Explicit Values for Gravitational and Hubble Constants from Cosmological Entropy Bound and Alpha-Quantization of Particle Masses

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In this study, we propose a method to derive expressions and numerical values for the gravitational and Hubble constants employing the "reverse engineering" path approach. Using the explicit form of Bekenstein bound for the cosmological horizon, we show that Nambu's mass-formula (the empirical alpha-quantization of elementary particle masses) is related to the proposed 3-D analogy of the holographic principle. The obtained form in the "median case" leads to the expression for the Hubble constant with the value of $H_0 = 71.995$ km/s/Mpc. The accuracy of obtained H_0 allows precise numerical calculation of the cosmological entropy bound, which coincides with the number of $\left(216 \times 2^{128}\right)^3$ bits. Conversely, the number leads to the expression for the gravitational constant resulting in $G = 6.67437305 \times 10^{-11}$, which exactly fits into the CODATA2018 value and the AAFII(2018) measurement [32]. As a coincidence, the proposed approach combined with the previous formulation of the LNNC (Large Numerical Number Coincidences) [10] allows obtaining the numerical value for the proton-to-electron mass ratio μ with an accuracy of 10^{-6}.

Note: SI units are deployed.

1 Introduction

Dimensionless numbers, including the mass ratios of the elementary particles and large numbers introduced by Dirac [45], remain unresolved puzzles in physics. To understand the significance of large number relations, the constant H_0 and G must be precisely measured, and their deviations in time and space must be constrained. However, at present, we know the value of the Hubble constant with a precision of $< 10\%$ [35]. Today, those using Planck and cosmic background data to measure a value for the Hubble constant arrive at a figure of 67.4 ± 0.5. However, the local approach provides a figure of 73.5 ± 2 [41, 43]. In contrast, the gravitational constant G is known to have better precision; however, its value has a relative accuracy of 2×10^{-5} depending on the measurement methods performed.

This paper presents an attempt to connect the Bekenstein cosmological entropy bound with the alpha-quantization of elementary particle masses. As a result, the Dirac large numbers appear as an intrinsic property of the cosmological entropy bound, which allows us to obtain the numerical value and to express G and H_0.

In 1952, Nambu proposed an empirical formula for the mass spectrum of elementary particles, known as "alpha quantization" [1]

$$m_n \approx \frac{n}{2}\, \alpha^{-1}\, m_e \tag{1}$$

where n is a natural number, α is the fine structure constant, and m_e is the electron mass. The mass interval is predicted as a half-integral multiple of approximately 70 MeV. It provides the muon mass with $n = 3$, the pion mass for $n = 4$, and the proton mass for $n = 27$ *etc*. Currently, at least 21 fundamental particles with lifetimes $>10^{-24}$ s are covered by this rule, with deviations of less than 1% [9]. The alpha quantization of elementary particle masses is extensively reviewed in the modern literature [16–28]. In particular, it is valid, for example, for the heaviest known particle, the top quark for which $n = 137 \times 36$ [20]. The Nambu formula was derived empirically and did not have any theoretical background. However, along with the new approaches to elaborate it in the frame of modern models, there were a few almost forgotten attempts to refine the formula, for example, by Nambu in 1966 [2], Hermann [3], and later [36–39] extending the quantum oscillator model, which led to clarifications and more accurate results for the mass ratios of elementary particles.

2 Bekenstein entropy bound for cosmological horizon

The cosmological (Hubble or de Sitter) horizon corresponds to the radius and volume.

