1 Introduction
The Antarctic Circumpolar Current (ACC), the largest ocean current on Earth, flows west to east at about 2 m/s faster than the Earth’s rotation at its latitude of about 40ºS to about 60ºS near the Antarctic continent [1, 2], as shown in Figure 1. Its mean transport is estimated to be about 134 sverdrup, i.e., $134 \times 10^6$ m$^3$/s. There are two different atmospheric winds to consider: the winds along the ACC and the winds along the contours of Antarctica, with variations in both able to cause robust changes in ACC transport. They are considered to be the major driving force of this enormous water current.

But the ACC current extends to the ocean floor, with a strong current velocity of about 2 cm/s at a depth of 3000 meters [3, 4]. So this approach becomes quite complicated by involving thermodynamic mixing vertically and horizontally, various wind strength and direction changes, Coriolis force effects, eddies, etc.

Ultimately, one might expect to identify a powerful and consistent energy source that would be capable of forcing such a large water transport at all depths as well as help drive the winds in the atmosphere.

Herein I apply Quantum Celestial Mechanics (QCM) to the binary system of the rotating Earth and the orbiting Moon, both objects providing the total system vector angular momentum required by QCM [5] to determine its gravitational stationary energy states exhibiting the quantization of angular momentum per unit mass. I can use the familiar general relativistic Schwarzschild metric because the QCM equilibrium radii $r_{eq}$ are much larger than the 9 millimeter Schwarzschild radius $r_g$ of the Earth. These QCM states at specific equilibrium radii in the plane of the Equator are assumed to define rotational cylinders co-axial to the Earth’s rotation axis that intersect the Earth’s surface. In particular, I am interested in determining whether the QCM angular momentum quantization per unit mass approach can be the source of the driving force responsible for the Antarctic Circumpolar Current.

2 QCM brief history review
In 2003 Howard G. Preston and I introduced [5] Quantum Celestial Mechanics (QCM) to explain the spacings of planetary orbits in the Solar System and in all known exoplanetary systems. In the Schwarzschild metric, the quantization of the total angular momentum per unit mass in a gravitationally bound system constrains the possible orbital radii to specific allowed values determined by quantization integers.

At that time, we were not successful in finding a system that could be a definitive test of QCM. Unfortunately, there existed no gravitationally bound system with three or more celestial objects for which the angular momentum was known to within 10%, not for the Solar System nor for the Jovian planets and their satellites. Therefore, we proposed several laboratory experiments to test for a repulsive gravitational QCM force, including the response of two pendulums in a microwave vacuum chamber and of the response of one LIGO interferometer to the slow one rotation per hour spin of a 10 kg mass several meters distant. Neither tests were
approved. However, the 2015 New Horizons flyby of Pluto and its 5 moons did provide the data [6] for the definitive test of QCM, with the predicted QCM orbital constraint relation verified to within 2.4%.

The QCM gravitational wave equation derived from the general relativistic Hamilton-Jacobi equation is

$$d^2\Psi + \frac{2}{r} \frac{d\Psi}{dr} + \left[\frac{E}{\mu} + \frac{r_g c^2}{2r} - \frac{\ell (\ell + 1) H^2 c^2}{2r^2}\right] \Psi = 0,$$

in which the scalar $\Psi = \exp[i S'/H]$, for $S' = S/\mu c$, with $S$ the classical action, $\mu$ the mass of the particle acted upon, and $c$ the speed of light in vacuum. The system scaling length is defined as $H = L_T/M_T c$, with $L_T$ the total vector angular momentum for the system of total mass $M_T$.

This QCM gravitational wave equation is not quantum gravity. However, there is a relationship to the Schrödinger equation in quantum mechanics that was derived from the normal Hamilton-Jacobi equation using the transformation $\Psi = \exp[i S'/h]$, with $S$ and the universal Planck’s constant $h$. Our $H$ is not a universal constant.

The inherent generality and power of this gravitational wave equation arises from its dependence upon only two important physical parameters that characterize the gravitationally bound system: the total mass $M_T$ and the total vector angular momentum $L_T$, both quantities defining $H$. In planetary systems, for example, the larger the value of $H$, the larger the spacings will be between the allowed QCM orbital equilibrium radii.

