D. Rabounski and L. Borissova

PARTICLES HERE AND BEYOND THE MIRROR

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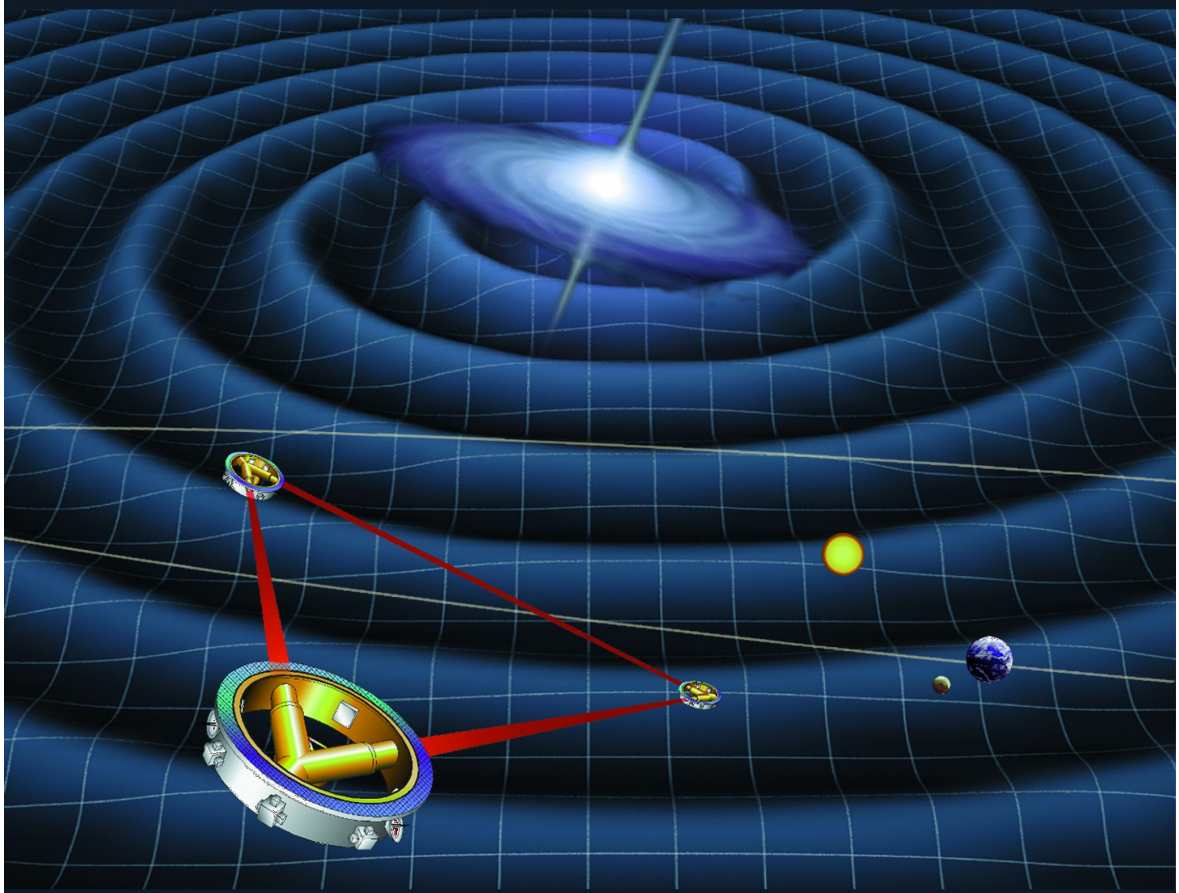

Dmitri Rabounski and Larissa Borissova

Particles Here and Beyond the Mirror

Second edition, revised and expanded by new chapters

Andra upplagan, reviderad och utvidgad med nya kapitel

Particlar här och bakom spegeln

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Foreword to the 2nd Edition

The background behind this book is as follows. In 1991 we initiated a study to find out what kinds of particles may theoretically inhabit the space-time of the General Theory of Relativity. As the instrument, we equipped ourselves with the mathematical apparatus of chronometric invariants (physically observable quantities) developed in the 1940's by Abraham Zelmanov.

The study was completed to reveal that aside for mass-bearing and massless (light-like) particles, those of the third kind may also exist. Their trajectories lie beyond the regular region in space-time. For a regular observer the trajectories are of zero four-dimensional length and zero three-dimensional observable length. Besides, along the trajectories the interval of observable time is also zero. Mathematically, this means that such particles inhabit a space-time with fully degenerate metric (fully degenerate space-time). We have therefore called such a space "zero-space" and such particles — "zero-particles".

For a regular observer their motion in zero-space is instantaneous, so zero-particles do realize long-range action. Through possible interaction with our-world's mass-bearing or massless particles, zero-particles may instantaneously transmit signals to any point in our three-dimensional space (a phenomenon we may call "non-quantum teleportation").

Considering zero-particles in the frames of the wave-particle duality, we have obtained that for a regular observer they are standing waves and the whole zero-space is filled with a system of standing light-like waves (zero-particles), i.e. standing light-holograms. This result corresponds to the known "stopped light experiment" (Harvard, USA, 2001).

Using the mathematical method of physically observable quantities, we have also showed that two separate regions in inhomogeneous space-time may exist, where the physically observable (proper) time flows into the future and into the past, while such a duality is not found in a homogeneous space-time. These regions are referred to as our world and the mirror world respectively; they are separated by a space-time membrane wherein observable time stops.

All the above results are derived exclusively from the application of Zelmanov's mathematical apparatus of physically observable quantities. Of course, these are not the final results which we could extract

from the theme, using Zelmanov's mathematical method. It is possible that further studies in this direction will give more theoretical and experimental results.

Now we present these results into your consideration. The second edition of this book is amended by new theoretical results and also two new chapters: a chapter on the theory of gravitational wave detectors and a chapter concerning virtual particles and non-quantum teleportation in the framework of the General Theory of Relativity.

In conclusion we would like to express belated thanks and sincere gratitude to Dr. Abraham Zelmanov (1913–1987) and Prof. Kyril Stanyukovich (1916–1989), teachers of ours, for countless hours of friendly conversations. We are also grateful to Kyril Dombrovski whose talks and friendly discussions greatly influenced our outlooks. Also we highly appreciate assistance from the side of our colleague Indranu Suhendro, whose editing has made this book much more accessible to a reader.

May 15, 2008 *Dmitri Rabounski and Larissa Borissova*

Chapter 1 Three Kinds of Particles According to Pseudo-Riemannian Space

§1.1 Problem statement

The main goal of the theory of motion of particles is to define the three-dimensional (spatial) coordinates of a particle at any given moment of time. In order to do this, one should be aware of three things. First, one should know in what sort of space-time the events take a place. That is, one should know the geometric structure of space and time, just as one should know the conditions of a road to be able to drive on it. Second, one should know the physical properties of the moving particle. Third, knowledge of the equations of motion of particles of a certain kind is necessary.

The first problem actually leads to the choice of a space from among the spaces known in mathematics, in order to represent just the right geometry for space and time which best fits the geometric representation of the observed world.

The view of the world as a space-time continuum takes its origin from the historical speech *Raum und Zeit* by Hermann Minkowski, which he gave on September 21, 1908, in Köln, Germany, at the 80th Assembly of the Society of German Natural Scientists and Physicians (Die Gesellschaft Deutscher Naturforscher und Ärzte). There he introduced the notion "space-time" into physics, and presented a geometric interpretation of the principle of invariance of the velocity of light and of Lorentz' transformations.

A few years later, in 1912, Marcel Grossmann, in his private conversation with Albert Einstein, a close friend of him, proposed Riemannian geometry as the geometry of the observed world. Later Einstein came to the idea that became the corner-stone of his General Theory of Relativity: that was the "geometric concept of the world", according to which the geometric structure of space-time determines all properties of the Universe. Thus Einstein's General Theory of Relativity, finalized by

him in 1915, is the first geometric theory of space-time and of motion of particles since the dawn of the contemporary science.

Consideration of the problem in detail led Einstein to the fact that the only way to represent space-time in the way that fits the existing experimental data is given by a *four-dimensional pseudo-Riemannian space* with the sign-alternating Minkowski signature $(+---)$ or $(-+++)$ (one time axis and three spatial axes). This is a particular case of the family of Riemannian spaces, i.e. spaces where geometry is Riemannian (the square of distance ds^2 between infinitely close points is set up by metric $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta = inv$). In a Riemannian space coordinate axes can be of any kind. Four-dimensional pseudo-Riemannian space is different on the account of the fact that there is a principal difference between the three-dimensional space, perceived as space, from the fourth axis — time. From the mathematical viewpoint this looks as follows: three spatial axes are real, while the time axis is imaginary (or vice versa), and choice of such conditions is arbitrary.

A particular case of a flat, uniform, and isotropic four-dimensional pseudo-Riemannian space is referred to as *Minkowski's space*. This is the basic space-time of the Special Theory of Relativity — the abstracted case, which is free of gravitational fields, rotation, deformation, and curvature. In the general case the real pseudo-Riemannian space is curved, non-uniform and anisotropic. This is the basic space-time of the General Theory of Relativity, where we meet both gravitational field, rotation, deformation, and curvature.

So, the General Theory of Relativity is built on view of the world as a four-dimensional space-time, where any and all objects possess not a three-dimensional volume alone, but their "longitude" in time. That is, any physical body, including ours, is a really existing four-dimensional instance with the shape of a cylinder elongated in time (cylinder of events of this body), created by perplexion of other cylinders at the moment of its "birth" and split into many other ones at the moment of its "death". For example, for a human the "time length" is the duration of their life from conception till death.

Very soon after Eddington in 1919 gave the first proof that Sun rays are curved by its gravitational field, many researchers faced strong obstacles in fitting together calculations made in the framework of the General Theory of Relativity with existing results of observations and experiments. Successful experiments verifying the General Theory of Relativity during the last 80 years have explicitly shown that the four-dimensional pseudo-Riemannian space is the basic space-time of the observed world (as far as the up-to-date measurement precision al-

lows us to judge). And if the inevitable evolution of human civilization and thought, and of experimental technology, shows that the four-dimensional pseudo-Riemannian space can no longer explain the results of new experiments, this will mean nothing other than the need to assume a more general space, which will include the four-dimensional pseudo-Riemannian space as a particular case.

This book will focus on the motion of particles based on the geometric concept of the world-structure: we will assume that the geometry of our space-time determined all properties of the observed world. Therefore, contrary to other researchers, we are not going to constrain the geometry of the space-time by any limitations and we will solve our problems the way the space-time geometry requires them to be solved.

Hence, any particle in the space-time corresponds to its own world-line, which sets up the three-dimensional (spatial) coordinates of this particle at any given moment of time. Subsequently, our goal to determine possible kinds of particles evolves into considering all allowable types of trajectories of motion in the four-dimensional space-time.

Generally speaking, referring to the equations of motion of free particles in a metric space (space-time) one actually refers to the *equations of geodesic lines*, which are the four-dimensional equations of the world-trajectory of a free particle*

$$\frac{d^2x^\alpha}{d\rho^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{d\rho}\frac{dx^\nu}{d\rho} = 0\,, \tag{1.1}$$

where $\Gamma^\alpha_{\mu\nu}$ are Christoffel's symbols of the 2nd kind and ρ is a parameter of derivation along to the geodesic line.

From the geometric viewpoint the equations of geodesic lines are equations of parallel transfer in the sense of Levi-Civita [1] of the four-dimensional kinematic vector

$$Q^\alpha = \frac{dx^\alpha}{d\rho}\,, \tag{1.2}$$

namely — the following equations

$$\frac{\mathrm{D}Q^\alpha}{d\rho} = \frac{dQ^\alpha}{d\rho} + \Gamma^\alpha_{\mu\nu}Q^\mu\frac{dx^\nu}{d\rho} = 0\,, \tag{1.3}$$

where $\mathrm{D}Q^\alpha = dQ^\alpha + \Gamma^\alpha_{\mu\nu}Q^\mu dx^\nu$ is the absolute differential of the kinematic vector Q^α transferred in parallel to itself and tangential to the trajectory of transfer (a geodesic line).

*Here and so forth space-time indices are Greek, for instance $\alpha, \beta, = 0, 1, 2, 3$, while spatial indices — Roman, for instance $i, k = 1, 2, 3$.

Levi-Civita parallel transfer means that the length of the transferred vector remains unchanged

$$Q_\alpha Q^\alpha = g_{\alpha\beta}\, Q^\alpha Q^\beta = const\,, \tag{1.4}$$

along the entire world-trajectory, where $g_{\alpha\beta}$ is the fundamental metric tensor of the space.

At this point, we note that the equations of geodesic lines are purely kinematic, as they do not contain the physical properties of the moving objects. Therefore to obtain the full picture of motion of particles we have to build *dynamic equations of motion*, solving which will give us not only the trajectories of the particles, but their properties (such as energy, frequency etc.) as well.

To do this we have to define: a) the possible types of trajectories in the four-dimensional space-time (pseudo-Riemannian space); b) the dynamical vector for each type of trajectory; c) the derivation parameter of each type of trajectory.

First we consider what types of trajectories are allowable in the four-dimensional pseudo-Riemannian space. Along a geodesic line the condition $g_{\alpha\beta}\, Q^\alpha Q^\beta = const$ is true. If along geodesic lines $g_{\alpha\beta}\, Q^\alpha Q^\beta \neq 0$, such lines are referred to as *non-isotropic geodesics*.

Along non-isotropic geodesics the square of the four-dimensional interval is not zero

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta \neq 0 \tag{1.5}$$

and the interval ds takes the form

$$ds = \sqrt{g_{\alpha\beta}\, dx^\alpha dx^\beta} \qquad \text{if} \quad ds^2 > 0\,, \tag{1.5}$$

$$ds = \sqrt{-\,g_{\alpha\beta}\, dx^\alpha dx^\beta} \quad \text{if} \quad ds^2 < 0\,. \tag{1.6}$$

If along geodesic lines $g_{\alpha\beta}\, Q^\alpha Q^\beta = 0$, such lines are referred to as *isotropic geodesics*. Along isotropic geodesics the square of the four-dimensional interval is zero

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta = c^2 d\tau^2 - d\sigma^2 = 0\,, \tag{1.7}$$

while the observable three-dimensional (spatial) interval $d\sigma$ and the interval of the physically observable time $d\tau$ are not zero (therefore isotropic trajectories are particularly degenerate).

This ends the list of types of trajectories in the four-dimensional pseudo-Riemannian space (the basic space-time of the General Theory of Relativity), known to scientists until the recent time.

We show in this book that other trajectories are theoretically allowable in the space, along which the four-dimensional interval, the interval of observable time and the observable three-dimensional interval are zero. Such trajectories lie beyond the four-dimensional pseudo-Riemannian space. These are trajectories in a fully degenerate space-time region. We call it "degenerate" because from the viewpoint of a regular observer, all distances and intervals of time in such a region degenerate into zero. Nevertheless, transition into such a degenerate space-time region from the regular space-time region is quite possible (provided certain physical conditions are achieved). And perhaps for the observer, who moves into such a degenerate space-time region, the terms "time" and "space" will not become void, but will be measured in different units.

Therefore we may consider the four-dimensional pseudo-Riemannian space (space-time) and the fully degenerate space-time region (henceforth referred to simply as space-time) in common as an *extended space-time*, in which both non-degenerate (isotropic and non-isotropic) and degenerate trajectories exist.

Hence in such an extended four-dimensional space-time, which is an actual "extension" of the basic space-time of the General Theory of Relativity to include the fully degenerate space-time region, three types of trajectories are allowable:

1) Non-isotropic trajectories (pseudo-Riemannian space). Motion along them is possible at subluminal and superluminal velocity;
2) Isotropic trajectories (pseudo-Riemannian space). Along such trajectories motion is possible at the velocity of light only;
3) Fully degenerate trajectories (zero-trajectories), which lie in the fully degenerate space-time.

According to these types of trajectories, three kinds of particles can be distinguished, which can exist in the four-dimensional space-time:

1) Mass-bearing particles (the rest-mass $m_0 \neq 0$) move along non-isotropic trajectories ($ds \neq 0$) at subluminal velocities (real mass-bearing particles) and at superluminal velocities (imaginary mass-bearing particles — tachyons);
2) Massless particles (the rest-mass $m_0 = 0$) move along isotropic trajectories ($ds = 0$) at the velocity of light. These are light-like particles, e.g. photons;
3) Particles of the 3rd kind move along trajectories in the fully degenerate space-time.

Besides, from the purely mathematical viewpoint, the equations of geodesic lines contain the same vector Q^α and the same parameter ρ irrespective of whether the considered trajectories are isotropic or non-isotropic. This shows that there must exist such equations of motion, which have a common form for mass-bearing and massless particles. We will proceed to search for such generalized equations of motion.

In the next paragraph, we will set forth the basics of the mathematical apparatus of physically observable quantities (chronometric invariants), which will serve as our main tool in this book. In §1.3 we will prove the existence of a generalized dynamical vector and derivation parameter, which are the same for mass-bearing and massless particles. §1.4 will focus on the physical conditions of the full degeneration of a pseudo-Riemannian space. §1.5 will consider the properties of particles in an extended four-dimensional space-time, which allows the full degeneration of the metric. In §1.6–§1.8 the chronometrically invariant dynamical equations of motion valid for all kinds of particles allowed in the extended four-dimensional space-time will be obtained. In §1.9 and §1.10 we will show that the equations of geodesic lines and Newton's laws of Classical Mechanics are particular cases of these dynamical equations. §1.11 and §1.12 will be devoted to two aspects of the obtained equations: 1) the conditions transforming the extended space-time into the regular space-time, and 2) the asymmetry of motion into the future (the direct flow of time) and into the past (the reverse flow of time). §1.13 and §1.14 will focus on the physical conditions of the direct flow of time and the reverse flow of time. §1.15 and §1.16 discuss certain specific cases such as a superluminal observer and gravitational collapse.

§1.2 Chronometrically invariant (observable) quantities

In order to build a descriptive picture of any physical theory, we need to express the results through real physical quantities, which can be measured in experiments (*physically observable quantities*). In the General Theory of Relativity, this problem is not a trivial one, because we are looking at objects in a four-dimensional space-time and we have to determine which components of the four-dimensional tensor quantities are physically observable.

Here is the problem in a nutshell. All equations in the General Theory of Relativity are put down in the *general covariant form*, which does not depend on our choice of the frame of reference. The equations, as well as the variables they contain, are four-dimensional. Which of those four-dimensional variables are observable in real physical experiments, i.e. which components are physically observable quantities?