$$R_H = \frac{c}{H_0}\,, \quad V_H = \frac{4\pi}{3}\left(\frac{c}{H_0}\right)^3, \tag{2}$$

where H_0 denotes the Hubble constant. Because we are looking for the upper limit of entropy, we shall consider the entire mass-energy content of the universe with $\Omega_{Tot} = 1$. Therefore, the critical density $\rho_{cr} = 3H_0^2/8\pi G$ within the Hubble volume provides the mass-energy

$$E = V_H\, \rho_{cr}\, c^2 = \frac{c^5}{2GH_0}\,. \tag{3}$$

It is easy to see that in such a case (i.e. $\Omega_{Tot} = 1$), the cosmological horizon also coincides with the Schwarzschild black hole radius*. The Bekenstein entropy bound for the black hole is

$$S = \frac{\lambda R E}{\hbar c} = \frac{4\pi R E}{\hbar c}. \tag{4}$$

The original Bekenstein formula [30] was derived based on the general considerations for "an arbitrary system of effective radius R" and contains factor $\lambda = 2\pi$. Recently the factor was clarified [31]; it was explicitly shown that particularly in the application to the Schwarzschild black hole case the factor is $\lambda = 4\pi$, which is strictly derived based on the entropy associated with the Hartle-Hawking state. Since the cosmological horizon coincides with the Schwarzschild black hole radius, as shown, the expression (4) has $\lambda = 4\pi$. The substitution of R and E from (3) leads to the value of the upper bound for the entropy of the universe

$$S = 2\pi \frac{c^5}{\hbar G H_0^2}. \tag{5}$$

The number (measured as the number of bits†) is known by its order and is also referred to as the computational capacity of the universe [44]. Notably, the critical mass of the universe can be written in terms of the obtained expression for the total entropy:

$$M_U = \frac{c^3}{2 G H_0} = S \times m_0 , \quad m_0 := \frac{\hbar H_0}{4\pi c^2}. \tag{6}$$

Hence, the mass m_0 can be interpreted as the minimal possible quanta of the mass-energy carrying one bit of information.

Note on Oldershaw-Fedosin scaling of the Planck constant

Using (5), one may consider the "scaled" Planck constant $\hbar^*$ such that‡

$$\hbar^* = \frac{S}{2\pi}\hbar , \quad h^* = S\,\hbar . \tag{7}$$

The constant $\hbar^*$ plays the role of the reduced Planck constant in a multiverse, where our universe represents an elementary particle or a quantum oscillation [4,5]. The Heisenberg uncertainty relation, which is hypothetically valid in a multiverse, is then given as

$$\Delta E\, \Delta\left(\frac{1}{H_0}\right) \geq \frac{\hbar^*}{2}. \tag{8}$$

On the other hand, the substitution of (7) into the expression leads to the Bekenstein law, which bounds the entropy by corresponding the total energy and time $1/H_0$ (or radius c/H_0) for the universe. Such notable correspondence to the Heisenberg uncertainty relation for the cosmological case is possible when using the above-mentioned factor $\lambda = 4\pi$ for the cosmological case.

3 3-D analogue of Holographic Principle with the "cell of space volume"

The Bekenstein bound implies the holographic principle [29]. Applying it to the cosmological horizon, the Hubble area can be represented as

$$A_H = 4\pi \left(\frac{c}{H_0}\right)^2 = S \times A_{Pl} , \tag{9}$$

where A_{Pl} is the Planck area§ and S is the Bekenstein cosmological bound (5). As the Plank area plays the role of a 1-bit unit of the area, analogous to that we may also introduce "the cell of space volume" V_0 such that the total Hubble volume is

$$V_H = \frac{4\pi}{3}\left(\frac{c}{H_0}\right)^3 = S \times V_0 . \tag{10}$$

Thus, the introduced V_0 shall play a similar role for 3-D being a 1-bit unit for the volume, as the Planck area does for 2-D. The substitution of (5) leads to the explicit form

$$V_0 = \frac{V_H}{S} = \frac{2}{3}\frac{G\hbar}{H_0 c^2}. \tag{11}$$

This parameter V_0 was introduced in the author's previous work [12, 13]. The new parameter of the space volume cell V_0 may imply a different sense than *the grain of space* used in the loop quantum gravity (LQG) approach [46, 47], where the grain of space is considered to be of the order of Planck length l_{Pl}^3. In contrast, the volume cell V_0 is of the order of the cube of the reduced Compton wavelength of an elementary particle. Simultaneously, similar constraints are given for the V_0-dependent uncertainty relation in the LQG approach [47].

