Twin Universes: a New Approach

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In this article, we derive a differential form of Einstein's field equations using Cartan's free coordinates calculus. Under this form, we see that it is possible to infer another set of field equations dual to the original one and which displays a negative sign. We may then relate this system to the equations sustaining the twin Universe of the Janus Cosmological Model developed by the astrophysicist J.-P. Petit.

Introduction

As early as 2014, the astrophysicist J.-P. Petit put forward a model of Universe which harbors two fields equations with two sources: it is referred to as *The Janus Cosmological Model* (JCM) [1] which is inspired by the twin Universes theory first proposed by A. Sakharov [2].

Such a bi-metric is shown to account for the Dark Energy description and other unsolved observational data [3], provided one distinguishes our Universe as filled with positives masses and energies, from another wherein negative masses and negative energies are assigned to.

From the quantum physics perspective, negative energies have always played an unsavory role.

However, following a recent publication, it appears that both negative energies and masses are physically compatible if the time reversal operator is kept unitary within the Dirac formalism [4].

This considerable mathematical progress lends support to the Janus Model which relies on this symmetry.

So far, the few theories exhibiting two opposite metrics have been arbitrarily assumed as a "natural" hypothesis with the confidence that subsequent results would eventually corroborate this postulate. In this paper, we tackle the problem at the very early stage: With the aid of the Cartan calculus and using the Hodge star operation, we rewrite the Einstein's field equations under a differential form.

With this preparation, we naturally infer another set of field equations which displays a negative sign. This differential procedure thus provides a straightforward basis wherefrom the *Janus Model* can be substantiated.

Notations

Space-time: Greek indices α, β run from 0, 1, 2, 3. Space-time signature: −2. In the present text, $\varkappa$ is the Einstein's constant: $8\pi G/c^4$ where G is Newton's gravitational constant, although we adopt here $c = 1$.

1 Differential form of Einstein's field equations

1.1 The Cartan procedure

Let us consider a 4-pseudo-Riemannian manifold referred to a general basis e_α. The dual basis θ^β of one-forms are related to the local (Roman) coordinates $\{a\}$ by:

$$\theta^\beta = a_a^\beta \, dx^a. \tag{1.1}$$

The (a_a^β) are called *vierbein* or *tetrad fields* [5].

We next define the *Cartan procedure*, a powerful coordinates free calculus which is extensively used in the foregoing.

Let us define the *connection forms* by:

$$\Gamma_\beta^\alpha = \left\{ {\alpha \atop \gamma\,\beta} \right\} \theta^\gamma. \tag{1.2}$$

The first Cartan structure equation is related to the torsion by [6, p.40]:

$$\Omega^\alpha = \frac{1}{2}\, T^\alpha_{\ \gamma\delta}\, \theta^\gamma \wedge \theta^\delta = d\theta^\alpha + \Gamma_\gamma^\alpha \wedge \theta^\gamma, \tag{1.3}$$

where $T^\alpha_{\ \gamma\delta} = \frac{1}{2}[\Gamma^\alpha_{\ [\gamma\delta]} - \Gamma^\alpha_{\ [\delta\gamma]}]$ is the torsion tensor.

In the Riemannian framework alone, it reduces obviously to:

$$d\theta^\alpha = -\Gamma_\gamma^\alpha \wedge \theta^\gamma. \tag{1.4}$$

The *second Cartan structure equation* is defined as [6, p.42]:

$$\Omega_\beta^\alpha = \frac{1}{2}\, R^\alpha_{\beta\gamma\delta}\, \theta^\gamma \wedge \theta^\delta = d\Gamma_\beta^\alpha + \Gamma_\gamma^\alpha \wedge \Gamma_\beta^\gamma, \tag{1.5}$$

$R^\alpha_{\beta\gamma\delta}$ are here the curvature tensor components.