Intuitively we may assume that the three-dimensional components of a four-dimensional tensor constitute a physically observable quantity. At the same time we cannot be absolutely sure that what we observe are the three-dimensional components *per se*, not more complicated variables which depend on other factors, e.g. on the properties of the physical standards of the space of reference.

Further, a four-dimensional vector (1st rank tensor) has as few as 4 components (1 time component and 3 spatial components). A 2nd rank tensor, e.g. a rotation or deformation tensor, has 16 components: 1 time component, 9 spatial components and 6 mixed (time-space) components. Are the mixed components physically observable quantities? This is another question that had no definite answer. Tensors of higher ranks have even more components; for instance the Riemann-Christoffel curvature tensor has 256 components, so the problem of the heuristic recognition of physically observable components becomes far more complicate. Besides there is an obstacle related to the recognition of observable components of covariant tensors (in which indices occupy the lower position) and of mixed type tensors, which have both lower and upper indices.

We see that the recognition of physically observable quantities in the General Theory of Relativity is not a trivial problem. Ideally we would like to have a mathematical technique to calculate physically observable quantities for tensors of any given ranks *unambiguously*.

Numerous attempts to develop such a mathematical method were made in the 1930's by outstanding researchers of that time. The goal was nearly attained by Landau and Lifshitz in their famous *The Classical Theory of Fields*, first published in Russian in 1939. Aside for discussing the problem of physically observable quantities itself, in §84 of their book, they introduced the interval of physically observable time and the observable three-dimensional interval, which depend on the physical properties (physical standards) of the space of reference of an observer. But all the attempts made in the 1930's were limited to solving certain particular problems. None of them led to development of a versatile mathematical apparatus.

A complete mathematical apparatus for calculating physically observable quantities in a four-dimensional pseudo-Riemannian space was first introduced by Abraham Zelmanov and is known as the *theory of chronometric invariants*. Zelmanov's mathematical apparatus was first presented in 1944 in his PhD thesis [2], where it is given in all details, then — in his short papers of 1956–1957 [3, 4].

A similar result was obtained by Carlo Cattaneo [5–8], an Italian

mathematician who worked independently of Zelmanov. Cattaneo published his first paper on this subject in 1958 [5]. He highly appreciated Zelmanov's theory of chronometric invariants, and referred to it in his last publication of 1968 [8].

The essence of Zelmanov's mathematical apparatus of physically observable quantities (chronometric invariants), designed especially for the four-dimensional, curved, non-uniform pseudo-Riemannian space (space-time), is as follows.

In any point of the space-time we can place a three-dimensional *spatial section* $x^0 = ct = const$ (three-dimensional space) orthogonal to a given *time line* $x^i = const$. If a spatial section is everywhere orthogonal to time lines, which pierce it at each point, such a space is referred to as *holonomic*. Otherwise, if the spatial section is non-orthogonal everywhere to the time lines, the space is referred to as *non-holonomic*.

Possible frames of reference of a real observer include a coordinate net spanned over a real physical body (the body of reference of the observer, which is located near him) and a real clock located at each point of the coordinate net. Both coordinate net and clocks represent a set of real references to which the observer refers his observations. Therefore, physically observable quantities, really registered by an observer, should be the result of projection of four-dimensional quantities onto the time line and onto the spatial section (time and three-dimensional space) of the reference body of the observer.

The operator of projection onto the time line of an observer is the world-vector of four-dimensional velocity

$$b^\alpha = \frac{dx^\alpha}{ds} \tag{1.8}$$

of his reference body with respect to him. This world-vector is tangential to the world-line of the observer in each point of his world-trajectory, so this is a unit-length vector

$$b_\alpha b^\alpha = g_{\alpha\beta}\,\frac{dx^\alpha}{ds}\,\frac{dx^\beta}{ds} = \frac{g_{\alpha\beta}\,dx^\alpha dx^\beta}{ds^2} = +1\,. \tag{1.9}$$

The operator of projection onto the spatial section of the observer (his local three-dimensional space) is determined as a four-dimensional symmetric tensor $h_{\alpha\beta}$, which is

$$\left.\begin{aligned} h_{\alpha\beta} &= -\,g_{\alpha\beta} + b_\alpha b_\beta \\ h^{\alpha\beta} &= -\,g^{\alpha\beta} + b^\alpha b^\beta \\ h^\beta_\alpha &= -\,g^\beta_\alpha + b_\alpha b^\beta \end{aligned}\right\}. \tag{1.10}$$

The world-vector b^α and the world-tensor $h_{\alpha\beta}$ are orthogonal to each other. Mathematically this means that their common contraction is zero

$$\left.\begin{aligned} h_{\alpha\beta}b^\alpha &= -\,g_{\alpha\beta}b^\alpha + b_\alpha b^\alpha b_\beta = -\,b_\beta + b_\beta = 0 \\ h^{\alpha\beta}b_\alpha &= -\,g^{\alpha\beta}b_\alpha + b^\beta b_\alpha b^\alpha = -\,b^\beta + b^\beta = 0 \\ h^\alpha_\beta b_\alpha &= -\,g^\alpha_\beta b_\alpha + b_\beta b^\alpha b_\alpha = -\,b_\beta + b_\beta = 0 \\ h^\beta_\alpha b^\alpha &= -\,g^\beta_\alpha b^\alpha + b^\beta b_\alpha b^\alpha = -\,b^\beta + b^\beta = 0 \end{aligned}\right\}. \tag{1.11}$$

So, the main properties of the operators of projection are commonly expressed, obviously, as follows

$$b_\alpha b^\alpha = +1\,, \qquad h^\beta_\alpha b^\alpha = 0\,. \tag{1.12}$$

If the observer rests with respect to his references (such a case is known as the *accompanying frame of reference*), $b^i = 0$ in his reference frame. The coordinate nets of the same spatial section are connected to each other through the transformations

$$\left.\begin{aligned} &\tilde{x}^0 = \tilde{x}^0\left(x^0, x^1, x^2, x^3\right) \\ &\tilde{x}^i = \tilde{x}^i\left(x^1, x^2, x^3\right), \quad \frac{\partial \tilde{x}^i}{\partial x^0} = 0 \end{aligned}\right\} \tag{1.13}$$

where the third equation means the fact that the spatial coordinates in the tilde-marked net are independent from time of the non-tilded net, that is equivalent to a coordinate net where the lines of time are fixed $x^i = const$ at any point. Transformation of the spatial coordinates is nothing but only transition from one coordinate net to another within the same spatial section. Transformation of time means changing the whole set of clocks, so this is transition to another spatial section (another three-dimensional space of reference). In practice this means replacement of one reference body with all of its physical references with another reference body that has its own physical references. But when using different references, the observer will obtain different results (other observable quantities). Therefore, the physically observable projections in an accompanying frame of reference should be invariant with respect to the transformation of time, i.e. they should be invariant with respect to the transformations (1.13). In other word, such quantities should possess the property of *chronometric invariance*.

We therefore refer to the physically observable quantities determined in an accompanying frame of reference as *chronometrically invariant quantities*, or *chronometric invariants* in short.

The tensor $h_{\alpha\beta}$, being considered in the space of a frame of reference accompanying an observer, possesses all properties attributed to the fundamental metric tensor, namely

$$h_i^\alpha h_\alpha^k = \delta_i^k - b_i b^k = \delta_i^k\,, \qquad \delta_i^k = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.14}$$

where δ_i^k is the unit three-dimensional tensor*. Therefore, in the accompanying frame of reference the three-dimensional tensor h_{ik} can lift or lower indices in chronometrically invariant quantities.

So, in the accompanying frame of reference the main properties of the operators of projection are

$$b_\alpha b^\alpha = +1\,, \qquad h_\alpha^i b^\alpha = 0\,, \qquad h_i^\alpha h_\alpha^k = \delta_i^k\,. \tag{1.15}$$

We calculate the components of the operators of projection in the accompanying frame of reference. The component b^0 comes from the obvious condition $b_\alpha b^\alpha = g_{\alpha\beta} b^\alpha b^\beta = 1$, which in the accompanying frame of reference ($b^i = 0$) is $b_\alpha b^\alpha = g_{00} b^0 b^0 = 1$. This component, in common with the rest components of b^α is

$$\left.\begin{aligned} &b^0 = \frac{1}{\sqrt{g_{00}}}\,, && b^i = 0 \\ &b_0 = g_{0\alpha} b^\alpha = \sqrt{g_{00}}\,, && b_i = g_{i\alpha} b^\alpha = \frac{g_{i0}}{\sqrt{g_{00}}} \end{aligned}\right\}, \tag{1.16}$$

while the components of $h_{\alpha\beta}$ are

$$\left.\begin{aligned} &h_{00} = 0\,, && h^{00} = -\,g^{00} + \frac{1}{g_{00}}\,, && h_0^0 = 0 \\ &h_{0i} = 0\,, && h^{0i} = -\,g^{0i}, && h_0^i = \delta_0^i = 0 \\ &h_{i0} = 0\,, && h^{i0} = -\,g^{i0}, && h_i^0 = \frac{g_{i0}}{g_{00}} \\ &h_{ik} = -\,g_{ik} + \frac{g_{0i} g_{0k}}{g_{00}}\,, && h^{ik} = -\,g^{ik}, && h_k^i = -\,g_k^i = \delta_k^i \end{aligned}\right\}. \tag{1.17}$$

Zelmanov developed a common mathematical method how to calculate the chronometrically invariant (physically observable) projections of any general covariant (four-dimensional) tensor quantity, and set it forth as a theorem (we refer to it as *Zelmanov's theorem*):

*This tensor δ_i^k is the three-dimensional part of the four-dimensional unit tensor δ_β^α, which can be used to replace indices in four-dimensional quantities.

ZELMANOV'S THEOREM: We assume that $Q^{ik\dots p}_{00\dots 0}$ are the components of the four-dimensional tensor $Q^{\mu\nu\dots\rho}_{00\dots 0}$ of the r-th rank, in which all upper indices are not zero, while all m lower indices are zero. Then, the tensor quantities

$$T^{ik\dots p} = (g_{00})^{-\frac{m}{2}}\, Q^{ik\dots p}_{00\dots 0} \tag{1.18}$$

constitute a chronometrically invariant three-dimensional contravariant tensor of $(r-m)$-th rank. Hence the tensor $T^{ik\dots p}$ is a result of m-fold projection onto the time line by the indices $\alpha, \beta \dots \sigma$ and onto the spatial section by $r-m$ the indices $\mu, \nu \dots \rho$ of the initial tensor $Q^{\mu\nu\dots\rho}_{\alpha\beta\dots\sigma}$.

According to the theorem, the chronometrically invariant (physically observable) projections of a four-dimensional vector Q^α are the quantities

$$b^\alpha Q_\alpha = \frac{Q_0}{\sqrt{g_{00}}}\,, \qquad h^i_\alpha Q^\alpha = Q^i, \tag{1.19}$$

while the chr.inv.-projections of a symmetric tensor of the 2nd rank $Q^{\alpha\beta}$ are the following quantities

$$b^\alpha b^\beta Q_{\alpha\beta} = \frac{Q_{00}}{g_{00}}\,, \quad h^{i\alpha} b^\beta Q_{\alpha\beta} = \frac{Q^i_0}{\sqrt{g_{00}}}\,, \quad h^i_\alpha h^k_\beta\, Q^{\alpha\beta} = Q^{ik}. \tag{1.20}$$

The chr.inv.-projections of a four-dimensional coordinate interval dx^α are the interval of the physically observable time

$$d\tau = \sqrt{g_{00}}\, dt + \frac{g_{0i}}{c\sqrt{g_{00}}}\, dx^i, \tag{1.21}$$

and the interval of the observable coordinates dx^i which are the same as the spatial coordinates. The physically observable velocity of a particle is the three-dimensional chr.inv.-vector

$$\mathrm{v}^i = \frac{dx^i}{d\tau}\,, \qquad \mathrm{v}_i \mathrm{v}^i = h_{ik}\, \mathrm{v}^i \mathrm{v}^k = \mathrm{v}^2, \tag{1.22}$$

which at isotropic trajectories becomes the three-dimensional chr.inv.-vector of the physically observable velocity of light

$$c^i = \mathrm{v}^i = \frac{dx^i}{d\tau}\,, \quad c_i c^i = h_{ik}\, c^i c^k = c^2. \tag{1.23}$$

Projecting the covariant or contravariant fundamental metric tensor onto the spatial section of an accompanying frame of reference ($b^i = 0$)

$$\left.\begin{aligned} h^\alpha_i h^\beta_k\, g_{\alpha\beta} &= g_{ik} - b_i b_k = -\,h_{ik} \\ h^i_\alpha h^k_\beta\, g^{\alpha\beta} &= g^{ik} - b^i b^k = g^{ik} = -\,h^{ik} \end{aligned}\right\}, \tag{1.24}$$

we obtain that the chr.inv.-quantity

$$h_{ik} = -\,g_{ik} + b_i b_k \tag{1.25}$$

is the *chr.inv.-metric tensor* (the *observable metric tensor*), using which we can lift and lower indices of any three-dimensional chr.inv.-tensorial object in the accompanying frame of reference. The contravariant and mixed components of the observable metric tensor are, obviously,

$$h^{ik} = -\,g^{ik}, \qquad h^i_k = -\,g^i_k = \delta^i_k\,. \tag{1.26}$$

Expressing $g_{\alpha\beta}$ through the definition of $h_{\alpha\beta} = -\,g_{\alpha\beta} + b_\alpha b_\beta$, we obtain the formula for the four-dimensional interval

$$ds^2 = b_\alpha b_\beta\, dx^\alpha dx^\beta - h_{\alpha\beta}\, dx^\alpha dx^\beta, \tag{1.27}$$

expressed through the operators of projection b_α and $h_{\alpha\beta}$. In this formula $b_\alpha dx^\alpha = c\,d\tau$, so the first term is $b_\alpha b_\beta\, dx^\alpha dx^\beta = c^2 d\tau^2$. The second term $h_{\alpha\beta}\, dx^\alpha dx^\beta = d\sigma^2$ in the accompanying frame of reference is the square of the observable three-dimensional interval*

$$d\sigma^2 = h_{ik}\, dx^i dx^k. \tag{1.28}$$

Thus, the four-dimensional interval, represented through the physically observable quantities, is

$$ds^2 = c^2 d\tau^2 - d\sigma^2. \tag{1.29}$$

The main physically observable properties attributed to the accompanying space of reference were deduced by Zelmanov in the framework of the theory, in particular — proceeding from the property of non-commutativity

$$\frac{{}^*\partial^2}{\partial x^i\,\partial t} - \frac{{}^*\partial^2}{\partial t\,\partial x^i} = \frac{1}{c^2}\,F_i\,\frac{{}^*\partial}{\partial t}\,, \tag{1.30}$$

$$\frac{{}^*\partial^2}{\partial x^i\,\partial x^k} - \frac{{}^*\partial^2}{\partial x^k\,\partial x^i} = \frac{2}{c^2}\,A_{ik}\,\frac{{}^*\partial}{\partial t} \tag{1.31}$$

of the chr.inv.-operators of derivation he introduced as follows

$$\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\,\frac{\partial}{\partial t}\,, \qquad \frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} - \frac{g_{0i}}{g_{00}}\,\frac{\partial}{\partial x^0}\,. \tag{1.32}$$

First two physically observable properties are characterized by the following three-dimensional chr.inv.-quantities: the vector of the *gravi-*

*This is due to the fact that $h_{\alpha\beta}$ in the accompanying frame of reference possesses all properties of the fundamental metric tensor $g_{\alpha\beta}$.

tational inertial force F_i and the antisymmetric tensor of the *angular velocities of rotation* of the space of reference A_{ik} which are

$$F_i = \frac{1}{1-\frac{\mathrm{w}}{c^2}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right), \tag{1.33}$$

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2\,c^2}\left(F_i v_k - F_k v_i\right). \tag{1.34}$$

Here w and v_i characterize the body of reference and its reference's space. These are the gravitational potential

$$\mathrm{w} = c^2\left(1-\sqrt{g_{00}}\right), \qquad 1-\frac{\mathrm{w}}{c^2} = \sqrt{g_{00}}\ , \tag{1.35}$$

and the linear velocity of rotation of the space

$$\left.\begin{aligned} v_i &= -\,c\,\frac{g_{0i}}{\sqrt{g_{00}}}\,, \qquad & v^i &= -\,c\,g^{0i}\sqrt{g_{00}} \\ v_i &= h_{ik}v^k, & v^2 &= v_k v^k = h_{ik}\,v^i v^k \end{aligned}\right\}. \tag{1.36}$$

We note that w and v_i don't possess the property of chronometric invariance, despite $v_i = h_{ik}v^k$ can be obtained as for a chr.inv.-quantity, through lowering the index by the chr.inv.-metric tensor h_{ik}.

Zelmanov had also found that the chr.inv.-quantities F_i and A_{ik} are linked to each other by two identities which are known as *Zelmanov's identities*

$$\frac{{}^{*}\partial A_{ik}}{\partial t} + \frac{1}{2}\left(\frac{{}^{*}\partial F_k}{\partial x^i} - \frac{{}^{*}\partial F_i}{\partial x^k}\right) = 0\,, \tag{1.37}$$

$$\frac{{}^{*}\partial A_{km}}{\partial x^i} + \frac{{}^{*}\partial A_{mi}}{\partial x^k} + \frac{{}^{*}\partial A_{ik}}{\partial x^m} + \frac{1}{2}\left(F_i A_{km} + F_k A_{mi} + F_m A_{ik}\right) = 0\,. \tag{1.38}$$

In the framework of quasi-Newtonian approximation, i.e. in a weak gravitational field at velocities much lower than the velocity of light and in the absence of rotation of space, F_i becomes a regular non-relativistic gravitational force $F_i = \frac{\partial \mathrm{w}}{\partial x^i}$.

Zelmanov had also proved the following theorem setting up the condition of holonomity of space:

> Zelmanov's theorem on holonomity of space: Identical equality of the tensor A_{ik} to zero in a four-dimensional region of the space (space-time) is the necessary and sufficient condition for the spatial sections to be everywhere orthogonal to the time lines in this region.

In other word, the necessary and sufficient condition of holonomity of a space should be achieved by equating to zero of the tensor A_{ik}. Naturally, if the spatial sections are everywhere orthogonal to the time lines (in such a case the space is holonomic), the quantities g_{0i} are zero. Since $g_{0i}=0$, we have $v_i=0$ and $A_{ik}=0$. Therefore, we will also refer the tensor A_{ik} to as the *space non-holonomity tensor*.