Successful applications of QCM have included the prediction of a Solar System total angular momentum of $1.86 \times 10^{45}$ kg-m^2/s, most of which is contributed by the Oort Cloud at about 40,000 AU, a value about 50 times the listed angular momentum of the Sun plus the 8 planets [7]. Compared to all the known exoplanetary systems, our Solar System is unique because no other system exhibits such large planetary spacings that require this large total system angular momentum value.

Successful applications to galaxies and clusters of galaxies describe how QCM can fit their almost constant rotational velocities without invoking dark matter. Also, QCM was shown to be able to derive the MOND relation, which fits the galaxy rotational data extremely well and is considered a viable competitor to dark matter approaches [8].

A new interpretation [9] of the redshifts of light from distance sources in the Universe was introduced by applying the interior metric in a static Universe, thereby revealing a possible negative QCM gravitational potential that becomes more negative non-linearly from the observer, meaning that the light source is in a deeper negative gravitational potential for all observers. As such, the clocks at the light source tick slower than at the observer and the observed redshifts are purely gravitational redshifts. No dark energy is required to agree with the measured SNe 1a redshifts that have been interpreted as a recently accelerating Universe, and the Hubble value becomes distance dependent.

### 3 QCM Schwarzschild metric radial equation

Applying the general relativistic Schwarzschild metric to the QCM wave equation for radius values beyond $r_g$, dropping very small terms, and then evaluating the angular equations in spherical polar coordinates, leaves the radial $r$ equation [5]

$$\frac{d^2\Psi}{dr^2} + \frac{2}{r} \frac{d\Psi}{dr} + \left[\frac{E}{\mu} + \frac{r_g c^2}{2r} - \frac{\ell (\ell + 1) H^2 c^2}{2r^2}\right] \Psi = 0,$$

with $\ell$ the angular momentum integer from the $\theta$ and $\phi$ coordinates.

From the energy expression in the square bracket, the effective potential

$$V_{eff} = -r_g c^2 + \frac{\ell (\ell + 1) H^2 c^2}{2r^2},$$

and the equilibrium radius for the QCM state $\ell$ is

$$r_{eq} = \ell (\ell + 1) \frac{2 H^2}{r_g}.$$  

If one decides to use the Schwarzschild metric in cylindrical coordinates instead, then the product $\ell (\ell + 1)$ usually becomes replaced by $m^2$, with $m$ the integer for the $\phi$ direction quantization.

I will take this $r_{eq}$ to be at the plane of the Equator for defining a cylinder co-axial with the Earth’s rotation axis that extends in both directions to intersect the Earth’s surface in North and South latitudes. Thus, by knowing the $H$ and $r_g$ values to calculate $r_{eq}$, one can predict the equilibrium radii of all the QCM states.

### 4 Results

#### 4.1 Earth spin only

The total vector angular momentum of the Earth-Moon system is required by QCM. However, a preliminary simple calculation that considers just the rotation of the Earth about its axis is instructive.

The pertinent physical parameters of the Earth-Moon system are listed in Table 1, including the Earth’s moment of inertia factor $\alpha = 0.827$ and the average angle factor $\beta = \cos(23.4^\circ)$ between the Earth’s equatorial plane and the plane of the Moon’s orbit. If only the Earth’s spin angular momentum is considered, $H = 3.26$ meters, so

$$r_{eq} = \ell (\ell + 1) 2.36 \times 10^3 \text{ m}.$$  

Beginning with the $\ell = 1$ state, all the $r_{eq}$ values will be too small for any important relationship to the ACC around Antarctica.
Table 1: Earth–Moon QCM parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Spin only $10^24$ kg</th>
<th>Earth–Moon $10^24$ kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>5.972</td>
<td>6.045</td>
</tr>
<tr>
<td>Radius</td>
<td>6.37</td>
<td>385.0</td>
</tr>
<tr>
<td>Period</td>
<td>0.08614</td>
<td>2.3605</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.827</td>
<td>—</td>
</tr>
<tr>
<td>$\beta$</td>
<td>—</td>
<td>0.918</td>
</tr>
<tr>
<td>$L_T$ ($10^3$ kg-m$^2$/s)</td>
<td>5.847</td>
<td>32.5</td>
</tr>
<tr>
<td>$H$ (m)</td>
<td>3.26</td>
<td>17.94</td>
</tr>
</tbody>
</table>