4 The Nambu formula for alpha-quantization of particle masses

The V_0-dependent uncertainty relation is:

$$\left(\frac{1}{2}\frac{\hbar}{mc}\right)^3 \geq V_0 . \tag{12}$$

Based on that, one may consider the quantization of elementary particle masses (1) as a classic quantum harmonic oscillator [40]. The particles' rest masses correspond to the oscillator eigenstates

$$E_n = m_n c^2 = \frac{n}{2}\hbar\omega , \quad \omega = \frac{c}{L},$$

where $L = V_0^{1/3}$ is the characteristic length of the oscillator, and n is a natural number for both parity cases with non-zero

*Since $R_S = (2G/c^2)(c^3/2GH_0)$.

†The entropy S is the number of states, the exact number of the Plank areas in covering area when using the holographic principle (9). Hence, factor ln(2) in the Bekenstein expression to obtain the number of bits, which appears in many textbooks is highly arguable and shall not be used.

‡Here it would be natural to introduce the "reduced" $\bar{S} = S/2\pi$ such that (7) takes the simpler form $\hbar^* = \bar{S}\hbar$, $h^* = \bar{S}\,h$.

§In such a way, the Plank area acquires a prefactor of two as $A_{Pl} = 2G\hbar/c^3$.

ground state ($n = 1$). Therefore, for elementary particles with mass m_n, the following condition holds:

$$\frac{n}{2}\,\frac{\hbar}{m_n c} = V_0^{1/3}\,. \tag{13}$$

The substitution of (11) for V_0 leads to particle masses that satisfy the above requirement

$$m_n = \frac{n}{2}\left(\frac{3}{2}\,\frac{\hbar^2 H_0}{Gc}\right)^{1/3}\,. \tag{14}$$

By direct calculation, it can be noted that

$$\left(\frac{3}{2}\,\frac{\hbar^2 H_0}{Gc}\right)^{1/3} \approx 137\, m_e\,,$$

where m_e is the mass of an electron. Thus, the obtained expression (14) represents Nambu's original mass formula (1), which is now related to the Bekenstein cosmological bound. The exact match to the factor to α^{-1} is achieved when the Hubble constant is $H_0 = 71.9949$ km/s/Mpc, as reviewed in the next section.

5 The Hubble constant, the Universe entropy number and *G* in the "median" case

Considering the "median case" or the "ideal" case when the exact equality in (14) holds as

$$\left(\frac{3}{2}\,\frac{\hbar^2 H_0}{Gc}\right)^{1/3} = \alpha^{-1}\, m_e\,, \tag{15}$$

it becomes possible to express the Hubble constant via better known G as

$$H_0 = \frac{2}{3}\,\frac{G\,c\,\alpha^{-3}\,m_e^3}{\hbar^2} = \frac{2}{3\alpha}\,\frac{Gm_e}{r_e^2\,c}\,, \quad \text{where } r_e = \frac{ke^2}{m_e c^2}\,, \tag{16}$$

which results in 71.9949 km/s/Mpc or 2.333×10^{-18} s^{-1} when using CODATA2018 for G. Substituting H_0 into (11) yields

$$V_0 = \left(\frac{ke^2}{m_e c^2}\right)^3 = r_e^3\,. \tag{17}$$

Furthermore, the substitution of the obtained H_0 (16) into (5) yields an explicit value for the universe total entropy bound:

$$S = \frac{4\pi}{3}\left(\frac{ke^2}{Gm_e^2}\,\frac{3\alpha}{2}\right)^3\,. \tag{18}$$