If somewhere the conditions $F_i=0$ and $A_{ik}=0$ are met in common, there the conditions $g_{00}=1$ and $g_{0i}=0$ are present as well (i.e. the conditions $g_{00}=1$ and $g_{0i}=0$ can be satisfied through the transformation of time in such a region). In such a region, according to (1.21), $d\tau=dt$: the difference between the coordinate time t and the physically observable time τ disappears in the absence of gravitational fields and rotation of the space. In other word, according to the theory of chronometric invariants, the difference between the coordinate time t and the physically observable time τ originates in both gravitation and rotation attributed to the space of reference of the observer (actually — his reference body, the Earth in the case of an Earth-bound observer), or in each of the motions separately.

On the other hand, it is doubtful to find such a region of the Universe wherein gravitational fields or rotation of the background space would be absent in clear. Therefore, in practice the physically observable time τ and the coordinate time t differ from each other. This means that the real space of our Universe is non-holonomic, and is filled with a gravitational field, while a holonomic space free of gravitation can be only a local approximation to it.

The condition of holonomity of a space (space-time) is linked direct to the problem of integrability of time in it. The formula for the interval of the physically observable time (1.21) has no an integrating multiplier. In other word, this formula cannot be reduced to the form

$$d\tau = A\,dt\,, \tag{1.39}$$

where the multiplier A depends on only t and x^i: in a non-holonomic space the formula (1.21) has non-zero second term, depending on the coordinate interval dx^i and g_{0i}. In a holonomic space $A_{ik}=0$, so $g_{0i}=0$. In such a case the second term of (1.21) is zero, while the first term is the elementary interval of time dt with an integrating multiplier

$$A=\sqrt{g_{00}}=f\left(x^0,x^i\right), \tag{1.40}$$

so we are allowed to write the integral

$$d\tau=\int\sqrt{g_{00}}\,dt\,. \tag{1.41}$$

Hence time is integrable in a holonomic space ($A_{ik}=0$), while it cannot be integrated in the case where the space is non-holonomic ($A_{ik}\neq 0$). In the case where time is integrable (a holonomic space), we can synchronize the clocks in two distantly located points of the space by moving a control clock along the path between these two points. In the case where time cannot be integrated (a non-holonomic space), synchronization of clocks in two distant points is impossible in principle: the larger is the distance between these two points, the more is the deviation of time on these clocks.

The space of our planet, the Earth, is non-holonomic due to the daily rotation of it around the Earth's axis. Hence two clocks located at different points of the surface of the Earth should manifest a deviation between the intervals of time registered on each of them. The larger is the distance between these clocks, the larger is the deviation of the physically observable time expected to be registered on them. This effect was sure verified by the well-known Hafele-Keating experiments [9–12] concerned with displacing standard atomic clocks by an airplane around the terrestrial globe, where rotation of the Earth's space sensibly changed the measured time. During a flight along the Earth's rotation, the observer's space on board of the airplane had more rotation than the space of the observer who stayed fixed on the ground. During a flight against the Earth's rotation it was vice versa. An atomic clock on board of such an airplane showed significant variation of the observed time depending on the velocity of rotation of the space.

Because synchronization of clocks at different locations on the surface of the Earth is a highly important problem in marine navigation and also aviation, in early time de-synchronization corrections were introduced as tables of the empirically obtained corrections which take the Earth's rotation into account. Now, thank to the theory of chronometric invariants, we know the origin of the corrections, and are able to calculate them on the basis of the General Theory of Relativity.

In addition to gravitation and rotation, the reference body can deform, changing its coordinate nets with time. This fact should also be taken into account in measurements. This can be done by introducing into the equations the three-dimensional symmetric chr.inv.-tensor of the *rates of deformation* the space of reference

$$\left.\begin{aligned} &D_{ik}=\frac{1}{2}\frac{{}^{*}\partial h_{ik}}{\partial t}\,, \qquad D^{ik}=-\frac{1}{2}\frac{{}^{*}\partial h^{ik}}{\partial t} \\ &D=h^{ik}D_{ik}=D^{n}_{n}=\frac{{}^{*}\partial \ln\sqrt{h}}{\partial t}\,, \qquad h=\det\|\,h_{ik}\| \end{aligned}\right\}. \tag{1.42}$$

Zelmanov had also deduced formulae for the four-dimensional quantities F_α, $A_{\alpha\beta}$, $D_{\alpha\beta}$ [13]

$$F_\alpha = -\,2\,c^2 b^\beta a_{\beta\alpha}\,, \tag{1.43}$$

$$A_{\alpha\beta} = c\,h^\mu_\alpha h^\nu_\beta\, a_{\mu\nu}\,, \tag{1.44}$$

$$D_{\alpha\beta} = c\,h^\mu_\alpha h^\nu_\beta\, d_{\mu\nu}\,, \tag{1.45}$$

which are the general covariant generalization of the chr.inv.-quantities F_i, A_{ik}, D_{ik}. The auxiliary quantities $a_{\alpha\beta}$ and $d_{\alpha\beta}$ here are

$$a_{\alpha\beta} = \frac{1}{2}\left(\nabla_\alpha b_\beta - \nabla_\beta b_\alpha\right), \quad d_{\alpha\beta} = \frac{1}{2}\left(\nabla_\alpha b_\beta + \nabla_\beta b_\alpha\right). \tag{1.46}$$

The usual Christoffel symbols of the 1st rank $\Gamma^\alpha_{\mu\nu}$ and the Christoffel symbols of the 1st rank $\Gamma_{\mu\nu,\sigma}$

$$\Gamma^\alpha_{\mu\nu} = g^{\alpha\sigma}\,\Gamma_{\mu\nu,\sigma} = \frac{1}{2}\,g^{\alpha\sigma}\left(\frac{\partial g_{\mu\sigma}}{\partial x^\nu} + \frac{\partial g_{\nu\sigma}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma}\right) \tag{1.47}$$

are linked to the respective chr.inv.-Christoffel symbols

$$\Delta^i_{jk} = h^{im}\Delta_{jk,m} = \frac{1}{2}\,h^{im}\left(\frac{{}^*\partial h_{jm}}{\partial x^k} + \frac{{}^*\partial h_{km}}{\partial x^j} - \frac{{}^*\partial h_{jk}}{\partial x^m}\right) \tag{1.48}$$

determined similarly to $\Gamma^\alpha_{\mu\nu}$. The only difference is that here instead of the fundamental metric tensor $g_{\alpha\beta}$ the chr.inv.-metric tensor h_{ik} is used. Components of the usual Christoffel symbols are linked to the other chr.inv.-chractersitics of the accompanying space of reference by the relations

$$D^i_k + A^{\cdot i}_{k\cdot} = \frac{c}{\sqrt{g_{00}}}\left(\Gamma^i_{0k} - \frac{g_{0k}\,\Gamma^i_{00}}{g_{00}}\right), \tag{1.49}$$

$$F^k = -\,\frac{c^2\,\Gamma^k_{00}}{g_{00}}\,, \tag{1.50}$$

$$g^{i\alpha}g^{k\beta}\,\Gamma^m_{\alpha\beta} = h^{iq}h^{ks}\Delta^m_{qs}\,. \tag{1.51}$$

Zelmanov had also deduced formulae for the chr.inv.-projections of the Riemann-Christoffel tensor. He followed the same procedure by which the Riemann-Christoffel tensor was built, proceeding from the non-commutativity of the second derivatives of an arbitrary vector taken in the given space. Taking the second chr.inv.-derivatives of an arbitrary vector

$${}^*\nabla_i\,{}^*\nabla_k\,Q_l - {}^*\nabla_k\,{}^*\nabla_i\,Q_l = \frac{2A_{ik}}{c^2}\,\frac{{}^*\partial Q_l}{\partial t} + H^{\cdot\cdot\cdot j}_{lki\cdot}Q_j\,, \tag{1.52}$$

he obtained the chr.inv.-tensor

$$H^{\cdots j}_{lki\cdot} = \frac{{}^*\partial\Delta^j_{il}}{\partial x^k} - \frac{{}^*\partial\Delta^j_{kl}}{\partial x^i} + \Delta^m_{il}\Delta^j_{km} - \Delta^m_{kl}\Delta^j_{im}\,, \tag{1.53}$$

which is like Schouten's tensor from the theory of non-holonomic manifolds [14]. The tensor $H^{\cdots j}_{lki\cdot}$ differs algebraically from the Riemann-Christoffel tensor because the presence of rotation of the space A_{ik} in the formula (1.52). Nevertheless its generalization gives the chr.inv.-tensor

$$C_{lkij} = \frac{1}{4}\left(H_{lkij} - H_{jkil} + H_{klji} - H_{iljk}\right), \tag{1.54}$$

which possesses all the algebraic properties of the Riemann-Christoffel tensor in this three-dimensional space. Therefore Zelmanov called C_{iklj} the *chr.inv.-curvature tensor*, which actually is the tensor of the physically observable curvature of the spatial section of the observer. Its contraction step-by-step

$$C_{kj} = C^{\cdots i}_{kij\cdot} = h^{im}C_{kimj}\,, \qquad C = C^j_j = h^{lj}C_{lj} \tag{1.55}$$

gives the chr.inv.-scalar C which means the *observable three-dimensional curvature* of this space.

Substituting the necessary components of the Riemann-Christoffel tensor into the formulae for its chr.inv.-projections

$$X^{ik} = -\,c^2\,\frac{R^{i\cdot k}_{0\cdot 0\cdot}}{g_{00}}\,, \quad Y^{ijk} = -\,c\,\frac{R^{\cdot ijk}_{0\cdots}}{\sqrt{g_{00}}}\,, \quad Z^{ijkl} = c^2R^{ijkl}, \tag{1.56}$$

and by lowering indices Zelmanov obtained the formulae

$$X_{ij} = \frac{{}^*\partial D_{ij}}{\partial t} - \left(D^l_i + A^{\cdot l}_{i\cdot}\right)\left(D_{jl} + A_{jl}\right) + \frac{1}{2}\left({}^*\nabla_i F_j + {}^*\nabla_j F_i\right) - \frac{1}{c^2}\,F_iF_j\,, \tag{1.57}$$

$$Y_{ijk} = {}^*\nabla_i\left(D_{jk} + A_{jk}\right) - {}^*\nabla_j\left(D_{ik} + A_{ik}\right) + \frac{2}{c^2}A_{ij}F_k\,, \tag{1.58}$$

$$Z_{iklj} = D_{ik}D_{lj} - D_{il}D_{kj} + A_{ik}A_{lj} - A_{il}A_{kj} + 2A_{ij}A_{kl} - c^2C_{iklj}\,, \tag{1.59}$$

where we have $Y_{(ijk)} = Y_{ijk} + Y_{jki} + Y_{kij} = 0$ just like in the Riemann-Christoffel tensor. Contraction of the spatial observable projection Z_{iklj} step-by-step gives

$$Z_{il} = D_{ik}D^k_l - D_{il}D + A_{ik}A^{\cdot k}_{l\cdot} + 2A_{ik}A^{k\cdot}_{\cdot l} - c^2C_{il}\,, \tag{1.60}$$

$$Z = h^{il}Z_{il} = D_{ik}D^{ik} - D^2 - A_{ik}A^{ik} - c^2C\,. \tag{1.61}$$

These are the basics of the mathematical apparatus of physically observable quantities — Zelmanov's chronometric invariants [2–4].

Given these definitions, we can find how any geometric object of the four-dimensional pseudo-Riemannian space (space-time) is seemed from the viewpoint of any observer whose location is this space. For instance, having any equation obtained in the general covariant tensor analysis, we can calculate the chr.inv.-projections of it onto the time line and onto the spatial section of any particular body of reference, then formulate the chr.inv.-projections in the terms of the physically observable properties of the space of reference. This way we will arrive at equations containing only quantities measurable in practice.

§1.3 Mass-bearing particles and massless particles

According to up-to-date physical concepts [15], mass-bearing particles are characterized by the four-dimensional vector of momentum P^α, while massless particles are characterized by the four-dimensional wave vector K^α

$$P^\alpha = m_0 \frac{dx^\alpha}{ds}\,, \qquad K^\alpha = \frac{\omega}{c}\frac{dx^\alpha}{d\sigma}\,, \tag{1.62}$$

where m_0 is the rest-mass that characterizes a mass-bearing particle, while ω is the frequency that characterizes a massless particle. The space-time interval ds is taken as the derivation parameter for mass-bearing particles (non-isotropic trajectories, $ds \neq 0$). Along isotropic trajectories $ds = 0$ (for massless particles), but the observable three-dimensional interval is $d\sigma \neq 0$. Therefore $d\sigma$ is taken as the derivation parameter for massless particles.

The square of the momentum vector P^α along the trajectories of mass-bearing particles is not zero, and is constant

$$P_\alpha P^\alpha = g_{\alpha\beta}\, P^\alpha P^\beta = m_0^2 = const \neq 0\,, \tag{1.63}$$

i.e. P^α is a non-isotropic vector. The square of the wave vector K^α is zero along the trajectories of massless particles

$$K_\alpha K^\alpha = g_{\alpha\beta}\, K^\alpha K^\beta = \frac{\omega^2}{c^2}\frac{g_{\alpha\beta}\, dx^\alpha dx^\beta}{d\sigma^2} = \frac{\omega^2}{c^2}\frac{ds^2}{d\sigma^2} = 0\,, \tag{1.64}$$

so K^α is an isotropic vector.

Since ds^2 in the chr.inv.-form (1.29) expresses itself through the square of the relativistic root as

$$ds^2 = c^2 d\tau^2 - d\sigma^2 = c^2 d\tau^2 \left(1 - \frac{\mathrm{v}^2}{c^2}\right), \quad \mathrm{v}^2 = h_{ik}\, \mathrm{v}^i \mathrm{v}^k, \tag{1.65}$$

we can put P^α and K^α down as

$$P^\alpha = m_0 \frac{dx^\alpha}{ds} = \frac{m}{c}\frac{dx^\alpha}{d\tau}\,, \qquad K^\alpha = \frac{\omega}{c}\frac{dx^\alpha}{d\sigma} = \frac{k}{c}\frac{dx^\alpha}{d\tau}\,, \tag{1.66}$$

where $k = \frac{\omega}{c}$ is the wave number and m is the relativistic mass. From the obtained formulae, we can see that the physically observable time τ can be used as a universal derivation parameter with respect to both isotropic and non-isotropic trajectories, i.e. as the single derivation parameter for mass-bearing and massless particles.

Calculation of the contravariant components of P^α and K^α gives

$$P^0 = m\frac{dt}{d\tau}\,, \qquad P^i = \frac{m}{c}\frac{dx^i}{d\tau} = \frac{1}{c}\,m\mathrm{v}^i\,, \tag{1.67}$$

$$K^0 = k\frac{dt}{d\tau}\,, \qquad K^i = \frac{k}{c}\frac{dx^i}{d\tau} = \frac{1}{c}\,k\mathrm{v}^i\,, \tag{1.68}$$

where $m\mathrm{v}^i$ is the three-dimensional chr.inv.-vector of the momentum of a mass-bearing particle, while $k\mathrm{v}^i$ is the three-dimensional wave chr.inv.-vector of a massless particle. The observable chr.inv.-velocity of massless particles equals the observable chr.inv.-velocity of light $\mathrm{v}^i = c^i$ (1.23).

The formula for $\frac{dt}{d\tau}$ can be obtained from the square of the vector of the four-dimensional velocity of a particle U^α, which for a subluminal velocity, the velocity of light, and a superluminal velocity is, respectively

$$g_{\alpha\beta}\,U^\alpha U^\beta = +1\,, \qquad U^\alpha = \frac{dx^\alpha}{ds}\,, \qquad ds = cd\tau\sqrt{1-\frac{\mathrm{v}^2}{c^2}}\,, \tag{1.69}$$

$$g_{\alpha\beta}\,U^\alpha U^\beta = 0\,, \qquad U^\alpha = \frac{dx^\alpha}{d\sigma}\,, \qquad ds = 0\,,\quad d\sigma = cd\tau\,, \tag{1.70}$$

$$g_{\alpha\beta}\,U^\alpha U^\beta = -1\,, \qquad U^\alpha = \frac{dx^\alpha}{|ds|}\,, \qquad ds = cd\tau\sqrt{\frac{\mathrm{v}^2}{c^2}-1}\,. \tag{1.71}$$

Substituting the definitions of §1.2 for h_{ik}, v_i, v^i into each formulae for $g_{\alpha\beta}\,U^\alpha U^\beta$, we arrive at three quadratic equations with respect to $\frac{dt}{d\tau}$. They are the same for subluminal velocities, the velocity of light, and superluminal velocities

$$\left(\frac{dt}{d\tau}\right)^2 - \frac{2v_i\mathrm{v}^i}{c^2\left(1-\frac{\mathrm{w}}{c^2}\right)}\frac{dt}{d\tau} + \frac{1}{\left(1-\frac{\mathrm{w}}{c^2}\right)^2}\left(\frac{1}{c^4}\,v_iv_k\mathrm{v}^i\mathrm{v}^k - 1\right) = 0\,. \tag{1.72}$$

This quadratic equation has two solutions

$$\left(\frac{dt}{d\tau}\right)_{1,2} = \frac{1}{1-\frac{\mathrm{w}}{c^2}}\left(\frac{1}{c^2}\,v_i\mathrm{v}^i \pm 1\right). \tag{1.73}$$

The function $\frac{dt}{d\tau}$ allows us to recognize what direction in time the particle takes. If $\frac{dt}{d\tau} > 0$, the time coordinate parameter t increases, i.e. the particle moves from the past into the future (the direct flow of time). If $\frac{dt}{d\tau} < 0$, the time coordinate parameter decreases, i.e. the particle moves from the future into the past (the reverse flow of time).

The quantity $1 - \frac{\mathrm{w}}{c^2} = \sqrt{g_{00}} > 0$, because the other cases $\sqrt{g_{00}} = 0$ and $\sqrt{g_{00}} < 0$ contradict the signature conditions $(+---)$. Therefore the coordinate time t stops $\frac{dt}{d\tau} = 0$ provided that

$$v_i \mathrm{v}^i = -\, c^2, \qquad v_i \mathrm{v}^i = +\, c^2. \tag{1.74}$$

The coordinate time t has the direct flow $\frac{dt}{d\tau} > 0$, if in the first and in the second solutions, are, respectively

$$\frac{1}{c^2}\, v_i \mathrm{v}^i + 1 > 0\,, \qquad \frac{1}{c^2}\, v_i \mathrm{v}^i - 1 > 0\,. \tag{1.75}$$

The coordinate time t has the reverse flow $\frac{dt}{d\tau} < 0$ at

$$\frac{1}{c^2}\, v_i \mathrm{v}^i + 1 < 0\,, \qquad \frac{1}{c^2}\, v_i \mathrm{v}^i - 1 < 0\,. \tag{1.76}$$

For subluminal particles, $v_i \mathrm{v}^i < c^2$ is always true. Hence the direct flow of time for regularly observed mass-bearing particles takes a place under the first condition from (1.75) while the reverse flow of time takes a place under the second condition from (1.76).