4.2 Earth–Moon total angular momentum

The QCM wave equation requires the total vector angular momentum of the gravitationally bound system in its applications. The orbital vector angular momentum value for the Moon is the much larger contributor in the Earth–Moon system but varies considerably, repeating every 18.6 years, because the angle between the Moon’s orbital plane and the Earth’s equatorial plane reaches a maximum difference angle of 28º36’ and a minimum of 18º20’.

Without accounting for this variation in the difference angle, the Moon’s orbital motion would contribute about $2.91 \times 10^{34}$ kg-m$^2$/s. Assuming an average difference angle of about 23.4º with respect to the Earth’s equatorial plane, with cosine $23.4º = 0.918$, the Moon’s average vector orbital angular momentum contribution is about $2.67 \times 10^{34}$ kg-m$^2$/s.

Therefore, the Earth-Moon $H = 17.94$ meters and $r_{eq} = \ell(\ell + 1) 71.52 \times 10^3$ m. (6)

The $r_{eq}$ calculated values and their surface intersection latitudes for $\ell = 1$ to 9 are listed in Table 2.

The two QCM equilibrium radii $r_{eq}$ for $\ell = 6$ and $\ell = 7$ intersect the surface at North and South latitudes of 61.9º and 51.0º, but only their South latitudes have a path that allows water to transport completely around the surface of the Earth just north of Antarctica.

Note that the $\ell = 1$ to 4 states have equilibrium radii that may be applicable in the Arctic Ocean at the North Pole. The remainder intersect land masses on the surface. All these QCM rotating cylinders could be creating mass currents underneath the crust in the mantle and within interior parts of the Earth, even supporting the generation of the magnetic field and the recent magnetic north pole’s rapid movement past the rotational North Pole toward Russia.

Qualitative radial probability distributions for the QCM cylinders that could be affecting the ACC are shown in Figure 2. The vertical line at $6.37 \times 10^6$ m, is the approximate Earth radius. Their wide radial distributions within the Earth adds to the complexity of interpreting their actions.

As determined below, all displacements from the equilibrium radius will experience a QCM driving force back toward $r_{eq}$, here interpreted as the distance from the Earth’s rotation axis for simplified discussion purposes only. This radial force keeps the ACC roughly localized in the $r$ coordinate, although the qualitative probability distributions shown in Figure 2 reveal a large spread in the radial direction underneath the surface. Moreover, displacements in latitude along the surface are also displacements in the $r$ coordinate, resulting in a complex dynamics to consider in any detailed analysis.

A fluid dynamics computer simulation would be needed to better understand the actual behavior of the ACC when QCM forces, winds, Coriolis effects, water density, and water viscosity are accounted for. The atmosphere above the ocean water would also be subject to the QCM forces in both the $r$ direction and the $\phi$ direction. A rough estimate of the dynamics is considered below.

4.3 Estimates of QCM forces

In the following simplified analysis of the Earth–Moon system, winds and Coriolis forces are ignored. In the $\phi$-direction, the QCM angular momentum per unit mass $L/\mu$ for a free particle at the equilibrium radius $r_{eq}$ is given by the QCM constraint,
Thus, substituting $m = \ell$ at the Equator, assuming a co-axial cylinder. Thus, substituting $L = \mu v_{\phi} r$ for the angular momentum produces the $\phi$ velocity

$$v_{\phi} = \frac{mcH}{r}. \quad (8)$$

QCM predicts for the $m = 6$ state a velocity $v_{\phi} \sim 1.1 \times 10^4$ m/s, and at the $m = 7$ state a $v_{\phi} \sim 1.26 \times 10^4$ m/s, values which can be compared to the actual average ACC velocity of about 212 m/s with respect to the stars. A large reduction in these predicted $\phi$-velocities would be required of viscosity effects in the water and impedance effects of the land interruption at the ocean bottom and at the edges of the continents.

