

Notes on Einstein's Gravitation

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Schwarzschild Class of Solutions

The Schwarzschild class of solutions are developed from a generalisation of Minkowski spacetime [1], where the latter is given by

$$ds^2 = dt^2 - dr^2 - |r - r_0|^2(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (1)$$

$$0 \leq |r - r_0| < \infty,$$

although Minkowski space appears in the literature by the following line element,

$$ds^2 = dt^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (2)$$

$$0 \leq r < \infty,$$

wherein $r_0 = 0$.

The generalisation of the Minkowski line element has the form,

$$ds^2 = A(\sqrt{C(r)}) dt^2 - B(\sqrt{C(r)}) d\sqrt{C(r)}^2 - C(r)(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (3)$$

$$C(r) \equiv C(|r - r_0|),$$

where $A(\sqrt{C(r)})$, $B(\sqrt{C(r)})$ and $C(r)$ are *a priori* unknown positive-valued analytic functions that must be determined by the intrinsic geometry of the line element and associated boundary conditions, satisfying the condition $R_{\mu\nu} = 0$. The function $\sqrt{C(r)} = R_c(r)$ is the radius of curvature. Using expression (3) in Einstein's field equations gives the following general line element in terms one unknown analytic function,

$$ds^2 = \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right) dt^2 - \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right)^{-1} d\sqrt{C(r)}^2 - C(r)(d\theta^2 + \sin^2 \theta d\varphi^2). \quad (4)$$

The admissible form of $C(r)$ that satisfies the intrinsic geometry of the line element and the required boundary conditions has been previously deduced [1], and is given by

$$\sqrt{C(r)} = R_c(r) = (|r - r_0|^n + \alpha^n)^{\frac{1}{n}}, \quad (5)$$

$$r \in \Re, \quad n \in \Re^+, \quad r \neq r_0,$$

where r_0 and n are *entirely arbitrary constants*, and α is a constant that depends upon the mass of the source of the gravitational field. Metric (4) is well defined on $-\infty < r < r_0 < r < \infty$, and has no singularity other than at $r = r_0$. Thus, there is no such thing as a black hole.

The metric appearing in the bulk of the literature under the name of the ‘‘Schwarzschild’’ solution¹, is *not* Schwarzschild’s solution, and is obtained from (4) and (5) by choosing $n = 1$, $r_0 = \alpha$, $r > r_0$. It should be noted that according to (5) the actual radius of curvature for the usual line element is $R_c = (r - \alpha) + \alpha$, so that α drops out of the expression for the radius of curvature, but that does not mean that R_c can then go down to zero. R_c must always obey expression (5), which generates it. There is no possibility of the so-called ‘‘black hole’’. It is the standard but erroneous *assumption* that R_c can go down to zero in the usual line element that has spawned the (fallacious) concept of the black hole. Schwarzschild’s true solution [3], although not well known, is obtained by choosing $n = 3$, $r_0 = 0$, $r > r_0$. Schwarzschild’s actual solution is well-defined on $0 < r < \infty$, and does not admit of a black hole. Schwarzschild in fact, never made any claims in relation to what has been called a black hole, notwithstanding it being so frequently attributed to him in the literature [4].

The infinite number of metrics obtained via (4) and (5) are equivalent, and so anything proved for any one of them necessarily holds for all of them. The simplest Schwarzschild-class metric is Brillouin’s solution [5, 6], obtained by choosing $n = 1$, $r_0 = 0$, $r > r_0$, giving the line element,

$$ds^2 = \left(1 - \frac{\alpha}{r + \alpha}\right) dt^2 - \left(1 - \frac{\alpha}{r + \alpha}\right)^{-1} dr^2 - (r + \alpha)^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (6)$$

$$0 < r < \infty.$$

The requirement that a solution for Einstein’s static vacuum field must admit of an infinite series of equivalent metrics was pointed out by Eddington as long ago as 1923 [7].