Note that we have looked at the problem of the direction of the coordinate time t assuming that the physically observable time is $d\tau > 0$ always due to the perception of any observer to see the events of his world in the order from the past to the future.

Now using formulae (1.67), (1.68), (1.73) we calculate the covariant components P_i and K_i, then — the projections of the four-dimensional vectors P^α and K^α onto the time line. We obtain

$$P_i = -\,\frac{m}{c}\,(\mathrm{v}_i \pm v_i)\,, \qquad K_i = -\,\frac{k}{c}\,(\mathrm{v}_i \pm v_i)\,, \tag{1.77}$$

$$\frac{P_0}{\sqrt{g_{00}}} = \pm m\,, \qquad \frac{K_0}{\sqrt{g_{00}}} = \pm k\,, \tag{1.78}$$

where the time projections $+m$ and $+k$ take a place during the observation of these particles moving into the future (the direct flow of time), while $-m$ and $-k$ take a place during the observation of these particles moving into the past (the reverse flow of time).

Therefore, the physically observable quantities are as follows. For a mass-bearing particle these are its relativistic mass $\pm m$ and the three-dimensional quantity $\frac{1}{c} m\mathrm{v}^i$, where $m\mathrm{v}^i$ is the observable vector of the momentum of the particle. For a massless particle these are the wave number of the particle $\pm k$ and the three-dimensional quantity $\frac{1}{c} k\mathrm{v}^i$, where $k\mathrm{v}^i$ is the observable wave vector of the particle.

From the obtained formulae (1.77) and (1.78), we can see that the observable wave vector $k\mathrm{v}^i$ characterizing massless particles is the complete analogue of the observable vector of the momentum $m\mathrm{v}^i$, which characterizes mass-bearing particles.

Substituting the obtained formulae for P^0, P^i, K^0, K^i, and also the formula for g_{ik} expressed through $h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k$ into the formulae for $P_\alpha P^\alpha$ (1.63) and $K_\alpha K^\alpha$ (1.64), we arrive at the relations between the physically observable energy and the physically observable momentum for a mass-bearing particle

$$\frac{E^2}{c^2} - m^2 \mathrm{v}_i \mathrm{v}^i = \frac{E_0^2}{c^2}\,, \tag{1.79}$$

and also that for a massless particle

$$h_{ik} \mathrm{v}^i \mathrm{v}^k = c^2, \tag{1.80}$$

where $E = \pm mc^2$ is the relativistic energy of the mass-bearing particle, while $E_0 = m_0 c^2$ is its rest-energy.

Therefore, by comparing the new common formulae for P^α and K^α (1.66) we have obtained, we arrive at an universal derivation parameter, which is the physically observable time τ, and is the same for both mass-bearing and massless particles. This is despite the fact that the four-dimensional dynamical vectors for particles of each of these two kinds, the vectors P^α and K^α, *differ* from each other.

Now we are going to find a universal dynamical vector, which in particular cases can be represented as the dynamical vector of a mass-bearing particle P^α and that of a massless particle K^α.

We will tackle this problem by assuming that the wave-particle duality, first introduced by Lois de Broglie for massless particles, is peculiar to particles of all kinds without any exception. That is, we will consider the motion of massless and mass-bearing particles as the propagation of waves in the approximation of geometric optics.

The four-dimensional wave vector of massless particles K^α in the approximation of geometric optics is [15]

$$K_\alpha = \frac{\partial \psi}{\partial x^\alpha}\,, \tag{1.81}$$

where ψ is the wave phase (known also as the *eikonal*) [15]. Following the same way, we set up the four-dimensional vector of the momentum of a mass-bearing particle

$$P_\alpha = \frac{\hbar}{c}\frac{\partial\psi}{\partial x^\alpha}\,, \tag{1.82}$$

where $\hbar$ is Planck's constant, while the coefficient $\frac{\hbar}{c}$ equates the dimensions of both parts of the equation. From these formulae we arrive at

$$\frac{K_0}{\sqrt{g_{00}}} = \frac{1}{c}\frac{{}^*\partial\psi}{\partial t}\,, \qquad \frac{P_0}{\sqrt{g_{00}}} = \frac{\hbar}{c^2}\frac{{}^*\partial\psi}{\partial t}\,. \tag{1.83}$$

Equating the quantities (1.83) to (1.78) we obtain

$$\pm\,\omega = \frac{{}^*\partial\psi}{\partial t}\,, \qquad \pm m = \frac{\hbar}{c^2}\frac{{}^*\partial\psi}{\partial t}\,. \tag{1.84}$$

From here we see that the value $+\omega$ for a massless particle and $+m$ for a mass-bearing particle take a place at the wave phase ψ which increases with time, while $-\omega$ and $-m$ take a place at the wave phase decreasing with time. From these expressions, we obtain a formula for the energy of both massless and mass-bearing particle, which takes the dual (wave-particle) nature of the particle into account. This is

$$\pm mc^2 = \pm\,\hbar\omega = \hbar\,\frac{{}^*\partial\psi}{\partial t} = E\,. \tag{1.85}$$

Now from (1.82) we obtain the dependence between the chr.inv.-momentum p^i of a particle and its wave phase ψ

$$p^i = m\mathrm{v}^i = -\,\hbar\,h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\,, \qquad p_i = m\mathrm{v}_i = -\,\hbar\,\frac{{}^*\partial\psi}{\partial x^i}\,. \tag{1.86}$$

Furthermore, as known [15], the condition $K_\alpha K^\alpha = 0$ can be presented in the form

$$g^{\alpha\beta}\,\frac{\partial\psi}{\partial x^\alpha}\,\frac{\partial\psi}{\partial x^\beta} = 0\,, \tag{1.87}$$

which is the basic equation of geometric optics known as the *eikonal equation*. Formulating the regular operators of derivation through the chr.inv.-differential operators $\frac{{}^*\partial}{\partial t}$ and $\frac{{}^*\partial}{\partial x^i}$, and taking into account that

$$g^{00} = \frac{1-\frac{1}{c^2}\,v_i v^i}{g_{00}}\,, \qquad g^{ik} = -\,h^{ik}, \qquad v^i = -\,c\,g^{0i}\sqrt{g_{00}}\,, \tag{1.88}$$

we arrive at the chr.inv.-eikonal equation for massless particles

$$\frac{1}{c^2}\left(\frac{{}^*\partial\psi}{\partial t}\right)^2 - h^{ik}\,\frac{{}^*\partial\psi}{\partial x^i}\,\frac{{}^*\partial\psi}{\partial x^k} = 0\,. \tag{1.89}$$

In the same way, we obtain the chr.inv.-eikonal equation for mass-bearing particles

$$\frac{1}{c^2}\left(\frac{{}^*\partial\psi}{\partial t}\right)^2 - h^{ik}\frac{{}^*\partial\psi}{\partial x^i}\frac{{}^*\partial\psi}{\partial x^k} = \frac{m_0^2c^2}{\hbar^2}\,, \tag{1.90}$$

which at $m_0 = 0$ becomes the same as the former one.

Substituting the relativistic mass m (1.84) into (1.66), we obtain the dynamical vector P^α which characterizes the motion of both massless and mass-bearing particles in geometric-optical approximation

$$P^\alpha = \frac{\hbar\omega}{c^3}\frac{dx^\alpha}{\partial\tau}\,, \qquad P_\alpha P^\alpha = \frac{\hbar^2\omega^2}{c^4}\left(1 - \frac{\mathrm{v}^2}{c^2}\right). \tag{1.91}$$

The length of the vector is a real quantity at $\mathrm{v} < c$, is zero at $\mathrm{v} = c$, and is an imaginary quantity at $\mathrm{v} > c$. Therefore, the obtained dynamical vector P^α characterizes particles with any rest-mass (real, zero, or imaginary).

The observable projections of the obtained universal vector P^α are

$$\frac{P_0}{\sqrt{g_{00}}} = \pm\,\frac{\hbar\omega}{c^2}\,, \qquad P^i = \frac{\hbar\omega}{c^3}\,\mathrm{v}^i, \tag{1.92}$$

where the observable time projection has the dimension of mass and the quantity $p^i = cP^i$ has the dimensions of momentum.

§1.4 Fully degenerate space-time. Zero-particles

As known, along the trajectories of massless particles (isotropic trajectories) the four-dimensional interval is zero

$$ds^2 = c^2d\tau^2 - d\sigma^2 = 0\,, \qquad c^2d\tau^2 = d\sigma^2 \neq 0\,. \tag{1.93}$$

But $ds^2 = 0$ not only at $c^2d\tau^2 = d\sigma^2$, but also when even a stricter condition is true, $c^2d\tau^2 = d\sigma^2 = 0$. The condition $d\tau^2 = 0$ means that the physically observable time τ has the same numerical value along the entire trajectory. The condition $d\sigma^2 = 0$ means that all three-dimensional trajectories have zero lengths. Taking into account the definitions of $d\tau$ (1.21), $d\sigma^2$ (1.28), and the fact that in any accompanying frame of reference $h_{00} = h_{0i} = 0$, we set down the conditions $d\tau^2 = 0$ and $d\sigma^2 = 0$ in the following, expanded form

$$c\,d\tau = \left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_iu^i\right)\right]c\,dt = 0\,, \qquad dt \neq 0\,, \tag{1.94}$$

$$d\sigma^2 = h_{ik}\,dx^i dx^k = 0\,, \tag{1.95}$$

where $u^i = \frac{dx^i}{dt}$ is the three-dimensional coordinate velocity of the particle, which isn't a physically observable chr.inv.-quantity.

As known, the necessary and sufficient condition of full degeneration of a metric means zero value of the determinant of the metric tensor. For the three-dimensional physically observable metric $d\sigma^2 = h_{ik}\, dx^i dx^k$ this condition is

$$h = \det ||\, h_{ik}|| = 0\,. \tag{1.96}$$

On the other hand, the determinant of the chr.inv.-metric tensor h_{ik} has the form [2–4]

$$h = -\,\frac{g}{g_{00}}\,, \tag{1.97}$$

where $g = \det ||\, g_{\alpha\beta}||$. Hence degeneration of the three-dimensional form $d\sigma^2$, i.e. $h = 0$, means degeneration of the four-dimensional form ds^2, i.e. $g = 0$. Therefore a four-dimensional space (space-time), wherein the conditions (1.94) and (1.95) are true, is a *fully degenerate space-time.*

Substituting $h_{ik} = -\,g_{ik} + \frac{1}{c^2}\, v_i v_k$ into (1.95), then dividing it by dt^2, we obtain (1.94) and (1.95), which are the *physical conditions of degeneration of the space* in the final form

$$\mathrm{w} + v_i u^i = c^2, \qquad g_{ik}\, u^i u^k = c^2 \left(1 - \frac{\mathrm{w}}{c^2}\right)^2, \tag{1.98}$$

where $v_i u^i$ is the scalar product of the linear velocity of the space rotation v_i and the coordinate velocity of the particle u^i.

If all quantities $v_i = 0$ (i.e. the space is holonomic), $\mathrm{w} = c^2$ and also $\sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2} = 0$. This means that the gravitational potential of the body of reference w is strong enough at the given point of the space (it is distant from the body) to bring the space to gravitational collapse at this point. This case will not be discussed here.

Below we shall look at the degeneration of the four-dimensional space (space-time) in the case, where the space is non-holonomic ($v_i \neq 0$), i.e. the spatial section of the observer experiences rotation.

Using the definition of $d\tau$ (1.21), we obtain the relation between the coordinate velocity u^i and the observable velocity v^i

$$\mathrm{v}^i = \frac{u^i}{1 - \frac{1}{c^2}\,(\mathrm{w} + v_k u^k)}\,. \tag{1.99}$$

Now we can write down ds^2 in a form in order to have the conditions of degeneration presented explicitly

$$ds^2 = c^2 d\tau^2 \left(1 - \frac{\mathrm{v}^2}{c^2}\right) = c^2 dt^2 \left\{ \left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)\right]^2 - \frac{u^2}{c^2} \right\}. \tag{1.100}$$

As obvious, the degenerate space-time can only host the particles for which the physical conditions of degeneration (1.98) are true.

We will refer to fully degenerate space-time as *zero-space*, while the particles allowed in a fully degenerate space-time (zero-space) will be referred to as *zero-particles*.

§1.5 An extended space for particles of all three kinds

When looking at the motion of mass-bearing and massless particles, we considered it in a four-dimensional space-time with the strictly non-degenerate metric ($g<0$). Now we are going to consider it in such a space-time wherein degeneration of metric is allowed ($g\leqslant 0$).

We already obtained, in the previous paragraph, the metric of such an extended space-time (see formula 1.100). Hence, the vector of the momentum of a mass-bearing particle P^α in such an extended space-time ($g\leqslant 0$) takes the form

$$P^\alpha = m_0\,\frac{dx^\alpha}{ds} = \frac{M}{c}\,\frac{dx^\alpha}{dt}\,, \tag{1.101}$$

$$M = \frac{m_0}{\sqrt{\left[1-\frac{1}{c^2}\,(\mathrm{w}+v_k u^k)\right]^2-\frac{u^2}{c^2}}}\,, \tag{1.102}$$

where M stands for the *gravitational rotational mass* of the particle. Such a mass M depends not only on the three-dimensional velocity of the particle with respect to the observer, but also on the gravitational potential w (the field of the body of reference) and on the linear velocity of rotation v_i of the space.

From the obtained formula (1.101) we see that in a four-dimensional space-time wherein degeneration of the metric is allowed ($g\leqslant 0$), the generalized parameter of derivation is the coordinate time t.

Substituting v^2 from (1.99) and $m_0 = m\sqrt{1-\mathrm{v}^2/c^2}$ into this formula, we arrive at the relationship between the relativistic mass of a particle m and its gravitational rotational mass M

$$M = \frac{m}{1-\frac{1}{c^2}\,(\mathrm{w}+v_i u^i)}\,. \tag{1.103}$$

From the obtained formula we see that M is a ratio between two quantities, each one is equal to zero in the case of degenerate metric ($g=0$), but the ratio itself is not zero $M\neq 0$.

This fact is no surprise. The same is true for the relativistic mass m in the case of $\mathrm{v}^2=c^2$. Once there $m_0=0$ and $\sqrt{1-\mathrm{v}^2/c^2}=0$, the ratio of these quantities is still $m\neq 0$.

Therefore, light-like (massless) particles are the ultimate case of mass-bearing particles at $\mathrm{v} \to c$. Zero-particles can be regarded the ultimate case of light-like ones that move in a non-holonomic space at the observable velocity v^i (1.99), which depends on the gravitational potential w of the body of reference and on the direction with respect to the linear velocity of rotation of the space.

The time component of the world-vector P^α (1.101) and the physically observable projection of the vector onto the time line are

$$P^0 = M = \frac{m}{1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)}\,, \tag{1.104}$$

$$\frac{P_0}{\sqrt{g_{00}}} = M\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right] = m\,, \tag{1.105}$$

while the spatial components of the vector are

$$P^i = \frac{M}{c}\,u^i = \frac{m}{c}\,\mathrm{v}^i, \tag{1.106}$$

$$P_i = -\,\frac{M}{c}\left[u_i + v_i - \frac{1}{c^2}\,v_i\left(\mathrm{w} + v_k u^k\right)\right]. \tag{1.107}$$

In a fully degenerate region of the extended space-time, i.e. under the physical conditions of degeneration (1.98), these components become

$$P^0 = M \neq 0\,, \qquad \frac{P_0}{\sqrt{g_{00}}} = m = 0\,, \tag{1.108}$$

$$P^i = \frac{M}{c}\,u^i, \qquad P_i = -\,\frac{M}{c}\,u_i\,, \tag{1.109}$$

i.e. particles of the degenerate space-time (zero-particle) bear zero relativistic mass, but their gravitational rotational masses are not zero.

Now we are going to look at mass-bearing particles in the extended space-time within the wave-particle duality concept. In such a case the components of the universal dynamical vector $P_\alpha = \frac{\hbar}{c}\frac{{}^*\partial\psi}{\partial x^\alpha}$ (1.82) are

$$\frac{P_0}{\sqrt{g_{00}}} = m = M\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right] = \frac{\hbar}{c^2}\frac{{}^*\partial\psi}{\partial t}\,, \tag{1.110}$$

$$P_i = \frac{\hbar}{c}\left(\frac{{}^*\partial\psi}{\partial x^i} - \frac{1}{c^2}\,v_i\,\frac{{}^*\partial\psi}{\partial t}\right), \tag{1.111}$$

$$P^i = \frac{m}{c}\,\mathrm{v}^i = \frac{M}{c}\,u^i = -\,\frac{\hbar}{c}\,h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\,, \tag{1.112}$$

$$P^0 = M = \frac{\hbar}{c^2\left(1 - \frac{\mathrm{w}}{c^2}\right)} \left(\frac{{}^*\partial\psi}{\partial t} - v^i \frac{{}^*\partial\psi}{\partial x^i} \right). \tag{1.113}$$

From these components, the following two formulae can be obtained

$$Mc^2 = \frac{1}{1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)} \hbar \frac{{}^*\partial\psi}{\partial t} = \hbar\,\Omega = E_{\mathrm{tot}}\,, \tag{1.114}$$

$$Mu^i = -\hbar\, h^{ik} \frac{{}^*\partial\psi}{\partial x^k}\,, \tag{1.115}$$

where Ω is the gravitational rotational frequency, while ω is the regular frequency

$$\Omega = \frac{\omega}{1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)}\,, \qquad \omega = \frac{{}^*\partial\psi}{\partial t}\,. \tag{1.116}$$

The first relation (1.114) links the gravitational rotational mass M to its corresponding total energy E_{tot}. The second relation (1.115) links the three-dimensional generalized momentum Mu^i to the gradient of the wave phase ψ.

The condition $P_\alpha P^\alpha = const$ in the approximation of geometric optics (the eikonal equation) takes the form (1.90). For the corpuscular form of this condition in the extended space-time we obtain the chr.inv.-formula

$$\frac{E^2}{c^2} - M^2u^2 = \frac{E_0^2}{c^2}\,, \tag{1.117}$$

where M^2u^2 is the square of the three-dimensional generalized momentum vector, $E = mc^2$, and $E_0 = m_0c^2$. Using this formula, we obtain the formula for the universal dynamic vector

$$P^\alpha = \frac{\hbar\,\Omega}{c^3}\frac{dx^\alpha}{dt} = \frac{\hbar \frac{{}^*\partial\psi}{\partial t}}{c^3\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right]}\frac{dx^\alpha}{\partial t}\,, \tag{1.118}$$

$$P_\alpha P^\alpha = \frac{\hbar^2\Omega^2}{c^4}\left\{ \left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right]^2 - \frac{u^2}{c^2} \right\}, \tag{1.119}$$

where the conditions of degeneration have been included.