The torque $\tau$ required to keep the volume flow $V \sim 4 \times 10^{26}$ m$^3$/s of ACC moving at approximately 2 m/s faster than the Earth’s rotational velocity can be estimated, using the viscosity $\eta = 1.6$ cP for water at about 2ºC, to be

$$\tau = 2\pi \eta V \approx 10^{14} \text{Nm}, \quad (9)$$

which translates to a force of about $10^{-12}$ N to keep a kilogram of water moving at 2 m/s faster than the Earth’s velocity.

Depending upon just where vertically and horizontally one calculates the driving torque pushing the water, QCM force values up to only about $10^{-9}$ N are estimated to be required. Any vertical water movement at the ACC latitude introduces displacement components in both the radial direction from the rotation axis and in the latitudinal direction. For simplicity, any latitudinal direction movement is assumed to be included within the $r$ direction movement for the spherical geometry of the Earth’s surface.

A small displacement from $r_{eq}$ in the radial direction results in an acceleration, calculated by taking the negative gradient of $V_{eff}$, to get

$$a_r = -\frac{r v_{\phi}^2}{2r^2} + \frac{m^2 H c^2}{r^3}. \quad (10)$$

So, if the water is at $r > r_{eq}$ or at $r < r_{eq}$, this QCM acceleration tries to move the water back to $r_{eq}$.

Water temperature differences are important. The surface water may be at a different temperature than the water below, so their density differences produce vertical mixing. Therefore, any water at the QCM equilibrium radius may move to a different radius value, with the radial velocity $v_r$ resulting in an acceleration in the $r$ direction. From Eq. 8, the $\phi$ acceleration

$$a_{\phi} = -\frac{mcH}{\ell^2}v_r. \quad (11)$$

Using the $m = 6$ and $m = 7$ parameters at the ACC, the expression becomes approximately

$$a_{\phi} \approx -0.003 v_r. \quad (12)$$

So both the sinking water and the rising water will experience a $\phi$ direction acceleration due to the QCM angular momentum per unit mass constraint, the accelerations depending upon the radial velocity directions and magnitudes.

These QCM forces and accelerations, when considered along with Coriolis forces and other influences, could be simulated on computer to determine their relative importance to the transport of the ACC.

Therefore, the estimated results of these QCM derivations suggest force and acceleration values strong enough to keep the ACC transport moving around the Antarctic continent, meaning that the ACC may be in a gravitational energy state dictated by the QCM quantization of angular momentum per unit mass constraint.

5 Conclusion

I have applied the QCM gravitational wave equation to the rotation of the Earth by utilizing both the total vector angular momentum of the Earth’s spin plus the larger value of the average angular momentum of the Moon in orbit. The QCM $\ell = 6$ state at $r_{eq} = 3.0 \times 10^6$ m intersects the Earth’s surface at 61.9ºS latitude, and the $\ell = 7$ state at $r_{eq} = 4.0 \times 10^6$ m intersects the Earth’s surface at 51.0ºS latitude. Both QCM cylinders intersect the surface in the wide latitude region where the ACC flows faster than the Earth’s rotation velocity by about 2 m/s.

The enormous QCM predicted velocity of about $1.1 \times 10^4$ m/s with respect to the stars is much larger than the actual ACC velocity of about 212 m/s. Viscosity effects on the water transport at all depths would need to be a significant opposing force to be able to reduce this QCM velocity to its actual velocity. Rough estimates of the pertinent forces suggest values on the order of $10^{-12}$ N to $10^{-9}$ N per kilogram are required.

Temperature differences produce mixing, which moves water away from the equilibrium radius measured from the rotation axis, resulting in an acceleration in both the $r$ direction and the $\phi$ direction. The QCM forces combined with the Coriolis force and other effects make for a complex transport of the ACC. However, a computer simulation that includes the QCM force driving the ACC would be necessary in order to evaluate the atmosphere and ocean behaviors in more detail.

Therefore, the QCM wave equation applied in the familiar Schwarzschild metric suggests that the true energy source for the ACC could be gravitational.

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References