Although the components of the metric tensor of (4), based upon the supposition of the line element (3), are determined by the field equations, the admissible form of the *a priori unknown* $C(r)$ is *not* determined by the field equations. It is determined by the intrinsic geometry of the line element, already fixed in (3) by the form of the line element for Minkowski spacetime itself, and the required boundary conditions. This illustrates that satisfaction of the field equations is a necessary but insufficient condition for a model of Einstein’s gravitational field. Indeed, one can substitute into (4) *any* analytic function for $C(r)$ without disturbing the spherical symmetry of the line element and without violating the field equations. However, not simply any analytic function will produce a meaningful model of Einstein’s gravitational field. For example, setting $C(r) = \exp(2r)$ produces a line element that is spherically symmetric and satisfies $R_{\mu\nu} = 0$, but it does not describe a model of Einstein’s gravitational field. To begin with, the resulting line element is not asymptotically Minkowski, and that is sufficient to invalidate the relevant line element as a model of Einstein’s gravitational field, notwithstanding its satisfaction of $R_{\mu\nu} = 0$ and its spherical symmetry.

The fundamental error in the usual analysis of spherically symmetric line elements of a Type 1 Einstein Space, has been its failure to apprehend the geometric fact that there is a distinction between the radius of curvature $R_c(r)$ and the geodesic proper radius $R_p(r)$ in a general spherically symmetric metric space such as that for Einstein’s gravitational field. In Minkowski space, $R_c(r)$ and $R_p(r)$ are identical, owing to the pseudo-Euclidean²

¹The first and correct form of this line element was in fact derived by Johannes Droste in 1916 [2].

²Actually, pseudo-Euclidean, after the geometry of Euclidean spaces.

nature of Minkowski space. But Einstein's gravitational field is non-Euclidean (it is a pseudo-Riemannian metric manifold), and so the familiar Euclidean relations do not apply. Nonetheless, the intrinsic geometry of the line element of Einstein's gravitational field for a spherically symmetric Type 1 Einstein Space is precisely the same as the line element for Minkowski space. In both cases the geodesic proper radius is given by the integral of the square root of the component of the line element that contains the square of the differential element of the radius of curvature and the radius of curvature is the square root of the coefficient of the collected infinitesimal angular terms. It is common in the literature to find the radius of curvature referred to as an "areal" radius, or in a certain case as a "Schwarzschild" radius. However, it is in fact *the radius of curvature*, owing to its formal geometric relationship to the Gaussian curvature [8, 9], and it does not determine the radial geodesic distance from the source of the gravitational field. In the case of (2), $R_c(r) = r$ and

$$R_p(r) = \int_0^r dr = r = R_c(r).$$

However, in the case of (4),

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(r_0)}^{R_c(r)} \sqrt{B(R_c(r))} dR_c(r) = \int_{r_0}^r \sqrt{B(R_c(r))} \frac{dR_c(r)}{dr} dr,$$

where $R_c(r_0)$ is *a priori unknown* owing to the fact that $R_c(r)$ is *a priori unknown*. One cannot simply assume that because $0 \leq r < \infty$ in (2) that it must follow that in (4) $0 \leq R_c(r) < \infty$. In other words, one cannot simply *assume* that $\sqrt{C(r_0)} = R_c(r_0) = 0$. In (4) and (5) the quantity r is just a parameter, and the radius of curvature and the proper radius are not the same in general. Furthermore, according to (4) and (5), $R_c(r_0) = \alpha$ and $R_p(r_0) = 0, \forall r_0$ [1, 8].

For the sake of completeness, a similar analysis extends the foregoing results to encompass the Reissner-Nordstrom, Kerr and Kerr-Newman configurations. The generalised line element, in Boyer Lindquist coordinates, is given by [10],

$$ds^2 = \frac{\Delta}{\rho^2} (dt - a \sin^2 \theta d\varphi)^2 - \frac{\sin^2 \theta}{\rho^2} [(R_c^2 + a^2) d\varphi - a dt]^2 - \frac{\rho^2}{\Delta} dR_c^2 - \rho^2 d\theta^2,$$

$$R_c = R_c(r) = \left(|r - r_0|^n + \beta^n \right)^{\frac{1}{n}}, \quad \beta = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - (q^2 + a \cos^2 \theta)},$$

$$a = \frac{2L}{\alpha}, \quad \rho^2 = R_c^2 + a^2 \cos^2 \theta, \quad a^2 + q^2 < \frac{\alpha^2}{4}, \quad \Delta = R_c^2 - \alpha R_c + q^2 + a^2,$$

$$r \in \mathfrak{R}, \quad n \in \mathfrak{R}^+,$$

wherein L is the angular momentum, q is the charge, and the constants r_0 and n are entirely arbitrary. In can be seen that when the charge $q = 0$, the Kerr configuration class is recovered; when the angular momentum is zero, the Reissner-Nordstrom configuration class is recovered; and when the charge and the angular momentum are both zero, the Schwarzschild class of solutions is recovered. In no case is a black hole possible. In all cases an infinite number of equivalent metrics is obtained.