In a degenerate region of the extended space-time we have $m_0 = 0$, $m = 0$, $\omega = \frac{{}^*\partial\psi}{\partial t} = 0$, and $P_\alpha P^\alpha = 0$. This means that from the viewpoint of an observer located in our world particles of a degenerate region (i.e. zero-particles) bear zero rest-mass m_0, zero relativistic mass m, zero relativistic frequency ω (which corresponds to the relativistic mass within

the wave-particle duality), while the length of the four-dimensional dynamical vector of any zero-particle is indeed conserved. Further for zero-particles, the gravitational rotational mass M (1.102), the three-dimensional generalized momentum Mu^i (1.115), and the gravitational rotational frequency Ω (1.116), which corresponds to the mass M according to the wave-particle duality, are not zero.

The zero-space metric $d\mu^2$ is not invariant from the viewpoint of an internal observer who is located in the zero-space. It can be proven proceeding from the 2nd condition of degeneration $d\sigma^2 = h_{ik}\, dx^i dx^k = 0$. Substituting here $h_{ik} = -\, g_{ik} + \frac{1}{c^2}\, v_i v_k$, dividing it by dt^2, and then substituting the 1st condition of degeneration $\mathrm{w} + v_i u^i = c^2$, we arrive at the internal zero-space metric

$$d\mu^2 = g_{ik}\, dx^i dx^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 \neq inv\,, \tag{1.120}$$

which is not invariant, as obvious. Hence, from the viewpoint of an observer located within the zero-space, the length of the four-dimensional vector of any zero-particle is not conserved along its trajectory

$$U_\alpha U^\alpha = g_{ik}\, u^i u^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 \neq const. \tag{1.121}$$

The eikonal equation for zero-particles can be obtained by substituting the conditions $m = 0$, $\omega = \frac{{}^*\partial\psi}{\partial t} = 0$, $P_\alpha P^\alpha = 0$ into the eikonal equation (1.89) or (1.90) we have obtained for mass-bearing and massless particles, respectively. As a result we obtain that the eikonal equation for zero-particles in the frame of reference of a regular observer whose location is our world is

$$h^{ik}\, \frac{{}^*\partial\psi}{\partial x^i}\, \frac{{}^*\partial\psi}{\partial x^k} = 0\,, \tag{1.122}$$

and this is a standing wave equation. This means that zero-particles may seem from the point of view of us *standing light waves* — the waves of stopped light (e.g. *information circles*).

As a result of our study of the extended space-time wherein full degeneration of the metric is allowed, we conclude that two ultimate space-time barriers exist in such a space-time:

1) *Light barrier*, to overcome which a particle should exceed the velocity of light;
2) *Zero-transition* for which a particle should be in a state of specific rotation depending on a particular distribution of matter (the conditions of degeneration).

§1.6 Equations of motion: general considerations

Now we are going to obtain the dynamical equations of motion of free particles in the extended space-time ($g \leqslant 0$), i.e. the equations of motion for mass-bearing, massless, and zero particles in a common form.

From the geometric viewpoint, the equations in question are those of parallel transfer in the sense of Levi-Civita applied to the universal dynamical vector P^α, i.e.

$$\mathrm{D}P^\alpha = dP^\alpha + \Gamma^\alpha_{\mu\nu} P^\mu dx^\nu = 0\,. \tag{1.123}$$

The equations of parallel transfer (1.123) are written in the general covariant form. In order for us to be able to use them in practice, the equations should contain only chronometrically invariant (physically observable) quantities. To bring the equations to the desired form we project them onto the time line and onto the spatial section of a frame of reference which accompanies to our references. We obtain

$$\left.\begin{aligned} b_\alpha \mathrm{D}P^\alpha &= \sqrt{g_{00}}\left(dP^0 + \Gamma^0_{\mu\nu} P^\mu dx^\nu\right) + \\ &\quad + \frac{g_{0i}}{\sqrt{g_{00}}}\left(dP^i + \Gamma^i_{\mu\nu} P^\mu dx^\nu\right) = 0 \\ h^i_\beta \mathrm{D}P^\beta &= dP^i + \Gamma^i_{\mu\nu} P^\mu dx^\nu = 0 \end{aligned}\right\}. \tag{1.124}$$

The Christoffel symbols found in these chr.inv.-equations (1.124) are not yet expressed in the terms of chr.inv.-quantities. We express the Christoffel symbols of the 2nd kind $\Gamma^\alpha_{\mu\nu}$ and those of the 1st kind $\Gamma_{\mu\nu,\sigma}$ included in them

$$\Gamma^\alpha_{\mu\nu} = g^{\alpha\sigma}\Gamma_{\mu\nu,\sigma}\,, \quad \Gamma_{\mu\nu,\rho} = \frac{1}{2}\left(\frac{\partial g_{\mu\rho}}{\partial x^\nu} + \frac{\partial g_{\nu\rho}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho}\right) \tag{1.125}$$

through the chr.inv.-properties of the accompanying frame of reference. Expressing the components $g^{\alpha\beta}$ and the first derivatives from $g_{\alpha\beta}$ through F_i, A_{ik}, D_{ik}, w, and v_i, after some algebra we obtain

$$\Gamma_{00,0} = -\frac{1}{c^3}\left(1 - \frac{\mathrm{w}}{c^2}\right)\frac{\partial \mathrm{w}}{\partial t}\,, \tag{1.126}$$

$$\Gamma_{00,i} = \frac{1}{c^2}\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 F_i + \frac{1}{c^4}\, v_i \frac{\partial \mathrm{w}}{\partial t}\,, \tag{1.127}$$

$$\Gamma_{0i,0} = -\frac{1}{c^2}\left(1 - \frac{\mathrm{w}}{c^2}\right)\frac{\partial \mathrm{w}}{\partial x^i}\,, \tag{1.128}$$

$$\Gamma_{0i,j} = -\frac{1}{c}\left(1 - \frac{\mathrm{w}}{c^2}\right)\left(D_{ij} + A_{ij} + \frac{1}{c^2}\, F_j v_i\right) + \frac{1}{c^3}\, v_j \frac{\partial \mathrm{w}}{\partial x^i}\,, \tag{1.129}$$

$$\Gamma_{ij,0} = \frac{1}{c}\left(1-\frac{\mathrm{w}}{c^2}\right)\left[D_{ij} - \frac{1}{2}\left(\frac{\partial v_j}{\partial x^i} + \frac{\partial v_i}{\partial x^j}\right) + \frac{1}{2\,c^2}\big(F_i v_j + F_j v_i\big)\right], \quad (1.130)$$

$$\Gamma_{ij,k} = -\,\Delta_{ij,k} + \frac{1}{c^2}\left[v_i A_{jk} + v_j A_{ik} + \frac{1}{2}\,v_k\left(\frac{\partial v_j}{\partial x^i} + \frac{\partial v_i}{\partial x^j}\right) - \right.$$
$$\left. - \frac{1}{2\,c^2}\,v_k\big(F_i v_j + F_j v_i\big)\right] + \frac{1}{c^4}\,F_k v_i v_j\,, \quad (1.131)$$

$$\Gamma^0_{00} = -\,\frac{1}{c^3}\left[\frac{1}{1-\frac{\mathrm{w}}{c^2}}\,\frac{\partial \mathrm{w}}{\partial t} + \left(1-\frac{\mathrm{w}}{c^2}\right)v_k F^k\right], \quad (1.132)$$

$$\Gamma^k_{00} = -\,\frac{1}{c^2}\left(1-\frac{\mathrm{w}}{c^2}\right)^2 F^k, \quad (1.133)$$

$$\Gamma^0_{0i} = \frac{1}{c^2}\left[-\frac{1}{1-\frac{\mathrm{w}}{c^2}}\,\frac{\partial \mathrm{w}}{\partial x^i} + v_k\left(D^k_i + A^{\cdot k}_{i\cdot} + \frac{1}{c^2}\,v_i F^k\right)\right], \quad (1.134)$$

$$\Gamma^k_{0i} = \frac{1}{c}\left(1-\frac{\mathrm{w}}{c^2}\right)\left(D^k_i + A^{\cdot k}_{i\cdot} + \frac{1}{c^2}\,v_i F^k\right), \quad (1.135)$$

$$\Gamma^0_{ij} = -\,\frac{1}{c\left(1-\frac{\mathrm{w}}{c^2}\right)}\left\{-\,D_{ij} + \frac{1}{c^2}\,v_n \times \right.$$
$$\times\left[v_j\left(D^n_i + A^{\cdot n}_{i\cdot}\right) + v_i\left(D^n_j + A^{\cdot n}_{j\cdot}\right) + \frac{1}{c^2}\,v_i v_j F^n\right] +$$
$$\left. + \frac{1}{2}\left(\frac{\partial v_i}{\partial x^j} + \frac{\partial v_j}{\partial x^i}\right) - \frac{1}{2\,c^2}\big(F_i v_j + F_j v_i\big) - \Delta^n_{ij}\,v_n\right\}, \quad (1.136)$$

$$\Gamma^k_{ij} = \Delta^k_{ij} - \frac{1}{c^2}\left[v_i\left(D^k_j + A^{\cdot k}_{j\cdot}\right) + v_j\left(D^k_i + A^{\cdot k}_{i\cdot}\right) + \frac{1}{c^2}\,v_i v_j F^k\right]. \quad (1.137)$$

Here Δ^i_{jk} stands for the chr.inv.-Christoffel symbols (1.48), which are determined similarly to $\Gamma^\alpha_{\mu\nu}$ with use of h_{ik} instead of $g_{\alpha\beta}$.

Having the regular operators of derivation expressed through the chr.inv.-differential operators, and writing down $dx^0 = c\,dt$ through $d\tau$ (1.21), we obtain a chr.inv.-formula for the regular differential

$$d = \frac{\partial}{\partial x^\alpha}\,dx^\alpha = \frac{{}^*\partial}{\partial t}\,d\tau + \frac{{}^*\partial}{\partial x^i}\,dx^i. \quad (1.138)$$

Now, having the chr.inv.-projections of P^α denoted as

$$\frac{P_0}{\sqrt{g_{00}}} = \varphi\,, \qquad P^i = q^i, \quad (1.139)$$

so that $P_0 = \varphi\sqrt{g_{00}}$ and $P^i = q^i$, we obtain the rest components of P^α

$$P^0 = \frac{1}{\sqrt{g_{00}}}\left(\varphi + \frac{1}{c}\, v_k q^k\right), \qquad P_i = -\frac{\varphi}{c}\, v_i - q_i\,. \tag{1.140}$$

With the formulae substituted into (1.124) we arrive at the chr.inv.-equations of parallel transfer of the vector P^α, which are

$$\left.\begin{aligned} &d\varphi + \frac{1}{c}\left(F_i q^i d\tau + D_{ik}\, q^i dx^k\right) = 0 \\ &dq^i + \left(\frac{\varphi}{c}\, dx^k + q^k d\tau\right)\left(D^i_k + A^{\cdot i}_{k\cdot}\right) - \frac{\varphi}{c}\, F^i d\tau + \Delta^i_{mk}\, q^m dx^k = 0 \end{aligned}\right\}. \tag{1.141}$$

From the obtained equations (1.141) we can make an easy transition to the desired dynamical equations of motion, with φ and q^i for the particles of different kinds substituted into (1.141) and divided by dt.

§1.7 Equations of motion in the extended space

The corpuscular and wave forms of the universal dynamical vector P^α for this case have been obtained in §1.5.

§1.7.1 Equations of motion of real mass-bearing particles

From (1.105) and (1.106) we obtain the chr.inv.-projections of P^α, taken the corpuscular form in the case of real mass-bearing particles,

$$\varphi = M\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)\right], \qquad q^i = M\,\frac{u^i}{c}\,, \tag{1.142}$$

where $\dfrac{u^2}{\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right]^2} < c^2$, $d\tau \neq 0$, $dt \neq 0$.

From here we immediately arrive at the corpuscular form of the dynamical equations of motion for real mass-bearing particles

$$\left.\begin{aligned} &\frac{d}{dt}\left\{M\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)\right]\right\} - \\ &\qquad - \frac{M}{c^2}\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)\right] F_i u^i + \frac{M}{c^2}\, D_{ik} u^i u^k = 0 \\ &\frac{d}{dt}\left(M u^i\right) + 2M\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)\right]\left(D^i_n + A^{\cdot i}_{n\cdot}\right) u^n - \\ &\qquad - M\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)\right] F^i + M\Delta^i_{nk}\, u^n u^k = 0 \end{aligned}\right\}, \tag{1.143}$$

where $d = \frac{{}^*\partial}{\partial t}\, d\tau + \frac{{}^*\partial}{\partial x^i}\, dx^i$, $\frac{d}{d\tau} = \frac{{}^*\partial}{\partial t} + \mathrm{v}^i \frac{{}^*\partial}{\partial x^i}$, and also

$$\frac{d}{dt} = \frac{{}^*\partial}{\partial t}\frac{d\tau}{dt} + u^i \frac{{}^*\partial}{\partial x^i} = \left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_m u^m\right)\right] \frac{{}^*\partial}{\partial t} + u^i \frac{{}^*\partial}{\partial x^i}\,. \qquad (1.144)$$

For the wave form of the universal dynamical vector P^α in the case of real mass-bearing particles we obtain, according to (1.110) and (1.112),

$$\varphi = \frac{\hbar}{c^2} \frac{{}^*\partial\psi}{\partial t}\,, \qquad q^i = -\frac{\hbar}{c}\, h^{ik} \frac{{}^*\partial\psi}{\partial x^k}\,, \qquad (1.145)$$

where the physically observable change of the wave phase ψ with time, i.e. the chr.inv.-derivative $\frac{{}^*\partial\psi}{\partial t}$, is positive for the particles moving from the past into the future, and is negative for those moving from the future into the past. From here we arrive at the wave form of (1.143), i.e. at the dynamical equations of propagation of waves, which correspond to real mass-bearing particles according to the wave-particle duality

$$\left.\begin{aligned}
&\pm \frac{d}{d\tau}\left(\frac{{}^*\partial\psi}{\partial t}\right) + \left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_p u^p\right)\right] F^i \frac{{}^*\partial\psi}{\partial x^i} - D^i_k u^k \frac{{}^*\partial\psi}{\partial x^i} = 0 \\
&\frac{d}{d\tau}\left(h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\right) \pm \frac{1}{c^2}\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_p u^p\right)\right] \frac{{}^*\partial\psi}{\partial t} F^i - \\
&\quad - \left\{\pm \frac{1}{c^2}\frac{{}^*\partial\psi}{\partial t}\, u^k - h^{km}\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_p u^p\right)\right] \frac{{}^*\partial\psi}{\partial x^m}\right\} \times \\
&\qquad\qquad \times \left(D^i_k + A^{\cdot i}_{k\cdot}\right) + h^{mn}\Delta^i_{mk} u^k \frac{{}^*\partial\psi}{\partial x^n} = 0
\end{aligned}\right\}. \qquad (1.146)$$

As seen, the first term in the time chr.inv.-equation and two terms in the spatial chr.inv.-equations of (1.146) are positive for particle-waves moving from the past into the future, while these terms are negative in the case of motion from the future into the past.

§1.7.2 Equations of motion of imaginary mass-bearing particles

In this case φ and q^i of the corpuscular form of P^α will differ from those presented for real mass-bearing particles (1.142) only by the presence of the multiplier $i = \sqrt{-1}$

$$\varphi = iM\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)\right], \qquad q^i = iM\frac{u^i}{c}\,, \qquad (1.147)$$

where $\dfrac{u^2}{\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right]^2} > c^2$, $d\tau \neq 0$, $dt \neq 0$.

Respectively, the corpuscular form of the dynamical equations of motion for imaginary mass-bearing particles (superluminal particles — tachyons) will differ from the equations obtained for real (subluminal) particles (1.143) by the presence of the coefficient i in the mass term M.

The chr.inv.-quantities φ and q^i for the wave form of the dynamical vector in the case of imaginary mass-bearing particles are the same as those for real mass-bearing particles (1.145). Hence, the wave form of the dynamical equations of motion is the same for both imaginary particle-waves and real particle-waves, and it is (1.146).

§ 1.7.3 Equations of motion of massless particles

According to (1.99), for massless (light-like) particles in the extended space-time (with taking the condition $\mathrm{v} = c$ into account) we have

$$\frac{u^2}{\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right]^2} = c^2, \quad d\tau \neq 0\,, \quad dt \neq 0\,. \tag{1.148}$$

Having this formula substituted into φ and q^i obtained for real mass-bearing particles being considered as corpuscles, i.e. into (1.142), we obtain

$$\varphi = M\frac{u}{c}\,, \qquad q^i = M\frac{u^i}{c}\,. \tag{1.149}$$

Respectively, the corpuscular form of the dynamical equations of motion for massless particles is

$$\left.\begin{aligned} &\frac{d}{dt}\left(Mu\right) + \frac{Mu}{c^2}\,F_i u^i + \frac{M}{c}\,D_{ik}\,u^i u^k = 0 \\ &\frac{d}{dt}\left(Mu^i\right) + 2M\frac{u}{c}\left(D^i_n + A^{\cdot i}_{n\cdot}\right)u^n - M\frac{u}{c}F^i + M\Delta^i_{nk}\,u^n u^k = 0 \end{aligned}\right\}. \tag{1.150}$$

The chr.inv.-quantities φ and q^i for the wave form of massless particles are the same that φ and q^i in the case of the wave form of mass-bearing particles (1.145). Respectively, the dynamical equations of propagation of waves, which correspond to massless particles in the framework of the wave-particle duality, are the same that those obtained for mass-bearing particle-waves (1.146).

