Remark to Approach to the Schwarzschild Metric with SL(2,R) Group Decomposition

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1 Remark to Section 5

1. The $SL(2,\mathbb{C})^*$ group definition. Let the group $SL(2,\mathbb{C})^*$ be a subgroup of $SL(2,\mathbb{C})$ with an element $Z' \in SL(2,\mathbb{C})^*$ such as

$$Z' = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} : a_1, a_4 \in Re,\ a_2, a_3 \in Im,\ \det(Z') = 1 \right\}.$$

The definition reflects the general Jacobian matrix form as given by (12) in [1].

2. The proof of the isomorphism of $SL(2,\mathbb{C})^*$ to $SL(2,\mathbb{R})$. The mapping (13) in [1] can be equivalently defined by the function that sends element of $Z' \in SL(2,\mathbb{C})^*$ to $Z \in SL(2,\mathbb{R})$

$$Z = T \cdot Z' \cdot T^{-1}$$

where

$$T = \begin{bmatrix} \sqrt{-i} & 0 \\ 0 & \sqrt{i} \end{bmatrix} \quad T^{-1} = \begin{bmatrix} \sqrt{i} & 0 \\ 0 & \sqrt{-i} \end{bmatrix} \quad \det(T) = 1.$$

The function is clearly a group homomorphism since

$$T \cdot Z_1' \cdot Z_2' \cdot T^{-1} = T \cdot Z_1' \cdot T^{-1} \cdot T \cdot Z_2' \cdot T^{-1} = Z_1 \cdot Z_2$$

for all $Z_1, Z_2 \in SL(2,\mathbb{R})$. It is obviously surjective. At last, as the inverse mapping

$$Z' = T^{-1} \cdot Z \cdot T$$

that sends any element of $SL(2,\mathbb{R})$ to $SL(2,\mathbb{C})^*$ is well defined it proves the injectivity. Hence, as a bijective homomorphism is shown, it finalizes the proof of $SL(2,\mathbb{C})^* \cong SL(2,\mathbb{R})$ mentioned in Section 5.

2 Corrections

The typo in the expression (10). The expression should evidently read with $\cosh^2(\beta)$ as follows

$$\begin{aligned} g_{\mu\nu} &= \begin{bmatrix} -\left(1 - v^2\right) & 0 \\ 0 & \left(1 - v^2\right)^{-1} \end{bmatrix} \\ &= \begin{bmatrix} -\cosh^{-2}(\beta) & 0 \\ 0 & \cosh^2(\beta) \end{bmatrix}. \end{aligned} \tag{10}$$

Section 5. A more appropriate notation for the Lorentz/ Minkowski basis for $\mathbb{SL}(2,\mathbb{R})$ is $\mathbb{R}^{1(2)}$ as the group consists of the real numbers.

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References

1. Kritov A. Approach to the Schwarzschild Metric with SL(2,R) Group Decomposition. *Progress in Physics*, 2020, v. 16 (2), 139–142.