Remark to Approach to the Schwarzschild Metric with SL(2,R) Group Decomposition

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1 Remark to Section 5

1. The SL(2, C)\(^*\) group definition. Let the group SL(2, C)\(^*\) be a subgroup of SL(2, C) with an element Z’ \(\in\) SL(2, C)\(^*\) such as
\[
Z' = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} : a_1, a_4 \in \text{Re}, \ a_2, a_3 \in \text{Im}, \ \det(Z') = 1 \right\}.
\]
The definition reflects the general Jacobian matrix form as given by (12) in [1].

2. The proof of the isomorphism of SL(2, C)\(^*\) to SL(2, R). The mapping (13) in [1] can be equivalently defined by the function that sends element of Z’ \(\in\) SL(2, C)\(^*\) to Z \(\in\) SL(2, R)
\[
Z = T \cdot Z' \cdot T^{-1}
\]
where
\[
T = \begin{bmatrix} \sqrt{-i} & 0 \\ 0 & \sqrt{i} \end{bmatrix} \quad T^{-1} = \begin{bmatrix} \sqrt{i} & 0 \\ 0 & \sqrt{-i} \end{bmatrix} \quad \det(T) = 1.
\]
The function is clearly a group homomorphism since
\[
T \cdot Z'_1 \cdot Z'_2 \cdot T^{-1} = T \cdot Z'_1 \cdot T^{-1} \cdot T \cdot Z'_2 \cdot T^{-1} = Z_1 \cdot Z_2
\]
for all Z_1, Z_2 \(\in\) SL(2, R). It is obviously surjective. At last, as the inverse mapping
\[
Z' = T^{-1} \cdot Z \cdot T
\]
that sends any element of SL(2, R) to SL(2, C)\(^*\) is well defined it proves the injectivity. Hence, as a bijective homomorphism is shown, it finalizes the proof of SL(2, C)\(^*\) \(\cong\) SL(2, R) mentioned in Section 5.

2 Corrections

The typo in the expression (10). The expression should evidently read with \(\cosh^2(\beta)\) as follows
\[
\begin{align*}
g_{\mu\nu} &= \begin{bmatrix} -\left(1 - \nu^2\right) & 0 \\ 0 & \left(1 - \nu^2\right)^{-1} \end{bmatrix} \\
\text{(10)}
\end{align*}
\]
Section 5. A more appropriate notation for the Lorentz/ Minkowski basis for SL(2, R) is \(\mathbb{R}^{1(2)}\) as the group consists of the real numbers.

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References