Defining the absolute exterior differential D of a tensor valued p-form of type (r, s)

$$(D\phi)_{j_1\ldots j_s}{}^{i_1\ldots i_r} = d\phi_{j_1\ldots j_s}{}^{i_1\ldots i_r} + \Gamma^{\,i_1}_{\,k} \wedge \phi_{j_1\ldots j_s}{}^{k\, i_2\ldots i_r} + \ldots - \Gamma^{\,k}_{\,j_1} \wedge \phi_{k\, j_2\ldots j_s}{}^{i_1\ldots i_r} - \ldots$$

we can write for example the *Bianchi identities* in a very simple way as:

$$D\Omega^\alpha = \Omega_\beta^\alpha \wedge \theta^\beta, \tag{1.6}$$

$$D\Omega_\beta^\alpha = 0. \tag{1.7}$$

1.2 The Einstein equations

1.2.1 The Einstein action

We first recall the *Hodge star* operator definition for an oriented n-dimensional *pseudo-Riemannian manifold* $(\mathrm{M}, \mathbf{g})$ whose volume element determined by $\mathbf{g}$ is:

$$\boldsymbol{\eta} = \sqrt{-\mathbf{g}}\; \theta^0 \wedge \theta^1 \wedge \theta^2 \wedge \theta^3.$$

Let $\Lambda_k(E)$ be the subspace of completely *antisymmetric multilinear forms* on the real vector space E.

The *Hodge star operator* * is a *linear isomorphism* *: $\Lambda_k(E) \to \Lambda_{n-k}(M)$ $(k \leqslant n)$. If $\theta^0, \theta^1, \theta^2, \theta^3$ is an oriented basis of 1-forms, this operator is defined by:

$$^*(\theta^{i_1} \wedge \theta^{i_2} \wedge \ldots \wedge \theta^{i_k}) = = \frac{\sqrt{-g}}{(n-k)!} \left[\epsilon_{j_1 \ldots j_n}\, g^{j_1 i_1} \ldots g^{j_k i_k}\, \theta^{j_{k+1}} \wedge \ldots \wedge \theta^{j_n}\right]. \tag{1.8}$$

With this preparation, the Einstein action simply reads:

$$^*R = R\,\boldsymbol{\eta}\,. \tag{1.9}$$

We shall need this action expressed in terms of tetrads.

Proof: With $\sigma^{\mu\nu} = {}^*(\theta^\mu \wedge \theta^\nu)$ and taking into account (1.8) we have

$$\sigma_{\beta\gamma} \wedge \Omega^{\beta\gamma} = \frac{1}{2}\,\sigma_{\beta\gamma}\, R^{\beta\gamma}_{\ \ \mu\nu}\, \theta^\mu \wedge \theta^\nu$$

and

$$^*(\theta^\mu \wedge \theta^\nu) = \frac{1}{2}\,\boldsymbol{\eta}_{\beta\alpha\sigma\rho}\, g^{\beta\mu} g^{\alpha\nu}\, \theta^\sigma \wedge \theta^\rho$$

i.e.

$$\sigma_{\beta\gamma} = \frac{1}{2}\,\boldsymbol{\eta}_{\beta\gamma\sigma\rho}\, \theta^\sigma \wedge \theta^\rho. \tag{1.10}$$

Thus,

$$\sigma_{\beta\gamma} \wedge \theta^\mu \wedge \theta^\nu = \frac{1}{2}\,\boldsymbol{\eta}_{\beta\gamma\sigma\rho}\, \theta^\sigma \wedge \theta^\rho \wedge \theta^\mu \wedge \theta^\nu = (\delta^\mu_\beta\, \delta^\nu_\gamma - \delta^\mu_\gamma\, \delta^\nu_\beta)\boldsymbol{\eta}$$

and:

$$\sigma_{\beta\gamma} \wedge \Omega^{\beta\gamma} = \frac{1}{2}\,(\delta^\mu_\beta\, \delta^\nu_\gamma - \delta^\mu_\gamma\, \delta^\nu_\beta)\, R^{\beta\gamma}_{\mu\nu}\boldsymbol{\eta} = R\,\boldsymbol{\eta} = {}^*R.$$

Taking into account (1.10) let us now compute the absolute exterior differential:

$$D\sigma_{\beta\gamma} = \frac{1}{2}\, D\,(\boldsymbol{\eta}_{\beta\gamma\sigma\rho}\, \theta^\sigma \wedge \theta^\rho).$$

In an orthonormal system $\boldsymbol{\eta}_{\beta\gamma\sigma\rho}$ is constant and: $D\,\boldsymbol{\eta}_{\beta\gamma\sigma\rho} = 0$.