The obtained expression allows the accurate calculation of the value as 3.9711×10^{122} till the 5th digit (corresponding to the accuracy of G). Moreover, because we expect the entropy S to be a *natural number* (number of bits of information), and as binary, it most probably should contain powers of 2. The search leads to the number that represents the cosmological entropy bound as a factor of two first primes

$$S = 3^9 \times 2^{393} = \left(216\times 2^{128}\right)^3\,. \tag{19}$$

Remarkably, the found number appears to be the cube of a natural number. The number provides a sufficient relative accuracy of 3×10^{-5} with (18) corresponding to the accuracy of G (see Section 8 for a more detailed discussion on this number). Furthermore, the reverse substitution of the number to (18) allows us to express the gravitational constant:

$$G = \frac{ke^2}{m_e^2}\left(\frac{3\alpha}{2}\right)\left(\frac{4\pi}{3S}\right)^{1/3} = \left(\frac{4\pi}{3}\right)^{1/3}\frac{\alpha}{144}\,\frac{ke^2}{m_e^2}\,2^{-128} \tag{20}$$

resulting in $G = 6.67437305\times10^{-11}$. This value perfectly fits the value of CODATA2018 for G. The obtained value also coincides with the AAFII(2018) measurement of 6.674375(82) performed with very high precision [32]. Moreover, the use of the obtained G in (16) results in the expression for the Hubble constant

$$H_0 = \frac{c}{r_e}\left[\left(\frac{4\pi}{3}\right)^{1/3}\frac{1}{216}\right]2^{-128}\,, \tag{21}$$

where r_e is the classical electron radius (16). Notably, to satisfy the equality to α in (15), the expressions acquire the factor given in square brackets. Denoting this factor as $\alpha_s = 1/133.995..$ (or "alpha-substitute"), both expressions can be written in the simpler form

$$H_0 = \alpha_s\,\frac{c}{r_e}\,2^{-128}\,, \quad G = \alpha_s\,\alpha\,\frac{3}{2}\,\frac{ke^2}{m_e^2}\,2^{-128}\,, \tag{22}$$

where

$$\alpha_s := \left(\frac{4\pi}{3}\right)^{1/3}\frac{1}{216}\,.$$

The significance of this parameter is reviewed further.

6 Proton to electron mass ratio from deviated *G* and H_0

We have considered the "median" or ideal case of exact equality to α in the extended Nambu's mass formula (14). In a real-life scenario, the masses of the elementary particles deviate from the median values by ±1%. There are two alternative ways to refine the Nambu mass formula to obtain more accurate masses for elementary particles. The first approach, as mentioned in Section 1, clarifies the quantum oscillator model. This leads to the appearance of eigenvalues or zeros of some functions instead of the natural number n in (1). The second alternative is to introduce the deviation of G and H_0 in the mass formula (14), which would also lead to non-constancy of V_0 and deviated states of the entropy from S depending on the nature of the particle. The first method appears to be preferable and requires further studies using QM.

However, in this section, we evaluate the second "heuristic" alternative.

One may recall the previously proposed expressions for G and the Hubble constants (LNNC) [10]. Denoting them with a prime (′) to distinguish them from the obtained "median" values, they are

$$H_0' = \frac{m_e c^2}{\hbar}\, 2^{-128}\,, \quad G' = \frac{3}{20}\, \frac{ke^2}{m_e m_p}\, 2^{-128}\,, \tag{23}$$

where m_p is the proton mass. The value for $H_0' = 70.39882$ km/s/Mpc differs by 2%* from the "median" case of H_0 (16). It is evident that the formula for H_0' can be treated as derived from the expression for H_0 (22), wherein the latter, α_s is simply *substituted* by α.

