplicity: how many points of the higher dimension space is displayed by one point of the lower dimension space. The instant fulfillment of the mapping (the multiplicity is infinite) is identical to the infinite speed of time, which in all cases is dimensionless. Hence, the complete numerical axis (line) in the set attributed to the metric space R^1 can be obtained by instant mapping one point $X_t \in R^0$ into a continuous set of points $X \subseteq R^1$ using two methods: the shift and carry methods.

In this case, metric spaces with an integer dimension can be represented as spaces with the zero time speed (that means that time is absent — there is no generation process, the number of displayed points is zero). The Hilbert space can be decomposed into an infinite number of metric spaces of a finite dimension (see §2.4 of Chapter 1 in [1]), and the following relation is fulfilled: $R^{n-1} \subseteq R^n$. And the cardinality of the set $\{R^{n-1}\}$ is equal to infinity: $|\{R^{n-1}\}| = \infty$. This assumes that the speed of time is infinite when creating a space, in which n is an integer, from a space of a lower dimension. Under the condition that the complete covering of R^n is not fulfilled (the speed of time is finite), the covered subset of R^n can be represented by a space R^d having a dimension d, where $(n-1) \leqslant rd \leqslant n$, i.e. a space with a fractional dimension. It is proposed to define spaces, in which the speed of time is finite and differs from zero, as fraction-dimensional spaces. The time speed function depends on the numerical value of the space dimension, which is a real number. It monotonically decreases within the interval of dimensions $(n-1) < d < n$, see Fig. 1 below.

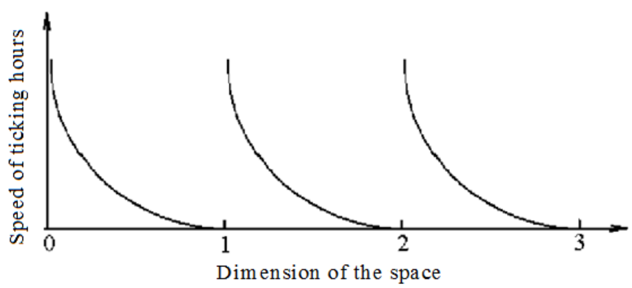

Fig. 1: The time speed of fraction-dimensional spaces.

The characteristic of the speed of time in the regions of our space is the speed of light. In this case, the distance from the point of radiation to the absorption point of a photon matches with the respective time duration registered by a remote clock. For example, the speed of light registered by our clock in this way on the boundary of the Universe exceeds the speed of light registered in our region of the space. When the numerical value of the time coordinate is reduced (with the respective numerical values the spatial coordinates) to the same measurement units, the magnitude of the speed of light is also dimensionless. Analysis of the speed of light in vacuum and material media shows that with the increasing density of matter the speed of light decreases. Reduction of the speed of light is accompanied by an increase in the dimension of space, see Fig. 1. This allows us to use the numerical value of the space dimension as the energy characteristic of the space. The density of matter has an inverse relation to the speed of time within an integer interval of the space dimension, see Fig. 2 below.

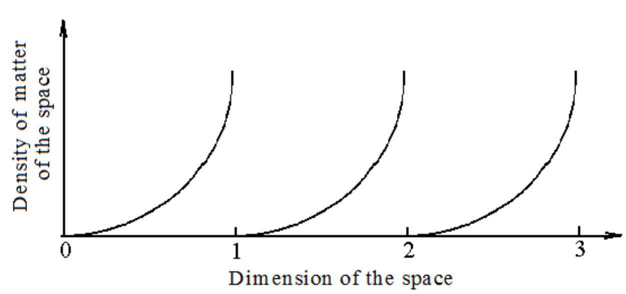

Fig. 2: The energy density of a fraction-dimensional space.

The boundary on the left of the fractional dimension (to an integer value of the space dimension) gives an infinite set of $(n-1)$-dimensional spaces having zero energy density. The boundary on the right is an n-dimensional space with an infinite density of matter, see Fig. 2. In this case, two versons representing a fraction-dimensional space are possible:

1. The space of an integer dimension R^{n-1} with the inclusions (domains or points and their neighborhoods, the set K) wherein the space is fraction-dimensional;
2. The space R_t^{n-1} containing the set S of points, each of which is a space of an integer dimension, and where covering Π by the set S of the space R_t^{n-1} is incomplete.