It must be emphasized that the Schwarzschild class of solutions, and also the Reissner-Nordstrom, Kerr and Kerr-Newman line elements, all describe the source of the gravitational field in terms of a *centre of mass*, and so the lone singularity that occurs in these line elements has *no physical significance*.

A full description of Einstein's gravitational field for $R_{\mu\nu} = 0$ requires *two* line elements – one for the voluminous interior of the source of the field and one for the region outside the source, the latter being a centre of mass description, such as one from the Schwarzschild class of solutions. This is illustrated further by the class of solutions for the idealised case of a sphere of homogeneous incompressible fluid.

The Homogeneous Incompressible Sphere of Fluid

Schwarzschild obtained, in 1916, a solution for a sphere of homogeneous incompressible fluid [11]. His solution has been generalised for an infinite class of equivalent metrics [12]. These solutions demonstrate that there is an upper bound and a lower bound on the size of a sphere of homogeneous incompressible fluid that can exist.

The generalised Schwarzschild line element for this configuration is [12],

$$ds^2 = \left[\frac{3 \cos |\chi_a - \chi_0| - \cos |\chi - \chi_0|}{2} \right]^2 dt^2 - \frac{3}{\kappa \rho_0} d\chi^2 - \frac{3 \sin^2 |\chi - \chi_0|}{\kappa \rho_0} (d\theta^2 + \sin^2 \theta d\varphi^2),$$

$$\sin |\chi - \chi_0| = \sqrt{\frac{\kappa \rho_0}{3}} \eta^{\frac{1}{3}}, \quad \eta = |r - r_0| + \rho, \quad \kappa = 8\pi k^2,$$

$$\rho = \left(\frac{\kappa \rho_0}{3} \right)^{-\frac{3}{2}} \left\{ \frac{3}{2} \sin^3 |\chi_a - \chi_0| - \frac{9}{4} \cos |\chi_a - \chi_0| \left[|\chi_a - \chi_0| - \frac{1}{2} \sin 2|\chi_a - \chi_0| \right] \right\},$$

$$r \in \mathfrak{R}, \quad \chi \in \mathfrak{R},$$

$$0 \leq |\chi - \chi_0| \leq |\chi_a - \chi_0| < \frac{\pi}{2},$$

where ρ_0 is the constant density of the sphere of fluid, the subscript a denotes values at the surface of the fluid sphere, k^2 is Gauss' gravitational constant, χ_0 denotes the arbitrary location of the centre of spherical symmetry of the sphere in the gravitational field, and r_0 the arbitrary parametric location of the centre of spherical symmetry. Schwarzschild's solution is recovered by choosing $\chi_0 = 0$, $r_0 = 0$, $\chi \geq 0$ and $r \geq 0$. The foregoing line element is non-singular.

Outside the sphere of fluid, where the sphere is described in terms of its *centre of mass*, the Schwarzschild class of line elements for $R_{\mu\nu} = 0$ is affected by the distribution of mass of the sphere of fluid, and becomes

$$ds^2 = \left(1 - \frac{\alpha}{R_c} \right) dt^2 - \left(1 - \frac{\alpha}{R_c} \right)^{-1} dR_c^2 - R_c^2 (d\theta^2 + \sin^2 \theta d\varphi^2),$$

$$R_c = (|r - r_0|^n + \epsilon^n)^{\frac{1}{n}}, \quad \alpha = \sqrt{\frac{3}{\kappa \rho_0}} \sin^3 |\chi_a - \chi_0|,$$

$$\epsilon = \sqrt{\frac{3}{\kappa\rho_0}} \left\{ \frac{3}{2} \sin^3 |\chi_a - \chi_0| - \frac{9}{4} \cos |\chi_a - \chi_0| \left[|\chi_a - \chi_0| - \frac{1}{2} \sin 2|\chi_a - \chi_0| \right] \right\}^{\frac{1}{3}},$$

$$r \in \mathfrak{R}, \quad 0 < |\chi_a - \chi_0| < \frac{\pi}{2}, \quad 0 < |r_a - r_0| < \infty,$$

where n , χ_0 and r_0 are entirely arbitrary constants. Schwarzschild's original solution for the region outside the sphere of fluid is recovered by choosing $n = 3$, $r_0 = 0$, $\chi_0 = 0$, $r > r_0$ and $\chi_a > \chi_0$.