The value of the gravitational constant is $G' = 6.6746305 \times 10^{-11}$, which deviates from the median G (22) by 3×10^{-5}, and is closer to the AAF-I and AAF-III measurements [32]. It is evident that these values (23) do not provide the equality to "alpha" in mass-formula (15); however remarkably H_0' and G' being substituted into (14) with $n = 27$ provide a good approximation of the proton mass, thus the ratio becomes

$$\mu = \frac{m_p}{m_e} = \left(\frac{5}{4}\, 3^9\, \alpha^{-1}\right)^{1/2}. \tag{24}$$

Moreover, it can be seen that both suggested formulas for the gravitational constant have relative deviations of 3×10^{-5}, equating G from (23) and (22) gives

$$\alpha^{-1} = \left(\frac{4\pi}{3}\right)^{1/3} \frac{5\mu}{108}\,, \tag{25}$$

where we expect the same relative error of 10^{-5}. The substitution of μ from (24) leads to

$$\alpha^{-1} = \left(\frac{4\pi}{3}\right)^{2/3} \left(\frac{15}{4}\right)^{3} = \mathbf{137.03}12258\,, \tag{26}$$

and substituting it again to (24) results in

$$\mu = \left(\frac{4\pi}{3}\right)^{1/3} \left(\frac{135}{4}\right)^{2} = \mathbf{1836.15}959\,, \tag{27}$$

which has a relative accuracy with the experimental value of the proton-to-electron ratio of 3×10^{-6}. The remarkable property of both expressions is their simple forms that involve powers of the first three primes as $15 = 5\times3$ and $135 = 3^3\times5$. The expression for μ can be assumed to be the best in terms of the precision-simplicity ratio (see [11] to see the complication level of formulae with comparable accuracy for μ). The expressions can also be rewritten in the following forms:

$$\mu = \frac{27}{2}\,(135\,\phi)\,, \quad \alpha^{-1} = 135\,\phi^2\,, \tag{28}$$

where

$$\phi := \frac{5}{8}\left(\frac{4\pi}{3}\right)^{1/3},$$

and the "alpha-substitute" is explicitly $\alpha_s^{-1} = 135\,\phi^{-1}$. Thus, the formula for μ restores the original form of Nambu's mass formula with $n = 27$. Hence, the factor $\phi \approx 1.0075$ plays the role of a small deviation and exhibits a deviation of α in Nambu's mass formula. Simultaneously, ϕ^2 shows how α deviates from an integer of 135. The deviation $\phi^3 \approx 1.02$ also provides the explicit ratios of the two values for the Hubble constants given by the expressions (21) and (23) as this deviation is given by the ratio of "alpha-substitute" in (21) to the exact "alpha" in (23).

7 Quantum number of the Universe and Eddington's number of particles

The paper would not be complete without reviewing the Eddington number of particles (pairs of protons and electrons), which he assumed to be $N = 2 \times 136 \times 2^{256}$ [6]. In Section 5, we review the number for the Bekenstein entropy S, which is also expressed by the power of 2 (19). Prior to the calculation of the Eddington number of particles, we calculate the n-number using the obtained mass formula (14) applied to the *entire* universe mass with $\Omega = 1$

$$M_u = \frac{c^3}{2GH_0} = \frac{n_u}{2}\left(\frac{3}{2}\,\frac{\hbar^2 H_0}{Gc}\right)^{1/3}. \tag{29}$$

Because we are applying it to "the median" case, it is clear that $M_u = (n_u/2)(\alpha^{-1} m_e)$. Using the obtained values for G and H_0 (22) after a few manipulations, the number becomes

$$n_u = \frac{2}{3}\left(\frac{3}{4\pi}\, S\right)^{2/3} = \frac{2}{3}\,\alpha_s^{-2}\, 2^{256}\,. \tag{30}$$

Using this number, it is evident that the Eddington number of protons can be expressed as

$$N = \frac{M_u \Omega_{M_p}}{m_p} = \Omega_{M_p}\,\frac{n_u}{2}\,\frac{\alpha^{-1}}{\mu}\,, \tag{31}$$

where Ω_{M_p} is the proton content of the universe. The obtained good approximations for α and μ of (28) provide the ratio

$$\frac{\alpha^{-1}}{\mu} = \frac{2}{27}\phi\,,$$

and substituting n from (30) results in the number of protons in the universe

$$N = \Omega_{M_p}\,\frac{10}{3}\,\alpha_s^{-1}\, 2^{256}\,, \tag{32}$$

where the second power of α_s^{-1} decreases with ϕ. This expression is fairly close to the famous Eddington number. However, the difference is that it contains the prefactor, and "alpha-substitute" ($\approx$ 134) instead of 136 in Eddington's expression.