This reflects the fact that in the *Riemannian framework* (metric connection), orthonormality is preserved under parallel transport as well as the transported vector magnitude. Therefore:

$$D\sigma_{\beta\gamma} = \boldsymbol{\eta}_{\beta\gamma\sigma\rho} D\theta^\sigma \wedge \theta^\rho.$$

Now, bearing in mind that the basis θ^σ is a *tensor valued 1-form of type (1,0)*, the first structure equation reads [7]:

$$D\theta^\sigma = \Omega^\sigma$$

and

$$D\sigma_{\beta\gamma} = \boldsymbol{\eta}_{\beta\gamma\sigma\rho}\, \Omega^\sigma \wedge \theta^\rho = \Omega^\sigma \wedge \sigma_{\beta\gamma\sigma}\,.$$

The latter is zero for the torsion free Riemann connection: $D\sigma_{\beta\gamma} = 0$.

In the same way, we can show that

$$D\sigma^{\beta\gamma}_{\ \ \ \alpha} = \mathrm{d}\sigma^{\beta\gamma}_{\ \ \ \alpha} + \Gamma^\beta_{\ \delta} \wedge \sigma^{\delta\gamma}_{\ \ \ \alpha} + \Gamma^\gamma_{\ \delta} \wedge \sigma^{\beta\delta}_{\ \ \ \alpha} - \Gamma^\delta_{\ \alpha} \wedge \sigma^{\beta\gamma}_{\ \ \ \delta} \tag{1.11}$$

with

$$\sigma^{\beta\gamma}_{\ \ \ \alpha} = {}^*(\theta^\beta \wedge \theta^\gamma \wedge \theta_\delta),$$

(where all indices are raised or lowered with $g_{\alpha\beta}$ from $\boldsymbol{g} = g_{\alpha\beta}\, \theta^\alpha \otimes \theta^\beta$).

1.2.2 The Einstein field equations

From (1.10), we infer:

$$\sigma_{\beta\gamma\delta} = \boldsymbol{\eta}_{\beta\gamma\delta\lambda}\, \theta^\lambda. \tag{1.12}$$

Under the variation of $\delta\theta^\beta$ of the orthonormal tetrad fields, we have

$$\delta(\sigma_{\beta\gamma} \wedge \Omega^{\beta\gamma}) = \delta\sigma_{\beta\gamma} \wedge \Omega^{\beta\gamma} + \sigma_{\beta\gamma} \wedge \delta\Omega^{\beta\gamma}\,.$$

Now, using (1.10) and (1.12) yields:

$$\delta\sigma_{\beta\gamma} = \frac{1}{2}\,\delta(\boldsymbol{\eta}_{\beta\gamma\delta\lambda}\, \theta^\delta \wedge \theta^\lambda) = \delta\theta^\delta \wedge \sigma_{\beta\gamma\delta}\,.$$

Then, applying the varied second structure equation

$$\delta\Omega^{\beta\gamma} = \mathrm{d}\delta\Gamma^{\beta\gamma} + \delta\Gamma^\beta_{\ \eta} \wedge \Gamma^{\eta\gamma} + \Gamma^\beta_{\ \eta} \wedge \delta\Gamma^{\eta\gamma}$$