It is clear from this solution that the constant α appearing in the Schwarzschild class of solutions is not the Newtonian mass. Only in a very weak field, such as that of the Sun is α approximately the Newtonian mass, but that mass is not assigned by comparison to a far field Newtonian potential, as is usually done. It is determined by calculation using quantities associated with the line element for the interior of the source of the gravitational field in all cases, be they weak or strong fields. In the former case the result differs little from the Newtonian value, but in strong fields the difference becomes significant. Furthermore, there are two masses to consider: the passive mass (substantial mass), as determined by the line element for the interior of the source of the gravitational field, and the active mass (gravitational mass) as determined for the line element for the region exterior to the source of the field but which is still obtained from an expression determined by quantities for the field inside the source of the gravitational field. These masses are not the same - the passive mass is greater than the active mass.

The passive mass of the sphere of fluid is determined by the line element for the interior of the sphere, by multiplying the constant density of the sphere into the volume V of the sphere, and is given by

$$M = \rho_0 V = \rho_0 \left(\frac{3}{\kappa\rho_0} \right)^{\frac{3}{2}} \int_{\chi_0}^{\chi_a} \sin^2 |\chi - \chi_0| \frac{(\chi - \chi_0)}{|\chi - \chi_0|} d\chi \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\varphi$$

$$= 2\pi\rho_0 \left(\frac{3}{\kappa\rho_0} \right)^{\frac{3}{2}} \left(|\chi_a - \chi_0| - \frac{1}{2} \sin 2|\chi_a - \chi_0| \right).$$

The active mass of the sphere is given by $2m = \frac{\alpha}{k^2}$, i.e.

$$m = \frac{\alpha}{2k^2} = \frac{1}{2k^2} \sqrt{\frac{3}{\kappa\rho_0}} \sin^3 |\chi_a - \chi_0|.$$

The ratio of the active to passive mass is,

$$\frac{m}{M} = \frac{2 \sin^3 |\chi_a - \chi_0|}{3 \left(|\chi_a - \chi_0| - \frac{1}{2} \sin 2|\chi_a - \chi_0| \right)}.$$

The escape velocity for the sphere of fluid is given by $v_a = \sin |\chi_a - \chi_0|$. Thus, as the escape velocity increases, the ratio $\frac{m}{M}$ decreases owing to the increase in the mass concentration.

In addition, the proper radius of the sphere of fluid can only be determined from the line elements for its interior [12]. The line elements for the region outside the source of the field can say nothing about the proper radius of the source of the field. This is not surprising,

since the line element for the region beyond the surface of the sphere of fluid describes the sphere in terms of its *centre of mass*, and as such treats the source as a point-mass, which has no extension.

Cosmological Models

A similar fundamental situation arises in the case of an Einstein cosmology. In the case of the FLRW model, for example, there is a line element containing an *a priori unknown* analytic function $\exp(g(t))$. This line element satisfies the field equations, but that does not of itself mean that it yields a valid cosmological model. The form of $\exp(g(t))$ must be determined by the intrinsic geometry of the line element and the boundary conditions. It is *not* determined by the field equations. One must demonstrate that there exists some $\exp(g(t))$ for the FLRW line element before any meaning can be given to it as an Einstein cosmological model. Therefore, any analysis that proceeds by utilising the a priori unknown function $\exp(g(t))$ may well be invalid since it has not been determined beforehand if $\exp(g(t))$ admits of a suitable form for an Einstein cosmological model. Now it has been shown [13] that $\exp(g(t))$ has only one form meeting the required boundary conditions. In fact, the intrinsic geometry of the FLRW line element implies, with the necessary boundary conditions, an infinite and unbounded Universe. From where does this infinity come? Precisely from $\exp(g(t))$, so that $\exp(g(t))$ is infinite for all values of t . In other words, the FLRW line element modelling an Einstein cosmology is actually independent of time. Therefore, any analysis that proceeds by treating $\exp(g(t))$ in the FLRW line element as finite at any given time, insofar as it is alleged to model an Einstein cosmology, must fail. The Standard Cosmological Model (Big Bang) has failed to correctly consider the intrinsic geometry of the line element and the boundary conditions on $\exp(g(t))$, and so it is invalid. It has merely been *assumed* in the Standard Cosmological Model that $\exp(g(t))$ can be well-defined, never proving that $\exp(g(t))$ has an admissible well-defined form.