*The ratio for the deviation is $(\alpha^{-2}/10\mu) = (4\pi/3)((5/8)^3)$, as can be seen later.

8 Discussion

In Section 5, we proposed the numerical value for the "median" cosmological entropy bound as the number of bits, which explicitly equals $(216 \times 2^{128})^3$. The number limits the upper bound of the informational capacity of the universe according to the Bekenstein law. When searching for the numbers of order 10^{122} with a relative accuracy of 10^{-5}, one must observe that there exist 10^{117} alternative natural numbers to choose between. Another good fit, for example, can be given by

$$137 \times \frac{81!}{2} \approx 3.971031 \times 10^{122}\,\text{bits}\,.$$

Notably, the number represents the order of the alternating group $A(81)$ with prefactor 137, which can be considered as a coincidence. However, the key advantage of (19) compared to other alternatives is that it is the simplest number composed of the product of only the first two primes. Second, it represents the cube of another natural number, which reveals its significance during the calculations. Moreover, the number (19) can also be represented by the Mersenne prime $M_{127} = 2^{127} - 1$, where M_{127} has the unique property of being the double Mersenne prime and fourth Catalan-Mersenne number* discovered by Catalan [34]. Hence, $S = (432 \times M_{127})^3$, which possibly connects the median entropy to the cyclic group† $\mathbb{Z}_{M_{127}}$.

Despite the presence of a power of two, the proposed number differs from the Eddington E-numbers [7, 8]. However, further study is required for a possible connection of the proposed number to the Clifford algebras and the finite groups of Lie type [48].

9 Conclusions

In 1935, Heisenberg [42] suggested using the number 432 to calculate the fine structure constant as $\alpha^{-1} = 432/\pi$. The paper has demonstrated that number 432 and its derivatives (108, 216) appear in the "median" or symmetric case of universe entropy bound, and further in the calculation for the dimensionless numbers (25), (19). An intriguing numerical expression for the total universe entropy for the Bekenstein cosmological bound is proposed (19), which contains only powers of 2 and 3. This allowed to construct an expression for the gravitational constant that gives a value of $G = 6.67437294 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$, which fits the range of CODATA2018 to the latest measurements. Along with the previous formulation for the Hubble constant, the approach provides a new alternative form (greater by 2% from the previous) as given by expressions (21), (16), resulting in $H_0 = 71.994$ km/s/Mpc, which corresponds to the "median" case of the universe entropy bound (19). The current accuracy of measurements of the Hubble constant H_0 limits the study. Further improvements in the measurements of the Hubble constant are required, as it will clarify the concordance of the value of the cosmological entropy bound S to the proposed number.

The paper proposes a path, using the explicit value for Bekenstein bound, to connect the maximum of the observable values such as the Hubble volume, the total mass of the universe with minimal measurable values (V_0 and m_0), which supposedly have to play a role in quantum mechanics. The approach can be extended towards a broader range of physical parameters, such as maximal and minimal force, maximal and minimal acceleration. The introduction of such parameters may lead to new approaches in quantum mechanics and cosmology. Further research is required in the frame of quantum mechanics, the LQG, which would utilize the introduced space volume V_0 parameter in connection to Clifford algebra $Cl_{3,0}$ (APS), where it has the correspondence to volume coordinate x_{123} [14]. Such a study may further refine the quantum oscillator model of elementary particle masses using the introduced parameters.

Received on July 9, 2021

*Since $127 = 2^7 - 1$, $7 = 2^3 - 1$, and $3 = 2^2 - 1$.

†Interestingly; this number also nearly coincides with the order of symplectic group Sp(n, q) with $q = 2^{43}$, $n = 1$ with prefactor 108, and the same for the Chevalley group $A_n(q)$ (PSL(n, q)).

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