we obtain

$$\delta(\sigma_{\beta\gamma} \wedge \Omega^{\beta\gamma}) = \delta\theta^\gamma \wedge (\sigma_{\beta\gamma\delta} \wedge \Omega^{\beta\gamma}) + \mathrm{d}(\sigma_{\beta\gamma} \wedge \delta\Gamma^{\beta\gamma}) - - \mathrm{d}\sigma_{\beta\gamma} \wedge \delta\Gamma^{\beta\gamma} + \sigma_{\beta\gamma} \wedge (\delta\Gamma^\beta_{\ \eta} \wedge \Gamma^{\eta\gamma} + \Gamma^\beta_{\ \eta} \wedge \delta\Gamma^{\eta\gamma}) \tag{1.13}$$

from the second line, we extract:

$$\mathrm{d}\sigma_{\beta\gamma} + \sigma_{\beta\gamma} \wedge (\Gamma^\eta_{\ \gamma} + \Gamma_{\beta\eta})$$

which is just: $D\sigma_{\beta\gamma}$. However, we know that: $D\sigma_{\beta\gamma} = 0$, and finally, the Einstein action variation is:

$$\delta(\sigma_{\beta\gamma} \wedge \Omega^{\beta\gamma}) = \delta\theta^\beta \wedge (\sigma_{\beta\gamma\delta} \wedge \Omega^{\gamma\delta}) + \mathrm{d}(\sigma_{\beta\gamma} \wedge \delta\Gamma^{\beta\gamma}) \tag{1.14}$$

(exact differential). The global Lagrangian density with matter is written:

$$\boldsymbol{L} = -\left(\frac{1}{2}\,\varkappa\right) {}^*\boldsymbol{R} + \boldsymbol{L}_{mat}\,.$$

Setting $^*T_\beta$ as the energy-momentum 3-form for *bare* matter we have the varied matter lagrangian density:

$$\boldsymbol{L}_{mat} = -\delta\theta^\beta \wedge {}^*T_\beta\,.$$

and taking into account (1.14) the global variation is:

$$\delta(\boldsymbol{L}) = -\delta\theta^\beta \wedge \left[\frac{1}{2}\,\varkappa\, \sigma_{\beta\gamma\delta} \wedge \Omega^{\gamma\delta} + {}^*T_\beta\right] + \text{(exact differential)}.$$

We eventually arrive at the field equations under the differential form:

$$-\frac{1}{2}\sigma_{\beta\gamma\delta}\wedge\Omega^{\gamma\delta} \;=\; \varkappa\,^{*}T_{\beta}\,, \tag{1.15}$$

where T_α is related to the energy-momentum tensor $T_{\alpha\beta}$ by $T_\alpha = T_{\alpha\beta}\,\theta^\beta$.

In the same manner, one has: $G_\alpha = G_{\alpha\beta}\,\theta^\beta$ so that these identifications lead to the field equations with a source in the classical form:

$$G_{\alpha\beta} \;=\; R_{\alpha\beta}-\frac{1}{2}\,g_{\alpha\beta}\,R \;=\; \varkappa\,T_{\alpha\beta}\,, \tag{1.16}$$

$G_{\alpha\beta}$ is conserved but not $T_{\alpha\beta}$, therefore we should look for the appropriate r.h.s. tensor.

To this effect we start by reformulating (1.15) as

$$-\frac{1}{2}\,\Omega_{\beta\gamma}\wedge\sigma^{\beta\gamma}{}_{\alpha} \;=\; \varkappa\,^{*}T_{\alpha} \tag{1.17}$$

and we use the second structure equation under the following form

$$\Omega_{\beta\gamma} \;=\; \mathrm{d}\Gamma_{\beta\gamma} \;-\; \Gamma_{\mu\beta}\wedge\Gamma^{\mu}{}_{\gamma} \tag{1.18}$$

so as to obtain:

$$\mathrm{d}\Gamma_{\beta\gamma}\wedge\sigma^{\beta\gamma}{}_{\alpha} = \mathrm{d}\left(\Gamma_{\beta\gamma}\wedge\sigma^{\beta\gamma}{}_{\alpha}\right) + \Gamma_{\beta\gamma}\wedge d\sigma^{\beta\gamma}{}_{\alpha}\,. \tag{18bis}$$