The FLRW line element is based upon the assertion that it is possible to express the spherically symmetric line element most generally in co-moving coordinates as [14]

$$ds^2 = e^\nu dt^2 + 2adr dt - e^\lambda dr^2 - e^\mu (r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2),$$

wherein ν, λ and μ are functions of the variables r and t . Then, by a series of transformations and use of the field equations, the FLRW line element is obtained:

$$ds^2 = dt^2 - \frac{e^{g(t)}}{\left[1 + \frac{k}{4}r^2\right]^2} (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2), \quad (7)$$

where k is a constant. Note that the field equations have *not* determined a form for $g(t)$. This must be determined from the intrinsic geometry of the line element and relevant boundary conditions. A question to be answered therefore is whether or not the intrinsic geometry and boundary conditions admit of a form for $g(t)$ that relates to an Einstein cosmological model. Furthermore, the range on the parameter r must also be determined from the intrinsic geometry of the line element and the boundary conditions. One cannot merely *assume* that in (7), $0 \leq r < \infty$. Indeed, the assumption is also demonstrably false.

Since a geometry is entirely determined by the *form* of its line element [14], everything must be determined from it. One cannot, as is usually done, merely foist assumptions upon it.

The intrinsic geometry of the line element and the consequent geometrical relations between the components of the metric tensor and associated boundary conditions determine all.

In (7) the quantity r is not a radial geodesic distance. It is not even a radius of curvature on (7). It is merely a parameter for the radius of curvature and the proper radius, both of which are well-defined by the *form* of the line element (describing a spherically symmetric metric manifold). The radius of curvature, R_c , for (7), is

$$R_c = e^{\frac{1}{2}g(t)} \frac{r}{1 + \frac{k}{4}r^2}. \quad (8)$$

The proper radius is

$$R_p = e^{\frac{1}{2}g(t)} \int \frac{dr}{1 + \frac{k}{4}r^2} = \frac{2e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left(\arctan \frac{\sqrt{k}}{2}r + n\pi \right), \quad n = 0, 1, 2, \dots \quad (9)$$

Since $R_p \geq 0$ by definition, $R_p = 0$ is satisfied when $r = 0 = n$. So $r = 0$ is the lower bound on r . The upper bound on r must also be ascertained from the line element and boundary conditions.

It is noted that the spatial component of (8) has a maximum of $\frac{1}{\sqrt{k}}$ at any time t , when $r = \frac{2}{\sqrt{k}}$. Thus, as $r \rightarrow \infty$, the spatial component of R_c runs from 0 (at $r = 0$) to the maximum $\frac{1}{\sqrt{k}}$ (at $r = \frac{2}{\sqrt{k}}$), then back to 0, since

$$\lim_{r \rightarrow \infty} \frac{r}{1 + \frac{k}{4}r^2} = 0.$$

Transform (7) by setting

$$R = R(r) = \frac{r}{1 + \frac{k}{4}r^2}, \quad (10)$$

which carries (7) into

$$ds^2 = dt^2 - e^{g(t)} \left[\frac{dR^2}{1 - kR^2} + R^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right]. \quad (11)$$

The quantity R appearing in (11) is not a radial geodesic distance. It is only a component of the radius of curvature in that it relates to the Gaussian curvature $G = \frac{1}{e^{g(t)}R^2}$. The radius of curvature of (11) is

$$R_c = \frac{1}{\sqrt{G}} = e^{\frac{1}{2}g(t)} R, \quad (12)$$

and the proper radius of Einstein's universe is, by (11),

$$R_p = e^{\frac{1}{2}g(t)} \int \frac{dR}{1 - kR^2} = \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left(\arcsin \sqrt{k}R + 2m\pi \right), \quad m = 0, 1, 2, \dots \quad (13)$$

Now according to (10), the minimum value of R is $R(r = 0) = 0$. Also, according to (10), the maximum value R is $R(r = \frac{2}{\sqrt{k}}) = \frac{1}{\sqrt{k}}$. $R = \frac{1}{\sqrt{k}}$ makes (11) singular, although (7) is not

singular at $r = \frac{2}{\sqrt{k}}$. Since by (10), $r \rightarrow \infty \Rightarrow R(r) \rightarrow 0$, then if $0 \leq r < \infty$ on (7), it follows that the proper radius of Einstein's universe is, according to (10),

$$R_p = e^{\frac{1}{2}g(t)} \int_0^{\infty} \frac{dR}{1 - kR^2} \equiv 0. \quad (14)$$