§ 1.7.4 Equations of motion of zero-particles

In the degenerate space-time, i.e. under the conditions of degeneration, the chr.inv.-projections of P^α, taken in the corpuscular form, are

$$\varphi = 0\,, \qquad q^i = M\frac{u^i}{c}\,, \tag{1.151}$$

where $\mathrm{w}+v_k u^k=c^2,\ d\tau=0,\ dt\neq 0$. Applying these to the common chr.inv.-equations of parallel transfer (1.141), we obtain the corpuscular form of the dynamical equations of motion for zero-particles

$$\left.\begin{aligned} &\frac{M}{c^2}\,D_{ik}u^iu^k=0\\ &\frac{d}{dt}\big(Mu^i\big)+M\Delta^i_{nk}u^nu^k=0 \end{aligned}\right\}. \tag{1.152}$$

The chr.inv.-projections φ and q^i for the wave form of the generalized dynamical vector P^α in the degenerate space-time are

$$\varphi=0\,,\qquad q^i=-\frac{\hbar}{c}\,h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\,, \tag{1.153}$$

from which we arrive at the wave form of the dynamical equations of motion of zero-particles

$$\left.\begin{aligned} &D^m_k u^k\frac{{}^*\partial\psi}{\partial x^m}=0\\ &\frac{d}{dt}\left(h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\right)+h^{mn}\Delta^i_{mk}u^k\frac{{}^*\partial\psi}{\partial x^n}=0 \end{aligned}\right\}, \tag{1.154}$$

i.e. the dynamical equations of propagation of waves which correspond to zero-particles in the framework of the wave-particle duality.

§1.8 Equations of geodesic motion in the strictly non-degenerate space

In this case, the corpuscular and the wave forms of the universal dynamical vector P^α were obtained earlier in §1.3.

§1.8.1 Equations of motion of real mass-bearing particles

According to (1.78) and (1.67), for the corpuscular form of P^α in the case of real mass-bearing particles we have

$$\varphi=\pm m\,,\qquad q^i=\frac{1}{c}\,m\mathrm{v}^i, \tag{1.155}$$

where $\mathrm{v}^2<c^2$, $d\tau\neq 0$, $dt\neq 0$. Using these chr.inv.-quantities on (1.141), we obtain the dynamical equations of motion of particles with the positive relativistic mass $m>0$ (they move from the past into the future)

$$\left.\begin{aligned} &\frac{dm}{d\tau}-\frac{m}{c^2}\,F_i\mathrm{v}^i+\frac{m}{c^2}\,D_{ik}\mathrm{v}^i\mathrm{v}^k=0\\ &\frac{d\big(m\mathrm{v}^i\big)}{d\tau}+2m\left(D^i_k+A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k-mF^i+m\Delta^i_{nk}\mathrm{v}^n\mathrm{v}^k=0 \end{aligned}\right\}, \tag{1.156}$$

and also the equations of motion of particles with the negative relativistic mass $m<0$ (they move from the future into the past)

$$\left.\begin{aligned} -\frac{dm}{d\tau}-\frac{m}{c^2}F_i\mathrm{v}^i+\frac{m}{c^2}D_{ik}\mathrm{v}^i\mathrm{v}^k=0 \\ \frac{d\big(m\mathrm{v}^i\big)}{d\tau}+mF^i+m\Delta^i_{nk}\mathrm{v}^n\mathrm{v}^k=0 \end{aligned}\right\}. \tag{1.157}$$

For the wave form of P^α, from (1.83) and (1.86) we obtain formulae which are similar to those we obtained earlier for the wave form of P^α in the extended space-time (1.145)

$$\varphi=\frac{\hbar}{c^2}\frac{{}^*\partial\psi}{\partial t}\,,\qquad q^i=-\frac{\hbar}{c}\,h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\,, \tag{1.158}$$

where $\frac{{}^*\partial\psi}{\partial t}$, i.e. the change of the physically observable wave phase with time, is positive for the motion from the past into the future, and is negative for the motion from the future into the past. Taking into account the fact that the chr.inv.-equations of parallel transfer of P^α (1.141) in the strictly non-degenerate space-time should be divided by the interval of the physically observable time $d\tau$, we obtain the wave form of the dynamical equations of motion of mass-bearing real particles

$$\left.\begin{aligned} &\pm\frac{d}{d\tau}\left(\frac{{}^*\partial\psi}{\partial t}\right)+F^i\frac{{}^*\partial\psi}{\partial x^i}-D^i_k\mathrm{v}^k\frac{{}^*\partial\psi}{\partial x^i}=0 \\ &\frac{d}{d\tau}\left(h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\right)-\big(D^i_k+A^{\cdot i}_{k\cdot}\big)\left(\pm\frac{1}{c^2}\frac{{}^*\partial\psi}{\partial t}\mathrm{v}^k-h^{km}\frac{{}^*\partial\psi}{\partial x^m}\right)\pm \\ &\qquad\pm\frac{1}{c^2}\frac{{}^*\partial\psi}{\partial t}F^i+h^{mn}\Delta^i_{mk}\mathrm{v}^k\frac{{}^*\partial\psi}{\partial x^n}=0 \end{aligned}\right\}. \tag{1.159}$$

The first term of the time chr.inv.-equation and the first two terms of the spatial chr.inv.-equations of (1.159) are positive for the motion from the past into the future, and are negative for the motion from the future into the past.

§1.8.2 Equations of motion of imaginary mass-bearing particles

In this case, the corpuscular form of φ and q^i is different from that obtained in the case of real mass-bearing particles (1.155) by only the presence of the multiplier $i=\sqrt{-1}$

$$\varphi=\pm im\,,\qquad q^i=\frac{1}{c}\,im\mathrm{v}^i, \tag{1.160}$$

where $\mathrm{v}^2 > c^2$, $d\tau \neq 0$, $dt \neq 0$. Respectively, the corpuscular form of the dynamical equations of motion of imaginary (superluminal) particles are different from those we have obtained for real (subluminal) particles by the presence of the coefficient i in the mass term m.

The wave form of φ and q^i for imaginary mass-bearing particles is similar to that of real mass-bearing particles (1.158). Respectively, the dynamical equations of propagation of waves, which correspond to imaginary mass-bearing particles, are similar to the dynamical equations of propagation of waves, which correspond to real mass-bearing particles (1.159).

We now see that in the framework of the wave concept there is no difference at what velocity a mass-bearing particle travels (a wave propagates) — below the velocity of light or above that. On the contrary, in the framework of the corpuscular concept the equations of motion of superluminal (imaginary) particles differ from those of subluminal ones by the presence of the coefficient i in the mass term m.

§1.8.3 Equations of motion of massless particles

In this case, the corpuscular form of φ and q^i takes the form

$$\varphi = \pm\,\frac{\omega}{c} = \pm\,k\,, \qquad q^i = \frac{1}{c}\,k\mathrm{v}^i = \frac{1}{c}\,kc^i, \tag{1.161}$$

where $\mathrm{v}^2 = c^2$, $d\tau \neq 0$, $dt \neq 0$, and also the physically observable chr.inv.-velocity of light c^i (1.23) is attributed to any massless particle

$$\mathrm{v}^i = \frac{dx^i}{d\tau} = c^i, \qquad c_i c^i = h_{ik}\,c^i c^k = c^2. \tag{1.162}$$

According to all the parameters, we obtain the corpuscular dynamical equations of motion of massless particles. They are

$$\left.\begin{aligned} &\frac{d\omega}{d\tau} - \frac{\omega}{c^2}\,F_i\,c^i + \frac{\omega}{c^2}\,D_{ik}\,c^i c^k = 0 \\ &\frac{d\left(\omega c^i\right)}{d\tau} + 2\omega\left(D^i_k + A^{\cdot i}_{k\cdot}\right)c^k - \omega F^i + \omega\Delta^i_{nk}\,c^n c^k = 0 \end{aligned}\right\} \tag{1.163}$$

in the case of massless particles which bear the positive relativistic frequency $\omega > 0$ (they move from the past into the future), and also

$$\left.\begin{aligned} &-\frac{d\omega}{d\tau} - \frac{\omega}{c^2}\,F_i\,c^i + \frac{\omega}{c^2}\,D_{ik}\,c^i c^k = 0 \\ &\frac{d\left(\omega c^i\right)}{d\tau} + \omega F^i + \omega\Delta^i_{nk}\,c^n c^k = 0 \end{aligned}\right\} \tag{1.164}$$

in the case of massless particles which bear $\omega < 0$ (they move from the future into the past)

The wave form of φ and q^i for massless particles is similar to that for mass-bearing particles (1.158). Therefore the dynamical equations of propagation of waves corresponding to massless (light-like) particles in the framework of the wave-particle duality are similar to those of mass-bearing particles in the framework of this concept (1.159). The only difference is in the observable velocity v^i, which should be replaced with the vector of the observable velocity of light c^i.

§1.9 A particular case: equations of geodesic lines

What are geodesic equations? As we mentioned in §1.1, these are the kinematic equations of motion of particles along the shortest (geodesic) trajectories. From the geometric viewpoint, geodesic equations are equations of the Levi-Civita parallel transfer

$$\frac{\mathrm{D}Q^\alpha}{d\rho} = \frac{dQ^\alpha}{d\rho} + \Gamma^\alpha_{\mu\nu} Q^\mu \frac{dx^\nu}{d\rho} = \frac{d^2x^\alpha}{d\rho^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{d\rho}\frac{dx^\nu}{d\rho} = 0 \qquad (1.165)$$

of the four-dimensional kinematic vector of a particle $Q^\alpha = \frac{dx^\alpha}{d\rho}$, which is directed tangential to the trajectory at its every point. Respectively, the non-isotropic geodesic equations (they set up the trajectories of mass-bearing free particles) are

$$\frac{\mathrm{D}Q^\alpha}{ds} = \frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0\,, \qquad Q^\alpha = \frac{dx^\alpha}{ds}\,, \qquad (1.166)$$

while the isotropic geodesic equations (they set up the trajectories of massless free particles) are

$$\frac{\mathrm{D}Q^\alpha}{d\sigma} = \frac{d^2x^\alpha}{d\sigma^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma} = 0\,, \qquad Q^\alpha = \frac{dx^\alpha}{d\sigma}\,. \qquad (1.167)$$

On the other hand any kinematic vector, similar to the dynamical vector P^α of a mass-bearing particle and to the wave vector K^α of a massless particle, is a particular case of the arbitrary vector Q^α, for which we have obtained the universal chr.inv.-equations of parallel transfer. Hence with the chr.inv.-projections φ and q^i of the kinematic vector of a mass-bearing particle, substituted into the universal chr.inv.-equations of parallel transfer (1.141), we should immediately arrive at the non-isotropic geodesic equations in the chr.inv.-form. Similarly, having substituted φ and q^i of the kinematic vector of a massless particle, we should arrive at the chr.inv.-equations of isotropic geodesics. This is what we are going to do now, in this paragraph.

For the kinematic vector of mass-bearing particles we have

$$\left.\begin{aligned} \varphi &= \frac{Q_0}{\sqrt{g_{00}}} = \frac{g_{0\alpha}Q^\alpha}{\sqrt{g_{00}}} = \pm\,\frac{1}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} \\ q^i &= Q^i = \frac{dx^i}{ds} = \frac{1}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}}\,\frac{dx^i}{c\,d\tau} = \frac{1}{c\sqrt{1-\frac{\mathrm{v}^2}{c^2}}}\,\mathrm{v}^i \end{aligned}\right\}. \tag{1.168}$$

For massless particles, taking into account that $d\sigma = c\,d\tau$ on isotropic trajectories, we have

$$\left.\begin{aligned} \varphi &= \sqrt{g_{00}}\,\frac{dx^0}{d\sigma} + \frac{1}{c\sqrt{g_{00}}}\,g_{0k}\,c^k = \pm 1 \\ q^i &= \frac{dx^i}{d\sigma} = \frac{dx^i}{c\,d\tau} = \frac{1}{c}\,c^i \end{aligned}\right\}. \tag{1.169}$$

With these φ and q^i substituted into the universal chr.inv.-equations of parallel transfer (1.141), we obtain the chr.inv.-geodesic equations of non-isotropic geodesics (mass-bearing free particles)

$$\left.\begin{aligned} &\pm\frac{d}{d\tau}\left(\frac{1}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}}\right) - \frac{F_i\mathrm{v}^i}{c^2\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} + \frac{D_{ik}\mathrm{v}^i\mathrm{v}^k}{c^2\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} = 0 \\ &\frac{d}{d\tau}\left(\frac{\mathrm{v}^i}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}}\right) \mp \frac{F^i}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} + \frac{\Delta^i_{nk}\mathrm{v}^n\mathrm{v}^k}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} + \\ &\qquad + \frac{(1\pm 1)}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}}\left(D^i_k + A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k = 0 \end{aligned}\right\} \tag{1.170}$$

and also the chr.inv.-geodesic equations of isotropic geodesics (massless free particles)

$$\left.\begin{aligned} &D_{ik}\,c^i c^k - F_i\,c^i = 0 \\ &\frac{dc^i}{d\tau} \mp F^i + \Delta^i_{nk}\,c^n c^k + (1\pm 1)\left(D^i_k + A^{\cdot i}_{k\cdot}\right)c^k = 0 \end{aligned}\right\}. \tag{1.171}$$

The upper sign in the alternating terms in these equations stands for the motion of particles from the past into the future (the direct flow of time), while the lower sign stands for the motion from the future into the past (the reverse flow of time). As seen, we again have the

asymmetry of motion along the axis of time. The same asymmetry was found in the dynamical equations of motion. We see that such an asymmetry does not depend on the physical properties of the moving particles, but rather on the properties of the space of reference of the observer (actually, on the properties of his body of reference), such as F^i, A_{ik}, D_{ik}. In the absence of gravitational inertial forces, rotation or deformation of the space of reference, the asymmetry vanishes.

§1.10 A particular case: Newtion's laws

In this paragraph we prove that the dynamical chr.inv.-equations of motion of mass-bearing particles are the four-dimensional generalizations of Newton's 1st and 2nd laws in the space (space-time), which is non-holonomic (i.e. is rotating, $A_{ik} \neq 0$) and deforming ($D_{ik} \neq 0$), and is also filled with a gravitational field ($F^i \neq 0$).

At low velocities we have $m = m_0$, so the general covariant dynamical equations of motion take the form

$$\frac{\mathrm{D}P^\alpha}{ds} = m_0 \frac{d^2x^\alpha}{ds^2} + m_0 \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = 0\,, \tag{1.172}$$

where having these equations divided by m_0, the dynamical equations turn immediately into kinematic ones, i.e. the regular non-isotropic geodesic equations.

These are the dynamic equations of motion of the so-called "free particles", i.e. particles which fall freely under the action of a gravitational field.

The motion of particles under the action of both gravitational field and additional force R^α not of gravitational nature, is not geodesic

$$m_0 \frac{d^2x^\alpha}{ds^2} + m_0 \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = R^\alpha. \tag{1.173}$$

All these are the dynamical equations of motion of particles in the *four-dimensional space-time*, while Newton's laws were set forth for the *three-dimensional space*. In particular, the derivation parameter we use in these equations is the space-time interval, not applicable to a three-dimensional space.

Let us now look at the dynamical chr.inv.-equations of motion of mass-bearing particles. At low velocities the equations are

$$\left.\begin{aligned} &\frac{m_0}{c^2}\left(D_{ik}\mathrm{v}^i\mathrm{v}^k - F_i\mathrm{v}^i\right) = 0 \\ &m_0\frac{d^2x^i}{d\tau^2} - m_0F^i + m_0\Delta^i_{nk}\mathrm{v}^n\mathrm{v}^k + 2m_0\left(D^i_k + A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k = 0 \end{aligned}\right\}, \tag{1.174}$$

where the spatial chr.inv.-projections are the actual dynamical equations of motion along the spatial section (three-dimensional space).

In a four-dimensional space (space-time), wherein the spatial sections have the Euclidean metric, all quantities $h_i^k = \delta_i^k$ and the tensor of the space deformation is zero $D_{ik} = \frac{1}{2}\frac{^*\partial h_{ik}}{\partial t} = 0$. In such a case $\Delta_{kn}^i = 0$, hence $m_0 \Delta_{nk}^i \mathrm{v}^n \mathrm{v}^k = 0$. If there also $F^i = 0$ and $A_{ik} = 0$, the spatial chr.inv.-projections of the equations of motion take the form

$$m_0 \frac{d^2 x^i}{d\tau^2} = 0\,, \tag{1.175}$$

or, in another form,

$$\mathrm{v}^i = \frac{dx^i}{d\tau} = const. \tag{1.176}$$

Hence the four-dimensional generalizations of *Newton's 1st law* for mass-bearing particles can be set forth as follows:

> Newton's 1st law: If a particle is free from the action of gravitational inertial forces (or such forces are balanced) and, at the same time, both rotation and deformation of the space is absent, it will experience straight, even motion.

Such a condition, as seen from the formulae for the Christoffel symbols (1.132 – 1.137), is only possible in the case where all $\Gamma_{\mu\nu}^{\alpha} = 0$, because any component of the Christoffel symbols is function of at least one of the quantities F^i, A_{ik}, D_{ik}.

Now let us assume that $F^i \neq 0$, but $A_{ik} = 0$ and $D_{ik} = 0$. In such a case, the spatial chr.inv.-equations of motion take the form

$$\frac{d^2 x^i}{d\tau^2} = F^i. \tag{1.177}$$

On the other hand, the gravitational potential and the force F^i as well as the quantities A_{ik} and D_{ik} according their definitions describe the body of reference and the local space connected to it. The quantity F^i sets up the gravitational inertial force acting on a unit-mass particle. The force acting on a particle with a mass m_0 is

$$\Phi^i = m_0 F^i, \tag{1.178}$$

so the spatial chr.inv.-equations of motion become

$$m_0 \frac{d^2 x^i}{d\tau^2} = \Phi^i. \tag{1.179}$$

Correspondingly, the four-dimensional generalizations of *Newton's 2nd law* for mass-bearing particles can be set forth as follows:

> NEWTON'S 2ND LAW: The acceleration that a particle gains from a gravitational field is proportional to the gravitational inertial force acting on the particle from the side of this field, and is reciprocal to its mass, in the absence of both rotation and deformation of the space.

Having any particular value of the gravitational inertial force Φ^i substituted into the spatial chr.inv.-equations of motion, which constitute the second equation of (1.174),

$$m_0 \frac{d^2x^i}{d\tau^2} + m_0 \Delta^i_{nk} \mathrm{v}^n \mathrm{v}^k + 2m_0 \left(D^i_k + A^{\cdot i}_{k\cdot}\right) \mathrm{v}^k = \Phi^i, \tag{1.180}$$

we can solve the equations in order to obtain the three-dimensional observable coordinates of a mass-bearing particle in the three-dimensional space at any moment of time (the trajectory of this particle).

As seen from the equations, the presence of the gravitational inertial force is not mandatory to make the motion curved and uneven. This happens if at least one of the quantities F^i, A_{ik}, D_{ik} is not zero. Hence, theoretically, a particle can be in the state of uneven and curved motion in even the absence of gravitational inertial forces, but in the case where the space rotates or deforms or both.