Then using (1.11) in (1.18bis), we infer:

$$\mathrm{d}\Gamma_{\beta\gamma}\wedge\sigma^{\beta\gamma}{}_{\alpha} = \mathrm{d}\left(\Gamma_{\beta\gamma}\wedge\sigma^{\beta\gamma}{}_{\alpha}\right) + \\ +\Gamma_{\beta\gamma}\wedge\left(\Gamma^{\beta}_{\ \delta}\wedge\sigma^{\delta\gamma}{}_{\alpha} - \Gamma^{\gamma}_{\ \delta}\wedge\sigma^{\beta\delta}{}_{\alpha} - \Gamma^{\delta}_{\ \alpha}\wedge\sigma^{\beta\gamma}{}_{\delta}\right). \tag{1.19}$$

Adding the second contribution ($\Gamma^{\alpha}\ \ \gamma\wedge\Gamma^{\gamma}\ \ \beta$) of (1.18) to (1.19), we obtain the Einstein field equations in a new form:

$$-\frac{1}{2}\,\mathrm{d}(\Gamma_{\beta\gamma}\wedge\sigma^{\beta\gamma}{}_{\alpha}) \;=\; \varkappa\,(^{*}T_{\alpha} \;+\; {}^{*}t_{\alpha}), \tag{1.20}$$

where

$$^{*}t_{\alpha} \;=\; \left(-\frac{1}{2}\,\varkappa\right)\Gamma_{\beta\gamma}\wedge(\Gamma_{\delta\alpha}\wedge\sigma^{\beta\gamma\delta} - \Gamma^{\gamma}{}_{\delta}\wedge\sigma^{\beta\delta}{}_{\alpha}), \tag{1.21}$$

where $^{*}t_\alpha$ should be here interpreted as *energy* and *momentum 3-form of the gravitational field* generated by this matter.

Equation (1.20) readily implies the conservation law:

$$\mathrm{d}(^{*}T_{\alpha} \;+\; {}^{*}t_{\alpha}) \;=\; 0\,. \tag{1.22}$$

Within the *Riemannian framework*, we know that the gravitational field cannot be localized, which is reflected by the fact that $^{*}t_\alpha$ does not transform as a tensor with respect to gauge transformations.

Indeed, as $\Gamma_{\beta\gamma}$ can be made zero at any given point of the Riemannian manifold, this 3-form vanishes.

To the 3-form $^{*}t_\alpha$ is thus associated the antisymmetric Einstein-Dirac pseudo-tensor $(\Theta^{a}_{b})_{ED}$ [8].

In order to explicitly write down (1.20) with a *true* 3-form on the r.h.s., one should add the *3-form of the energy-momentum for the vacuum* denoted by $(^{*}t_\alpha)_{vac}$.

Equation (1.22) eventually satisfies the conservation law:

$$\mathrm{d}[^{*}T_{\alpha} \;+\; (^{*}t_{\alpha})_{gravity}] \;=\; 0 \tag{1.23}$$

with:

$$(^{*}t_{\alpha})_{gravity} \;=\; {}^{*}t_{\alpha} \;+\; (^{*}t_{\alpha})_{vac}\,. \tag{1.24}$$

To the 3-form $(^{*}t_\alpha)_{vac}$ corresponds the tensor

$$(t_{\alpha\beta})_{vac} \;=\; \left(-\frac{1}{2}\,\varkappa\right)\Xi\;g_{\alpha\beta}\,, \tag{1.25}$$

where Ξ is the variable cosmological term which replaces the cosmological constant Λ as [9]:

$$G_{\alpha\beta} = R_{\alpha\beta}-\frac{1}{2}\,g_{\alpha\beta}\,R = \varkappa\left[T_{\alpha\beta}+(t_{\alpha\beta})_{ED}\right]+\Xi\,g_{\alpha\beta}\,. \tag{1.26}$$

2 Two opposite field equations

Since we deal with a Lorentzian manifold $n = 4$, repeated application of the duality operation *, gives:

$$^{*}(^{*}G_{\beta}) \;=\; -\,^{*}G_{\beta}\,, \tag{2.1}$$

$$^{*}(\varkappa\,^{*}T_{\beta}) \;=\; -\,(\varkappa\,^{*}T_{\beta})\,. \tag{2.2}$$

The Cartan formalism thus allows for two "opposite" field equations to appear.