Therefore, $0 \leq r < \infty$ on (7) is false. Furthermore, since the proper radius of Einstein's universe cannot be zero and cannot depend upon a set of coordinates (it must be an invariant), expressions (9) and (13) must agree. Similarly, the radius of curvature of Einstein's universe must be an invariant (independent of a set of coordinates), so expressions (8) and (12) must also agree, in which case $0 \leq R < \frac{1}{\sqrt{k}}$ and $0 \leq r < \frac{2}{\sqrt{k}}$. Then by (9), the proper radius of Einstein's universe is

$$R_p = \lim_{\alpha \rightarrow \frac{2}{\sqrt{k}}} e^{\frac{1}{2}g(t)} \int_0^{\alpha} \frac{dr}{1 + \frac{k}{4}r^2} = \frac{2e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left[\left(\frac{\pi}{4} + n\pi \right) - m\pi \right], \quad n, m = 0, 1, 2, \dots \quad (15)$$

$$n \geq m.$$

Setting $p = n - m$ gives for the proper radius,

$$R_p = \frac{2e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left(\frac{\pi}{4} + p\pi \right), \quad p = 0, 1, 2, \dots \quad (16)$$

Now by (13), the proper radius of Einstein's universe is

$$R_p = \lim_{\alpha \rightarrow \frac{1}{\sqrt{k}}} e^{\frac{1}{2}g(t)} \int_0^{\alpha} \frac{dR}{\sqrt{1 - kR^2}} = \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left[\left(\frac{\pi}{2} + 2n\pi \right) - m\pi \right], \quad n, m = 0, 1, 2, \dots$$

$$2n \geq m.$$

Setting $q = 2n - m$ gives the proper radius of Einstein's universe as,

$$R_p = \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left(\frac{\pi}{2} + q\pi \right), \quad q = 0, 1, 2, \dots \quad (17)$$

Expressions (16) and (17) must be equal for all values of p and q . This can only occur if $g(t)$ is infinite for all values of t . Thus, the proper radius of Einstein's universe is infinite, and hence, the radius of curvature of Einstein's universe is also infinite. In addition, it follows from the line elements, that the volume and the area of Einstein's universe are infinite for all time t . Thus, Einstein's universe is infinite and unbounded and independent of time. Therefore, the Standard Cosmological Model (Big Bang) is inconsistent with General Relativity and is therefore invalid.

The standard static cosmological models suffer from the same fundamental defects, and are therefore invalid. The line element for Einstein's cylindrical model is,

$$ds^2 = dt^2 - [1 - (\lambda - 8\pi P_0) R_c^2]^{-1} dR_c^2 - R_c^2(d\theta^2 + \sin^2 \theta d\varphi^2).$$

This has no Lorentz signature solution for $\frac{1}{\sqrt{\lambda-8\pi P_0}} < R_c(r) < \infty$ [15]. For $1 - (\lambda - 8\pi P_0) R_c^2 > 0$ and $R_c = R_c(r) \geq 0$,

$$0 \leq R_c < \frac{1}{\sqrt{\lambda - 8\pi P_0}}.$$

The proper radius is

$$R_p = \lim_{\alpha \rightarrow \frac{1}{\sqrt{\lambda-8\pi P_0}}} \int_0^\alpha \frac{dR_c}{\sqrt{1 - (\lambda - 8\pi P_0) R_c^2}} = \frac{(1 + 4n)\pi}{2\sqrt{\lambda - 8\pi P_0}}, \quad n = 0, 1, 2, \dots$$

which is arbitrarily large.

The spherical model of de Sitter is given by the line element

$$ds^2 = \left(1 - \frac{\lambda + 8\pi\rho_{00}}{3} R_c^2\right) dt^2 - \left(1 - \frac{\lambda + 8\pi\rho_{00}}{3} R_c^2\right)^{-1} dR_c^2 - R_c^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

where ρ_{00} is the macroscopic density of the Universe. This line element has no Lorentz signature solution on $\sqrt{\frac{3}{\lambda+8\pi\rho_{00}}} < R_c < \infty$ [15], so $0 \leq R_c < \sqrt{\frac{3}{\lambda+8\pi\rho_{00}}}$. The proper radius is

$$R_p = \lim_{\alpha \rightarrow \sqrt{\frac{3}{\lambda+8\pi\rho_{00}}}} \int_0^\alpha \frac{dR_c}{R_c \sqrt{\lambda + 8\pi\rho_{00}}} = \sqrt{\frac{3}{\lambda + 8\pi\rho_{00}}} \frac{(1 + 4n)\pi}{2}, \quad n = 0, 1, 2, \dots$$

which is arbitrarily large.