If a particle moves under the joint action of the gravitational inertial force Φ^i and another force R^i not of gravitational nature, the spatial chr.inv.-equations of motion of the particle take the form

$$m_0 \frac{d^2x^i}{d\tau^2} + m_0 \Delta^i_{nk} \mathrm{v}^n \mathrm{v}^k + 2m_0 \left(D^i_k + A^{\cdot i}_{k\cdot}\right) \mathrm{v}^k = \Phi^i + R^i. \tag{1.181}$$

Given a flat three-dimensional space, there $\Delta^i_{kn} = 0$ is true: the second term in the equations vanishes. Due to the fact that such a space is free of rotation and deformation, the spatial chr.inv.-equations of motion of the particle take the form

$$m_0 \frac{d^2x^i}{d\tau^2} = \Phi^i, \qquad m_0 \frac{d^2x^i}{d\tau^2} = \Phi^i + R^i, \tag{1.182}$$

respectively in the case of only the force of gravitation and inertia Φ^i, and in the case of that in common with an additional force R^i not of gravitational nature, which deviates particles from a geodesic line.

Thus we have obtained that motion under the action of gravitational inertial forces is possible in either curved or flat space. Why?

As known, the curvature of a Riemannian space is characterized by the Riemann-Christoffel curvature tensor $R_{\alpha\beta\gamma\delta}$ consisting of the second derivatives of the fundamental metric tensor $g_{\alpha\beta}$ and the first derivatives

of it. The necessary and sufficient condition of a Riemannian space to be curved is $R_{\alpha\beta\gamma\delta} \neq 0$. To have non-zero curvature, it is necessary and sufficient that the second derivatives of $g_{\alpha\beta}$ are non-zero.

On the other hand we also know that the first derivatives of the fundamental metric tensor $g_{\alpha\beta}$ in a flat space may not be zero.

The chr.inv.-equations of motion contain the quantities Δ^i_{kn}, F^i, A_{ik}, D_{ik}, which depend on the first derivatives of $g_{\alpha\beta}$. Therefore at $R_{\alpha\beta\gamma\delta} = 0$ (a flat space) the Christoffel symbols Δ^i_{kn}, the gravitational inertial force F^i, the space rotation A_{ik}, and the space deformation D_{ik} may not be equal to zero.

§1.11 Analysis of the equations: the ultimate transitions between the basic space and zero-space

As we can see, at $\mathrm{w} = -v_i u^i$ in our formulae the quantities of the extended space-time ($g \leqslant 0$) are replaced by those of the strictly non-degenerate space-time ($g < 0$)

$$d\tau = \left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right] dt = dt\,, \tag{1.183}$$

$$u^i = \frac{dx^i}{dt} = \frac{dx^i}{d\tau} = \mathrm{v}^i, \tag{1.184}$$

$$M = \frac{m}{1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)} = m\,, \tag{1.185}$$

$$P^0 = M = m\,, \qquad P^i = \frac{1}{c} M u^i = \frac{1}{c} m\mathrm{v}^i, \tag{1.186}$$

and in this transition the coordinate time t coincides with the physically observable time τ.

Of course, once $\mathrm{w} \to 0$ (weak gravitational field) and $v_i = 0$ (no rotation of the space) at the same time, the transformation also occurs under a narrower condition $\mathrm{w} = -v_i u^i = 0$. On the other hand, it is doubtful to find a region free of rotation and gravitational fields in the observed part of the Universe. We therefore see that the transition to the regular (strictly non-degenerate) space-time always happens at

$$\mathrm{w} = -v_i u^i = -v_i \mathrm{v}^i. \tag{1.187}$$

Substituting this condition into the equations of motion we have obtained in §1.7 and §1.8, we arrive at the following conclusions on the geometrical structure of the extended space-time.

The corpuscular equations of motion (*particle-balls*) in the extended space-time transform into those in the regular (strictly non-degenerate) space-time *in full*, i.e. no terms are vanished or new terms are added up, only in the case of motion from the past into the future ($m>0$, $im>0$, $\omega>0$). For particle-balls, which move from the future into the past ($m<0$, $im<0$, $\omega<0$), such a transformation is *incomplete*.

On the other hand, the wave equations of motion (*particle-waves*) in the extended space-time transform into those in the regular space-time *in full* for both particles with $m>0$, $im>0$, $\omega>0$ (they move from the past into the future) and particles with $m<0$, $im<0$, $\omega<0$ (they move from the future into the past).

In the next §1.12 we are going to find out why this happens.

In the regular space-time ($g<0$) we have P^0 (1.67), which after substitution of $\frac{dt}{d\tau}$ (1.73) and the transition condition $\mathrm{w}=-v_i u^i=-v_i \mathrm{v}^i$ becomes the sign-alternating relativistic mass

$$P^0 = m\,\frac{dt}{d\tau} = \frac{m}{1-\frac{\mathrm{w}}{c^2}}\left(\frac{1}{c^2}\,v_i \mathrm{v}^i \pm 1\right) = \pm\, m\,. \tag{1.188}$$

In the extended space-time ($g\leqslant 0$) we have obtained $P^0=M$, but through another method (1.104), without use of $\frac{dt}{d\tau}$ which is the source of the alternating sign in the formula (1.188).

Hence the component $P^0=\pm m$ obtained in the regular space-time (1.188), taking two numerical values, can not be a particular case of the single value $P^0=M$ obtained in the extended space-time.

To understand the reason why, we turn from the sign-alternating formula $P^0=\pm m$ of the regular space-time to the formula $P^0=M$ of the extended space-time. This can be easily done by substituting the already known relationship between the physically observable velocity v^i and the coordinate velocity u^i (1.99) into the sign-alternating formula $P^0=\pm m$ (1.188).

As a result, we obtain the expanded relation for the component P^0 in the extended space-time

$$P^0 = \frac{m}{1-\frac{\mathrm{w}}{c^2}}\left[\frac{1}{c^2}\,\frac{v_i u^i}{1-\frac{1}{c^2}\,(\mathrm{w}+v_i u^i)} \pm 1\right], \tag{1.189}$$

which evidently takes the alternating sign. For particles, which move in the extended space-time from the past into the future, P^0 becomes

$$P^0 = \frac{m}{1-\frac{1}{c^2}\,(\mathrm{w}+v_i u^i)} = +M\,, \tag{1.190}$$

which is the same that (1.104). For particles, which move move from the future into the past, P^0 becomes

$$P^0 = \frac{m\left[\frac{1}{c^2}\left(2v_i u^i + \mathrm{w}\right) - 1\right]}{\left(1 - \frac{\mathrm{w}}{c^2}\right)\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right]} = -M\,. \tag{1.191}$$

These are two funally generalized formulae for P^0. Naturally, in the framework of the regular space-time the first formula $P^0 = +M$ (1.190) unambiguously transforms into $P^0 = +m$, while the second formula $P^0 = -M$ (1.191) transforms into $P^0 = -m$.

It should be noted that the remarks made on the sign-alternating formulae for P^0 do not affect all dynamical equations of motion we have obtained in the extended space-time. This is because the obtained equations of motion include the gravitational rotational mass in the general notation, M, without any respect to a particular composition of it. Substituting these two values of M into the equations of motion, we arrive at mere the equations of two kinds: the equation of motion from the past into the future, and the equation of motion from the future into the past.

Let us now come back to the physical condition $\mathrm{w} = -v_i u^i$ (1.187), which manifests the transition from the dynamical equations of motion in the extended space-time to those in the regular space-time. We have also found that $d\tau = dt$ (1.183) under this condition. On the other hand we know that the equality $d\tau = dt$ is not imperative in the regular space-time. On the contrary, in the observed Universe the interval of the physically observable time $d\tau$ is almost always a bit different from the interval of the coordinate time dt.

Therefore, the ultimate transition from the extended space-time to the regular space-time that occurs under the condition $\mathrm{w} = -v_i u^i$ is not a case of the conditions usual in the regular space-time.

Does that contain a contradiction between the equations of motion in the regular space-time and those in the extended space-time?

No it doesn't. All laws applicable to the regular space-time ($g < 0$) are as well true in a non-degenerate region ($g < 0$) of the extended space-time $g \leqslant 0$. At the same time those two non-degenerate regions *are not the same*. That is, the degenerate space-time added up to the regular space-time produces two absolutely segregate manifolds. The extended space-time is a different manifold and is absolutely independent of either strictly non-degenerate space-time or degenerate one. So there is no surprise in the found fact that the transition from one to another occurs under very limited particular conditions.

The only question is what configuration of those manifolds exists in the observable Universe. Two options are possible here:

a) The non-degenerate space-time ($g<0$) and the degenerate space-time ($g=0$) exist as *two segregate manifolds*: the regular space-time of the General Theory of Relativity with a small "add-on" of zero-space;

b) The non-degenerate space-time and the degenerate space-time exist as *two internal regions of the same manifold* — the extended space-time ($g\leqslant 0$) which we have looked at.

In either case, the ultimate transition from the non-degenerate space-time into the degenerate space-time occurs under the physical conditions of degeneration (1.98). Future experiments and astronomical observations will show which one of these two options exists in reality.

§1.12 Analysis of the equations: asymmetry of the space and the world beyond the mirror

Compare the corpuscular equations of motion for particles with $m>0$ (1.156) and $\omega>0$ (1.163) with those for particles with $m<0$ (1.157) and $\omega<0$ (1.164).

Even a first look manifests the obvious fact that the corpuscular equations of motion (particle-balls) from the past into the future are different from those for from the future into the past. The same asymmetry exists for the wave form of the equations (particle-waves). Why?

From the purely geometrical viewpoint, asymmetry of the equations of motion into the future or into the past manifests follows:

> There in the four-dimensional, curved, inhomogeneous space-time (pseudo-Riemannian space) is a *primordial asymmetry* of the directions into the future and into the past.

To understand the origin of such a primordial asymmetry we consider an example.

Assume that there in the four-dimensional space-time is a *mirror*, which coincides with the spatial section and, hence, separates the past from the future. Assume also that the mirror reflects all particles and waves coming on it from the past and from the future. In such a case the particles and waves which move from the past into the future ($m>0$, $im>0$, $\omega>0$) always hit the mirror, then bounce back into the past so that their properties reverse ($m<0$, $im<0$, $\omega<0$). At the same time, *vice versa*, the particles and waves which move from the future into the past ($m<0$, $im<0$, $\omega<0$) hitting the mirror change the sign of their properties ($m>0$, $im>0$, $\omega>0$) to bounce back into the future.

With the aforementioned *concept of the mirror* everything becomes easy to understand. Look at the wave form of the equations of motion (1.159). After reflection from the mirror, the quantity $\frac{^*\partial\psi}{\partial t}$ changes its sign. Hence the equations of propagation of waves into the future ("plus" in the equations) become those of the same wave propagating into the past ("minus" in the equations), and *vice versa* the equations of propagation of waves into the past ("minus") after reflection become those of the same wave propagating into the future ("plus").

Noteworthy, the equations of propagation of waves into the future and those into the past transform into each other *in full*, i.e. no terms are vanished and no new terms are added up. Hence the wave form of matter *fully reflects* from the mirror.

This is not the case for the corpuscular equations of motion. After reflection from the mirror, the quantity $\varphi = \pm m$ for mass-bearing particles and also $\varphi = \pm k = \pm \frac{\omega}{c}$ for massless particles change their signs. However the corpuscular equations of motion into the future transform into those of motion into the past *not in full*. In the spatial chr.inv.-equations of motion into the future, there is an additional term present. This term is not found in the spatial chr.inv.-equations of motion into the past. This term for mass-bearing and massless particles is, respectively,

$$2m\left(D^i_k + A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k, \qquad 2k\left(D^i_k + A^{\cdot i}_{k\cdot}\right)c^k. \tag{1.192}$$

Hence a particle which moves from the past into the future hits the mirror and bounces back to lose a term in its spatial chr.inv.-equations of motion, and *vice versa* a particle moving from the future into the past bounces from the mirror to acquire an additional term in the spatial chr.inv.-equations of motion. So, we have obtained that the mirror itself affects the trajectories of particles!

As a result, a particle with negative mass or frequency *is not* a simple mirror reflection of a particle whose mass or frequency is positive. Either in the case of particle-balls or in the case of particle-waves we do not deal with simple reflection or bouncing from the mirror, but with *entering* through the mirror into the *mirror world*. There in the mirror world all particles bear negative masses or frequencies, and move from the future into the past (it is from the viewpoint of an observer whose location is our world) .

Particle-waves of our world have no effect on the mirror world as well as particle-waves of the mirror world have no effect on us. On the contrary, particle-balls of our world may affect the mirror world, and particle-balls of the mirror world may have some effect on our world.

Full isolation of our world from the mirror world, i.e. the absence of mutual influence between particles of both worlds, takes a place under the obvious condition

$$D^i_k \mathrm{v}^k = -A^{\cdot i}_{k\cdot} \mathrm{v}^k, \tag{1.193}$$

which sets up the asymmetric term (1.192) of the corpuscular equations of motion to be zero. This happens only if $A^{\cdot i}_{k\cdot} = 0$ and $D^i_k = 0$, i.e. in a region of the space which is free of rotation and deformation.

Noteworthy, if particles of positive mass (frequency) were co-existing in our world with those of negative mass (frequency), they would interfere to destroy each other inevitably so no particles would be left in our world. On the contrary, we observe nothing of the kind.

Therefore in the second part of our analysis of the obtained equations of motion we arrive at the following conclusions:

1) The primordial (fundamental) asymmetry of the space-time directions into the future and into the past is due to a certain space-time mirror, which coincides geometrically with the spatial section of the observer, and reflects all particles or waves which bounce it from the side of the past or of the future. At the same time the space-time mirror maintains such physical conditions, which very differ from those in the regular space-time, and meet the particular physical conditions in a fully degenerate region of the space-time (zero-space), wherein the physically observable time stops. We therefore arrive at an obvious conclusion that the rôle of such a space-time mirror is played by the whole zero-space or by a particular region in it;
2) The space-time falls apart into our world and the mirror world. In our world (positive relativistic masses and frequencies) all particles and waves move from the past into the future. In the mirror world (negative relativistic masses and frequencies) all particles and waves move from the future into the past;
3) If entering into the mirror world through the mirror, particles and waves of our world become seemed having negative masses and frequencies, and moving from the future into the past;
4) We observe neither particles with negative masses or frequencies nor waves with negative phases, because they exist in the mirror world, i.e. beyond the mirror. Particles or waves we can observe are those of our world, or those at the exit from the mirror (or when bouncing the mirror, as it seems to us) as they have come from the mirror world, so all particles and waves we can observe move from the past into the future.

§1.13 The physical conditions characterizing the direct and reverse flow of time

In this paragraph, we are going to look at physical conditions under which: a) time has direct flow, i.e. from the past into the future, b) time has reverse flow, i.e. from the future into the past, and c) time stops.

In the General Theory of Relativity, time is determined as the fourth coordinate $x^0 = ct$ of the four-dimensional space-time, where c is the velocity of light, while t is the coordinate time. The formula itself manifests the fact that t changes even with the velocity of light and does not depend on the physical conditions of observation. Therefore the coordinate time t is also referred to as the *ideal time*. Aside for the ideal time, there is the physically observable time τ (*real time*), which very depends on the conditions of observation. The theory of chronometric invariants determines the interval of the physically observable time as the chr.inv.-projection of the increment of the four-dimensional coordinates dx^α on the line of time of the observer

$$d\tau = \frac{1}{c}\, b_\alpha dx^\alpha. \tag{1.194}$$

In the frame of reference, which accompanies to a regular subluminal (substantial) observer, $d\tau$ is, according to (1.21),

$$d\tau = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt - \frac{1}{c^2}\, v_i dx^i = dt - \frac{1}{c^2}\, \mathrm{w} dt - \frac{1}{c^2}\, v_i dx^i. \tag{1.195}$$

From here we see that $d\tau$ consists of three parts: a) the interval of the coordinate time dt, b) the interval of the "gravitational" time $dt_{\mathrm{g}} = \frac{1}{c^2}\,\mathrm{w}dt$, and c) the interval of the "rotational" time $dt_{\mathrm{r}} = \frac{1}{c^2}\, v_i dx^i$. The stronger is the field of gravity of the body of reference and the faster rotates the space of the body (the space of reference of the observer), the slower flows the observable time $d\tau$ of the observer. Theoretically the strong enough gravitational field and the fast enough rotation of the space may stop the flow of the physically observable time.

We define the *mirror world* as the space-time where time flows backward with respect to that in our own frame of reference, located in our space-time.

The direction of the coordinate time t, which describes the displacement along the time coordinate axis $x^0 = ct$, is displayed by the sign of the derivative $\frac{dt}{d\tau}$. Respectively, the sign of the derivative $\frac{d\tau}{dt}$ displays the direction of the physically observable time τ.

In §1.3 we have obtained the coordinate time function $\frac{dt}{d\tau}$ (1.73), which comes from the condition of conservation of the four-dimensional

velocity of a subluminal, light-like and superluminal particle along its four-dimensional trajectory (1.69–1.71). On the other hand, the coordinate time function can also be obtained in another way by presenting the space-time interval $ds^2 = c^2 d\tau^2 - d\sigma^2$ as

$$ds^2 = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 - 2\left(1 - \frac{\mathrm{w}}{c^2}\right) v_i\, dx^i dt + g_{ik}\, dx^i dx^k. \qquad (1.196)$$

From here we see that the elementary space-time distance between two infinitely adjacent world-points consists of the three-dimensional distance $g_{ik}\, dx^i dx^k$ and two terms, which depend on the physical properties of the space (space-time).

The term $\left(1 - \frac{\mathrm{w}}{c^2}\right) c\, dt$ is due the fourth dimension (time) and the gravitational potential w which characterizes the field of the body of reference. In the absence of gravitational fields, the time coordinate $x^0 = ct$ changes evenly with the velocity of light. Once $\mathrm{w} \neq 0$, the coordinate x^0 changes in a "slower" manner by the quantity $\frac{\mathrm{w}}{c^2}$. The stronger gravitational potential w, the slower time flows. At $\mathrm{w} = c^2$ the coordinate time t stops completely. As well-known, such a condition is implemented in the state of gravitational collapse.

The term $\left(1 - \frac{\mathrm{w}}{c^2}\right) v_i\, dx^i dt$ is due to the joint action of the gravitational inertial force and the space rotation. This term is not zero only if $\mathrm{w} \neq c^2$ (i.e. out gravitational collapse) and also $v_i \neq 0$ (the space is non-holonomic, i.e. the three-dimensional space experiences rotation).