Can we find its physical meaning? A straightforward justification can be provided by the *Janus model* of J.P. Petit whose universes exhibit opposite energy/masses.

This model is characterized by two types of distinct metric tensors $^{(+)}g_{\mu\nu}$ and $^{(-)}g_{\mu\nu}$, which imply two distinct field equations:

$$^{(+)}G_{\beta\mu} = {}^{(+)}R_{\beta\mu}-\frac{1}{2}\,{}^{(+)}g_{\beta\mu}{}^{(+)}R = \varkappa\left[^{(+)}T_{\beta\mu}+\varpi^{(-)}T_{\beta\mu}\right], \tag{2.3}$$

$$^{(-)}G_{\beta\mu} = {}^{(-)}R_{\beta\mu}-\frac{1}{2}\,{}^{(-)}g_{\beta\mu}{}^{(-)}R = \varkappa\left[^{(-)}T_{\beta\mu}+\omega^{(+)}T_{\beta\mu}\right], \tag{2.4}$$

where $^{(+)}g_{\mu\nu}$ refers to positive mass/energy particles while $^{(-)}g_{\mu\nu}$ refers to negative mass/energy particles with the corresponding Ricci tensors $^{(+)}R_{\mu\nu}$ and $^{(-)}R_{\mu\nu}$.

Here $^{\pm}T_{\mu\nu}$ is the massive tensor which implicitly contains the gravitational field tensor defined from (1.24).

With our definition, we then have the obvious correspondences:

$$^{*}G_{\beta} \;\to\; {}^{(+)}G_{\beta\mu}\,, \qquad {}^{*}T_{\beta} \;\to\; {}^{(+)}T_{\beta\mu}+\varpi^{(-)}T_{\beta\mu}\,,$$

$$^{*}(^{*}G_{\beta}) \;\to\; {}^{(-)}G_{\beta\mu}\,, \quad {}^{*}(^{*}T_{\beta}) \;\to\; -\left(^{(-)}T_{\beta\mu}+\omega^{(+)}T_{\beta\mu}\right).$$

Each solution of (2.3) and (2.4) is a *Friedmann-Lemaitre-Roberston-Walker metric*

$$^{(\pm)}\mathrm{d}s^2 = \mathrm{d}t^2 - {}^{(\pm)}a(t)^2\,\frac{\mathrm{d}u^2 + u^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)}{\left(1 + \dfrac{k\,u^2}{4}\right)^2}\,, \qquad (2.5)$$

where k is referred to as the *curvature index*: $\{-1, 0, 1\}$.

Ultimately, inspection shows that:

$$\varpi = \frac{^{(-)}a^3}{^{(+)}a^3} \quad \text{and} \quad \omega = \frac{^{(+)}a^3}{^{(-)}a^3}\,, \quad \omega = \varpi^{-1}. \qquad (2.6)$$

3 Conclusions and outlook

According to the Cosmological Janus Model, mass and charge inversions simultaneously result from time reversal which grant the theory a particularly simple and exhaustive symmetry.

As a final point, let us emphasize that the *JCM* bi-metric scheme is far from being an arbitrary postulate as it proves consistent with the newest developments in astrophysics.

It is also formally sustained by a specific splitting of the *Riemann tensor* in two 2nd rank tensor field equations as shown in [10]. This 4th rank tensor theory eventually leads to the space-time of constant curvature (i.e. in vacuum). It thereby copes with the recent view suggesting that the laws of physics are invariant under the symmetry group of *De Sitter* space (maximally symmetric space), rather than the *Poincaré* group of Special Relativity [11–14].

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