It is also worth noting that it has recently been shown that the likely source of the Cosmic Microwave Background (CMB) is not the Cosmos but the oceans of the Earth [15–23], and therefore the CMB has nothing to do with the Standard Cosmological Model (Big Bang). It is anticipated that the PLANCK satellite, soon to be launched to the 2nd Lagrange Point, will verify the oceans, the Earth Microwave Background (EMB), as the source of the CMB. The PLANCK satellite is equipped with absolute measuring instruments whereas the WMAP satellite has only differential instruments and so cannot take an absolute measurement, which simply means that the interpretation of its data (and that of COBE) as a verification of the Big Bang source of the CMB is invalid. Indeed, the WMAP data appears to have no relevance for cosmology at all.

Translation of the centre of symmetry

The usual interpretations confound the location of the centre of spherical symmetry of the gravitational field with the origin of a coordinate system associated with the parameter r whose corresponding centre of spherical symmetry is not at its origin of coordinates. In Efcleethean 3-space the equation of a sphere of radius D and centre C located at the extremity of the fixed vector \vec{r}_0 , may be written

$$(\vec{r}(u) - \vec{r}_0) \bullet (\vec{r}(u) - \vec{r}_0) = D^2. \quad (18)$$

where u is some parameter upon which \vec{r} depends. The centre of the sphere is not at the origin of the coordinate system unless $\vec{r}_0 = \vec{0}$. The usual interpretations treat the origin of the

parametric coordinate system as the centre of a non-Euclidean sphere (for the gravitational field) when the centre of the said sphere is not in fact located at the origin of the parametric coordinate system or in the gravitational field. The centre of spherical symmetry of the source of the gravitational field is actually a *centre of mass* and as such is not a physical object. The black hole results from the construction of a completely different and irrelevant manifold, with the claim that the singularity at $r = \alpha = 2m$ in the usual line element is a “coordinate” singularity, and the claim that the origin of the parametric coordinate system at $r = 0$ is the “true” singularity for the gravitational field given by the usual line element, in the belief that the origin of the parametric coordinate system is the location of the centre of spherical symmetry for Einstein’s gravitational field. The inadmissibility of this usual conception, from which the black hole is actually obtained, can be readily seen by claiming that the “true” centre of the sphere described by (18) is not at the extremity of the fixed non-zero vector \vec{r}_0 , but at the origin of the coordinate system to which the vectors are referred, and then construct a “transformation of coordinates”, in the fashion of the Kruskal-Szekeres procedure, that actually retains the centre of the sphere at the extremity of the fixed non-zero vector \vec{r}_0 , but interprets the “centre” of the sphere to be at the origin of coordinates, so that its actual centre, the centre of mass, is a “removable” coordinate artifact.

Note that if $\vec{r}(u)$ and \vec{r}_0 are collinear, then the vector notation can be dropped, whereupon equation (18) takes the scalar form

$$(r(u) - r_0)(r(u) - r_0) = D^2,$$

where D is now more readily seen to be precisely that quantity appearing in expression (5), being the parametric *distance* $|r(u) - r_0|$ between the parametric points $r(u)$ and r_0 on the real line (the radial line in Minkowski space that passes through the origin of the coordinate system and the points $r(u)$ and r_0 on that radial line), where r_0 is the location of the centre of symmetry in Minkowski space corresponding to the centre of symmetry $R_p(r_0) = 0$ in the gravitational field.

References

- [1] Crothers S. J. On the geometry of the general solution for the vacuum field of the point-mass. *Progress in Physics*, v. 2, 3–14, 2005, (www.ptep-online.com/index_files/2005/PP-02-01.PDF).
- [2] Droste J. The field of a single centre in Einstein’s theory of gravitation, and the motion of a particle in that field. *Ned. Acad. Wet., S. A.*, v. 19, 197, 1917, (www.geocities.com/theometria/Droste.pdf).
- [3] Schwarzschild K. On the gravitational field of a mass point according to Einstein’s theory. *Sitzungsber. Preuss. Akad. Wiss., Phys. Math. Kl.*, 189, 1916, (arXiv: physics/9905030, www.geocities.com/theometria/schwarzschild.pdf).
- [4] Crothers S. J. A Brief History of Black Holes. *Progress in Physics*, v. 2, 54–57, 2006, (www.ptep-online.com/index_files/2006/PP-05-10.PDF).
- [5] The singular points of Einstein’s Universe. *Journ Phys. Radium*, v. 23, 43, 1923, (www.geocities.com/theometria/brillouin.pdf).
- [6] Abrams L. S. Black holes: the legacy of Hilbert’s error. *Can. J. Phys.*, v. 67, 919, 1989, (arXiv:gr-qc/0102055, www.geocities.com/theometria/Abrams1989.pdf).