The first assumes that there exists a single integer-dimensional space containing a set of inclusions. The second — a set of integer-dimensional spaces. The latter is impossible under the previously stated assumption that an integer space contains infinite dense matter or is a continuum of integer spaces of lower dimensions. It is more preferable to assume that at all points of S we have the same space, but for each matter density d (time speed) the respective subset of points of this space is R_d^{n-1}. These subsets do not intersect with each other:

$$\bigcup R_{di}^{n-1} \cap R_{dj}^{n-1} = \emptyset, \quad \forall\, i \neq j;\ i, j = 1,\ 2,\ \ldots\ \infty.$$

All this is equivalent to the fact that each point of the space has one numerical value of the matter density parameter, i.e.

$$R^{n-1} \cap \bigcup R_{di}^{n-1} = R^{n-1}, \quad i = 1,\ 2,\ \ldots\ \infty.$$

In the case, where is a chain of the sets of points with zero numerical value of the matter density, interaction between the points at the ends of this chain occurs without time i.e. instantly (the speed of time is infinite there). However, the density of matter at each point of this chain is zero in this case as well as the space dimension of this set R^n. This is limiting and unreachable by definition. Moreover, the set R_d^{n-1} is uniquely mapped into one point of the space R_t^{n-1}. This implies the continuity of the mapping of R^{n-1} into R_t^{n-1}. The region of the set R_t^{n-1}, for given numerical values of d, belongs to the set of positive values of the numerical axis. Boundary of this region is the set of points in which the matter density of the space (i.e. the speed of time) is not defined. This corresponds to whole-dimensional spaces in which the time factor is absent. Suppose that the covering of Π remains unchanged. Corresponding to this covering, the average fractional dimension is $m_d = \mathrm{M}[d] = const$. The numerical characteristic of the coating, in turn, is proportional to m_d. If density of the coating is $d_\Pi = f(\Pi)$, then $m_d = d_\Pi$. From here, in a fraction-dimensional space, two time processes are possible:

1. The process of convergence of points of the set $S \subset R_t^{n-1}$ with each other upto coincidence (absorption), which makes possible to equalize the matter density throughout the entire space R^{n-1};

2. The process inverse to the convergence of points. Separation of one point into at least two points.

These two processes compete and provide a mapping of K into S, previously considered in two ways: the shift and carry methods. Due to the shift and transfer of points of the set R_t^{n-1}, mutual absorption of the points is possible. This should be accompanied with the reverse: the generation of points. This condition ensures that the covering of the same set R_t^{n-1} — the conservation law of the dimension (covering) of the space — remains constant. On the other hand, the covering Π is incomplete, but ensures the mapping S into the range of possible numerical values of the set R_t^{n-1}, — the positive numerical axis. This mapping is also determined by the fractional dimension through the time flow, and determines dynamics of the interaction processes of points of the set $K = \{R_d^{n-1}\}$ with each other. Therefore, **the space of a fractional dimension is dynamic**. The point of the set R_t^{n-1} corresponds to R_d^{n-1} — the set of points with the same density of matter of the space R^{n-1}. That is, in the absence of interaction with the remaining points, its position is determined only upto the set R_d^{n-1}. In this case, the point of the latter can be defined (perhaps) simultaneously at all points of R_d^{n-1}. That is, each such point has no distinctive features over the others. In the case of absorption (synthesis), it is possible and necessary to generate (divide) points of the spaces R_t^{n-1} and K. This is a necessary condition for generating a space (i.e. transfer). On the other hand, at a sufficiently high density of matter in the localization region of the point, the time speed is sufficiently small: displacement or transfer in this case almost does not require time. This also gives rise to the effect of supposedly simultaneous finding of one point in all places (points) of the localization region.

Results

Spaces of fractional dimensions contain local inhomogeneities in which the fractional dimension of the space differs from the fractional dimension of the vacuum region (which is the neighborhood of the inhomogeneity, the localization space). These are material objects. A local inhomogeneity is manifested in the numerical values of the parameters of the fields of a material object. The numerical values of the field parameters show the energy distribution of the space in the object's localization region. Combinations of the fields as the distribution characteristics of energies of the space give a description to the whole variety of the material objects. Degeneration of a fraction-dimensional space in the part of material objects leads to the appearance of zero-dimensional parameters that is quantum numbers. This quantum mechanism determines the discreteness of the set of phenomena there. A space with a unit inhomogeneity is an integer (for example, the three-dimensional space) everywhere, except for the heterogeneity itself. For an observer, it turns into a point because transition from one point to another does not require time. The very region of heterogeneity is a point at which the density of matter is infinite high. Passage through this point requires an infinite amount of time. Such a point is limiting, boundary, open, that is unreachable. Another boundary, with a uniform density of matter throughout the space, is also unreachable. Hence, we have an open interval for describing the entire set of material objects in a fraction-dimensional space.

Conclusion

So, a model of the cosmological system of spaces is proposed here. When considering this model, evolution of the Universe is discussed as well as the problem of description of fraction-dimensional spaces. Such spaces are defined as a results of energy conversion from the moment of inflation to R^3. The concept of singularity as a space in which time is absent is proposed. A "fractional space" is defined as a space in which the process of energy conversion from one type to another takes place. In this case, time is a factor of the process of energy conversion. Dynamics of fraction-dimensional spaces is predicted. These research results are a basis to calculate numerical values of the characteristics of such spaces.

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