- [7] Eddington A. S. The mathematical theory of relativity, Cambridge University Press, Cambridge, 2nd edition, 1960.
- [8] Crothers S. J. Spherically Symmetric Metric Manifolds and the Black Hole Catastrophe. (www.aias.us)
- [9] Levi-Civita T. The Absolute Differential Calculus, Dover Publications Inc., New York, 1977.
- [10] Crothers S. J. On the Ramifications of the Schwarzschild Space-time Metric. *Progress in Physics*, v. 1, 74–80, 2005, (www.ptep-online.com/index_files/2005/PP-01-10.PDF).
- [11] Schwarzschild K. On the gravitational field of a sphere of incompressible fluid according to Einsteins theory. *Sitzungsber. Preuss. Akad. Wiss., Phys. Math. Kl.*, 1916, 424 ([arXiv: physics/9912033](http://arXiv:physics/9912033), www.geocities.com/theometria/Schwarzschild2.pdf).
- [12] Crothers S. J. On the Vacuum Field of a Sphere of Incompressible Fluid. *Progress in Physics*, v. 2, 76–81, 2005, (www.ptep-online.com/index_files/2005/PP-02-06.PDF).
- [13] Crothers S. J. On the ‘Size’ of Einstein’s Spherically symmetric Universe. *Progress in Physics*, v. 3, p–pp, 2007, (www.ptep-online.com/index_files/2007/PP-11-10.PDF).
- [14] Tolman R. C. Relativity Thermodynamics and Cosmology, Dover Publications Inc., New York, 1987.
- [15] Crothers S. J. On the general solution to Einstein’s vacuum field for the point-mass when $\lambda \neq 0$ and its consequences for relativistic cosmology. *Progress in Physics*, 2005, v. 3, 7-14, (www.ptep-online.com/index_files/2005/PP-03-02.PDF).
- [16] Robitaille P.-M. L. WMAP: a radiological analysis. Spring Meeting of the American Physical Society Ohio Section, S1.00003, March 31-April 1, 2006.
- [17] Robitaille P.-M. L. WMAP: a radiological analysis II. Spring Meeting of the American Physical Society Northwest Section, G1.0005, May 19-20, 2006.
- [18] Robitaille P.-M. L. WMAP: a radiological analysis. *Progress in Physics*, 2007, v. 1, 3-18, (www.ptep-online.com/index_files/2007/PP-08-01.PDF).
- [19] Robitaille P.-M. L. On the origins of the CMB: insight from the COBE, WMAP, and Relikt-1 Satellites. *Progress in Physics*, 2007, v. 1, 19-3, (www.ptep-online.com/index_files/2007/PP-08-02.PDF).
- [20] Robitaille P.-M. L. On the Earth Microwave Background: absorption and scattering by the atmosphere. *Progress in Physics*, 2007, v. 3, 3-4, (www.ptep-online.com/index_files/2007/PP-010-01.PDF).
- [21] Robitaille P.-M. L., Rabounski D. COBE and the absolute assignment of the CMB to the Earth. American Physical Society March Meeting, L20.00007, March 5-9, 2007.
- [22] Rabounski D. The relativistic effect of the deviation between the CMB temperatures obtained by the COBE satellite. *Progress in Physics*, 2007, v. 1, 24-26, (www.ptep-online.com/index_files/2007/PP-08-03.PDF).
- [23] Robitaille P.-M. L., On the Nature of the Microwave Background at the Lagrange 2 Point. Part I. *Progress in Physics*, 2007, v. 4., 74-83, (www.ptep-online.com/index_files/2007/PP-11-11.PDF).
- [24] Borissova L., Rabounski D. On the nature of the Microwave Background at the Lagrange 2 Point. Part II. *Progress in Physics*, 2007, v. 4., 84-95, (www.ptep-online.com/index_files/2007/PP-11-12.PDF).