Having both parts of (1.196) divided by $ds^2 = c^2 d\tau^2 \left(1 - \frac{\mathrm{v}^2}{c^2}\right)$, we obtain a quadratic equation the same that (1.72), which has two solutions (1.73). Proceeding from the solutions (1.73), we see that the coordinate time increases $\frac{dt}{d\tau} > 0$, stops $\frac{dt}{d\tau} = 0$, and decreases $\frac{dt}{d\tau} < 0$ under the following conditions

$$\frac{dt}{d\tau} > 0 \qquad \text{if} \quad v_i \mathrm{v}^i > \pm c^2, \qquad (1.197)$$

$$\frac{dt}{d\tau} = 0 \qquad \text{if} \quad v_i \mathrm{v}^i = \pm c^2, \qquad (1.198)$$

$$\frac{dt}{d\tau} < 0 \qquad \text{if} \quad v_i \mathrm{v}^i < \pm c^2. \qquad (1.199)$$

As known, the regular (substantional) particles we observe move at velocities which are slow to the velocity of light. So the physical condition under which the coordinate time stops $v_i \mathrm{v}^i = \pm c^2$ (1.198) can not be met in the world of substance, but is permitted for the other states of matter (light-like matter, for instance).

The coordinate time increases ($\frac{dt}{d\tau} > 0$) at $v_i \mathrm{v}^i > \pm c^2$. In a regular laboratory, the linear velocity of the space rotation (e.g. the linear velocity of the daily rotation of the earth) is also slow to the velocity of light. Hence in a regular laboratory we have $v_i \mathrm{v}^i > -c^2$ (the angle α between the linear velocity of the space rotation and the observable velocity of the particle we observe is within the limits $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$). In such a regular case the flow of the coordinate time is direct, i.e. the particle moves from the past into the future.

The coordinate time decreases ($\frac{dt}{d\tau} < 0$) at $v_i \mathrm{v}^i < \pm c^2$.

Until now we have only looked at the flow of the coordinate time t. Now we are going to analyze the possible directions of the physically observable time τ, which depends on the sign of the derivative $\frac{d\tau}{dt}$. To obtain a formula for this derivative, we divide the formula we have obtained for $d\tau$ (1.195) by dt. We obtain

$$\frac{d\tau}{dt} = 1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right). \tag{1.200}$$

By definition, the clock of any regular observer registers always positive intervals of time irrespective of in what direction the clock's hands rotate. Therefore in a regular laboratory bound on the Earth, the physically observable time may increase or stop, but it never decreases. Nevertheless, the decrease of the observable time ($\frac{d\tau}{dt} < 0$) is possible in certain circumstances.

From (1.200) we see that the observable time increases, stops, or decreases under the following conditions, respectively

$$\frac{d\tau}{dt} > 0 \qquad \text{if} \quad \mathrm{w} + v_i u^i < c^2, \tag{1.201}$$

$$\frac{d\tau}{dt} = 0 \qquad \text{if} \quad \mathrm{w} + v_i u^i = c^2, \tag{1.202}$$

$$\frac{d\tau}{dt} < 0 \qquad \text{if} \quad \mathrm{w} + v_i u^i > c^2. \tag{1.203}$$

As obvious, the condition under which the observable time stops $\mathrm{w} + v_i u^i = c^2$ is also the condition of degeneration of the space-time (1.98). In a particular case, where the space is free of rotation, the observable time stops following gravitational collapse $\mathrm{w} = c^2$.

Generally speaking, the state of zero-space can be given by any of the whole scale of physical conditions represented as $\mathrm{w} + v_i u^i = c^2$. The state of gravitational collapse ($\mathrm{w} = c^2$) is only a particular case in the scale of the conditions, which occurs in the absence of the space rotation ($v_i = 0$). In other words, the *mirror membrane* between the world with

the direct flow of time and the world with the reverse flow of time (the mirror world) is not a particular region of a zero-space wherein gravitational collapse occurs, but the whole zero-space in general.

So what is the flow of the coordinate time t and what is the flow of the physically observable time τ?

In the function of the coordinate time $\frac{dt}{d\tau}$ we assume that the real time measured by any observer, the quantity τ, is the standard to which the coordinate time t is determined. In any calculation or observation we are linked to the observer himself. So, the function of the coordinate time $\frac{dt}{d\tau}$ manifests the motion of the observer along the time axis $x^0 = ct$, observed from his own viewpoint.

In the function of the observable time $\frac{d\tau}{dt}$ the standard to which the observer compares his measurements is the time coordinate t of him. That is, the physically observable time τ registered by the observer is determined with respect to the motion of the whole spatial section of the observer along the axis of time (this motion occurs evenly at the velocity of light). Therefore the function of the observable time $\frac{d\tau}{dt}$ gives a view of the observer from "outside", showing his *true motion* with respect to the time axis.

In other words, the function of the coordinate time $\frac{dt}{d\tau}$ shows the membrane between our world and the mirror world from the viewpoint of an observer himself (his logic recognizes always the observed time as that flowing from the past into the future). The function of the observable time $\frac{d\tau}{dt}$ gives an abstracted glimpse at the membrane from "outside". This means that the function of the observable time manifests the *true structure* of the space-time membrane between our world and the mirror world, where time flows at the opposite direction.

§1.14 Basic introduction into the mirror world

To obtain a more detailed view of the space-time membranes, we are going to use a *local geodesic frame of reference*. The fundamental metric tensor within the infinitesimal vicinity of a point in such a frame is

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{2}\left(\frac{\partial^2 \tilde{g}_{\mu\nu}}{\partial\tilde{x}^\rho\,\partial\tilde{x}^\sigma}\right)(\tilde{x}^\rho - x^\rho)\,(\tilde{x}^\sigma - x^\sigma) + \dots\ , \qquad (1.204)$$

i.e. the numerical values of its components in the vicinity of a point differ from those at this point itself only in the 2nd order terms or the higher other terms, which can be neglected. Therefore at any point in a local geodesic frame of reference the fundamental metric tensor (within the 2nd order terms withheld) is a constant, while the first derivatives of the metric, i.e. the Christoffel symbols, are zero [2–4].

As obvious, within the infinitesimal vicinity of any point in a Riemannian space a local geodesic frame of reference can be set up. As a result, at any point belonging to the local geodesic frame of reference, a flat space can be set up tangential to the Riemannian space so that the local geodesic frame of reference in the Riemannian space is a global geodesic frame in the tangential flat space. Because in a flat space the fundamental metric tensor is constant, in the vicinity of a point in the Riemannian space, the quantities $\tilde{g}_{\mu\nu}$ converge to those of the tensor $g_{\mu\nu}$ in the tangential flat space. That means that, in the tangential flat space, we can set up a system of the basis vectors $\vec{e}_{(\alpha)}$ tangential to the curved coordinate lines of the Riemannian space. Because the coordinate lines of a Riemannian space are curved (in a general case), and, in the case where the space is non-holonomic, are not even orthogonal to each other, the lengths of the basis vectors are sometimes substantially different from the unit length.

Consider the world-vector $d\vec{r}$ of an infinitesimal displacement, i.e. $d\vec{r} = (dx^0, dx^1, dx^2, dx^3)$. Then $d\vec{r} = \vec{e}_{(\alpha)} dx^\alpha$, where the components are

$$\left.\begin{aligned} \vec{e}_{(0)} &= (e^0_{(0)}, 0, 0, 0)\,, \qquad \vec{e}_{(1)} = (0, e^1_{(1)}, 0, 0) \\ \vec{e}_{(2)} &= (0, 0, e^2_{(2)}, 0)\,, \qquad \vec{e}_{(3)} = (0, 0, 0, e^3_{(3)}) \end{aligned}\right\}. \tag{1.205}$$

The scalar product of the vector $d\vec{r}$ with itself gives $d\vec{r}d\vec{r} = ds^2$. On the other hand, it is $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$. So, we obtain a formula

$$g_{\alpha\beta} = \vec{e}_{(\alpha)}\vec{e}_{(\beta)} = e_{(\alpha)} e_{(\beta)} \cos\left(x^\alpha; x^\beta\right), \tag{1.206}$$

which facilitates our better understanding of the geometric structure of different regions within the Riemannian space and even beyond it. According to (1.206),

$$g_{00} = e^2_{(0)}\,, \tag{1.207}$$

while, on the other hand, $\sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}$. Hence the length of the time basis vector $\vec{e}_{(0)}$ (it is tangential to the real line of time $x^0 = ct$) is

$$e_{(0)} = \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}\,, \tag{1.208}$$

so the lesser it is than one, the greater the gravitational potential w. In the case of gravitational collapse ($\mathrm{w} = c^2$), the length of the time basis vector $\vec{e}_{(0)}$ becomes zero.

According to (1.206) the quantity g_{0i} is

$$g_{0i} = e_{(0)} e_{(i)} \cos\left(x^0; x^i\right), \tag{1.209}$$

on the other hand, $g_{0i} = -\frac{1}{c}\, v_i \left(1 - \frac{\mathrm{w}}{c^2}\right) = -\frac{1}{c}\, v_i\, e_{(0)}$. Hence

$$v_i = -\, c\, e_{(i)} \cos\left(x^0; x^i\right). \tag{1.210}$$

Then according to the general formula (1.206)

$$g_{ik} = e_{(i)} e_{(k)} \cos\left(x^i; x^k\right), \tag{1.211}$$

we obtain the chr.inv.-metric tensor $h_{ik} = -\, g_{ik} + \frac{1}{c^2}\, v_i v_k$ in the form

$$h_{ik} = e_{(i)} e_{(k)} \Big[\cos\left(x^0; x^i\right) \cos\left(x^0; x^k\right) - \cos\left(x^i; x^k\right) \Big]. \tag{1.212}$$

From (1.210), we see that, from the geometrical viewpoint, v_i is the projection (scalar product) of the spatial basis vector $\vec{e}_{(i)}$ onto the time basis vector $\vec{e}_{(0)}$, multiplied by the velocity of light. If the spatial sections are everywhere orthogonal to the lines of time (giving holonomic space), $\cos\left(x^0; x^i\right) = 0$ and $v_i = 0$. In a non-holonomic space, the spatial sections are not orthogonal to the lines of time, so $\cos\left(x^0; x^i\right) \neq 0$. Generally $\left|\cos\left(x^0; x^i\right)\right| \leqslant 1$, hence the linear velocity of the space rotation v_i (1.210) can not exceed the velocity of light.

If $\cos\left(x^0; x^i\right) = \pm 1$, the velocity of the space rotation is

$$v_i = \mp\, c e_{(i)}\,, \tag{1.213}$$

and the time basis vector $\vec{e}_{(0)}$ coincides with the spatial basis vectors $\vec{e}_{(i)}$ (time "falls" into space). At $\cos\left(x^0; x^i\right) = +1$ the time basis vector is co-directed with the spatial ones $\vec{e}_{(0)} \uparrow\uparrow \vec{e}_{(i)}$. In the case $\cos\left(x^0; x^i\right) = -1$ the time and spatial basis vectors are oppositely directed $\vec{e}_{(0)} \uparrow\downarrow \vec{e}_{(i)}$.

Let us have a closer look at the condition $\cos\left(x^0; x^i\right) = \pm 1$. If any spatial basis vector is co-directed (or oppositely directed) relative to the time basis vector, the space is degenerate. Maximum degeneration occurs when all three vectors $\vec{e}_{(i)}$ coincide with each other and with the time basis vector $\vec{e}_{(0)}$.

The terminal condition of the coordinate time $v_i \mathrm{v}^i = \pm c^2$ presented through the basis vectors is

$$e_{(i)}\, \mathrm{v}^i \cos\left(x^0; x^i\right) = \mp\, c \tag{1.214}$$

and becomes true when $e_{(i)} = 1$, $\mathrm{v} = c$, and $\cos\left(x^0; x^i\right) = \pm 1$. In such a case, once the linear velocity of the space rotation reaches the velocity of light the angle between the time line and the spatial lines becomes either zero or π depends on the direction of the space rotation.

Let us illustrate this with a few examples.

Space does not rotate, i.e. is holonomic In this case $v_i = 0$, so the spatial sections are everywhere orthogonal to the lines of time and the angle between them is $\alpha = \frac{\pi}{2}$. Hence, in the absence of the space rotation, the time basis vector $\vec{e}_{(0)}$ is orthogonal to all spatial basis vectors $\vec{e}_{(i)}$. This means that all clocks can be synchronized, and will display the same time (synchronization of clocks at different points in the space does not depend on the path of synchronization). The linear velocity of the space rotation is $v_i = -c\, e_{(i)} \cos\alpha = 0$. At $v_i = 0$ we have

$$d\tau = \left(1 - \frac{\mathrm{w}}{c^2}\right) c\, dt\,, \qquad\qquad h_{ik} = -\, g_{ik}\,, \tag{1.215}$$

and the metric of the space-time $ds^2 = c^2 d\tau^2 - d\sigma^2$ becomes

$$ds^2 = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\, dx^i dx^k, \tag{1.216}$$

i.e. the observable time depends only on the gravitational potential w. Two options are possible here:

a) The gravitational inertial force is $F_i = 0$, and also the linear velocity of the space rotation is $v_i = 0$. In such a case, according to the definitions of F_i and v_i (see §1.2), we obtain $\sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2} = 1$ and $g_{0i} = -\frac{1}{c}\sqrt{g_{00}}\, v_i = 0$. The fact that the gravitational potential w vanishes means, in particular, that it does not depend on the three-dimensional coordinates (a homogeneous distributed gravitational field). In this case the motion of an observer across the space leaves the rates of clocks the same (the global synchronization of clocks remains unchanged with time).

b) Once $F_i \neq 0$ and $v_i = 0$, we have the derivative $\frac{\partial \mathrm{w}}{\partial x^i} \neq 0$ in the formula for F_i (1.33). This means that the gravitational potential w depends on the three-dimensional coordinates, i.e. the rate of clocks differs at different points of the space. Hence at $F_i \neq 0$ the synchronization of clocks at different points of a holonomic space (a space free of rotation) does not preserve with time.

In a holonomic space (a space free of rotation) gravitational collapse may occur ($\mathrm{w} = c^2$) only if $F_i \neq 0$. If $F_i = 0$ in a holonomic space, according to the definition of F_i (1.33) we have $\mathrm{w} = 0$ so gravitational collapse is not possible.

Space rotates at subluminal velocity In such a case the spatial sections are not orthogonal to the lines of time $v_i = -c\, e_{(i)} \cos\alpha \neq 0$. Because $-1 \leqslant \cos\alpha \leqslant +1$, we have $-c \leqslant v_i \leqslant +c$. Hence $v_i > 0$ at $\cos\alpha > 0$, and also $v_i < 0$ at $\cos\alpha < 0$.

Space rotates at the velocity of light (1st case) The lesser α, the greater v_i. In the ultimate case, where $\alpha = 0$, the linear velocity of the space rotation is $v_i = -c$. In such a case the spatial basis vectors $\vec{e}_{(i)}$ coincide with the time basis vector $\vec{e}_{(0)}$ (space coincides with time).

Space rotates at the velocity of light (2nd case) If $\alpha = \pi$, $v_i = +c$ and the time basis vector $\vec{e}_{(0)}$ also coincides with the spatial basis vectors $\vec{e}_{(i)}$, but is oppositely directed. This case may be understood as space coinciding time flowing from the future into the past.

§ 1.15 Who is a superluminal observer?

We can outline a few types of the frames of reference which may exist in the space-time of the General Theory of Relativity. Particles including an observer himself moving at a subluminal velocity ("inside" the light cone) bear real relativistic masses. In other words, such particles, body of reference, and observer are in the state of matter commonly referred to as "substance". Therefore any observer whose frame of reference is subluminal will be referred to as *subluminal observer* or *substantional observer* in other word.

Particles and an observer moving at the velocity of light (i.e. over the surface of the light cone) bear $m_0 = 0$, but their relativistic masses (masses of motion) are $m \neq 0$. They are in the light-like state of matter. Hence we will call an observer whose frame of reference is characterized by the light-like state a *light-like observer.*

Accordingly, we will call particles and an observer moving at a superluminal velocity *superluminal particles* and *superluminal observer.* They are in the state of matter where $m_0 \neq 0$, while their relativistic masses are imaginary.

It is intuitively clear who a subluminal observer is, this term requires no further explanation. The same more or less applies to a light-like observer. From the viewpoint of a light-like observer, the world around looks like a colourful system of light waves. But who is a superluminal observer? To understand this let us give an example.

Imagine a new supersonic jet airplane to be commissioned into operation. All members of the commission are inborn blind. And so is the pilot. Thus we may assume that all information about the surrounding world the pilot and the members of the commission gain from sound, that is from sound waves in air. It is sound waves that build a picture that those people will perceive as their "real world".

Now the airplane has taken off and begun to accelerate. As long as its velocity is less than the velocity of sound, the blind members

of the commission match up its "heard" position in the sky with the one we can see. But once the sound barrier is overcome, everything changes. The blind members of the commission still perceive the velocity of the airplane as equal to the velocity of sound regardless of its real velocity. For them the velocity of propagation of sound waves in the air is the *ultimate high speed of propagation of information*, while the real supersonic jet airplane is beyond their "real world", it is located in the world of "imaginary objects" and all properties of it are imaginary from their viewpoint. The blind pilot will hear nothing as well. Not a single sound will reach him from the past reality, and only local sounds from the cockpit (which also moves at the supersonic velocity) will break the silence. Once the velocity of sound is overcome, the blind pilot leaves the subsonic world for a new supersonic one. From his new viewpoint (the supersonic frame of reference), the old subsonic fixed world that contains the airport and the members of the commission will simply disappear, becoming a region of "imaginary quantities".

What is light? Transverse waves that run across a certain medium at a constant velocity. We perceive the world around through sight, receiving light waves from other objects. It is waves of light that build our picture of the "true real world".

Now imagine a spaceship which accelerates faster and faster to eventually overcome the light barrier at still growing velocity. From the purely mathematical viewpoint, this is quite possible in the space-time of the General Theory of Relativity. For us the velocity of the spaceship will still be equal to the velocity of light whatever is its real velocity. For us the velocity of light will be the ultimate high speed of propagation of information, while the real spaceship for us will stay in another "unreal" world of superluminal velocities wherein all properties are imaginary. The same is true for the spaceship's pilot. From his viewpoint, having the light barrier overcome brings him into a new superluminal world which becomes his "true reality", while the old world of subluminal velocities is gone, left behind in the region of "imaginary reality".

§ 1.16 Gravitational collapse in different regions of space

We will call *gravitational collapsar* a region of the space-time wherein the gravitational collapse condition $g_{00} = 0$ is true.

According to the theory of chronometric invariants $\sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}$. So, the condition of collapse $g_{00} = 0$ also means $\mathrm{w} = c^2$. We will look at such a collapsed region from outside, from the viewpoint of a regular observer who stays distant from such a region.

We put down the formula for the four-dimensional interval so that it contains an explicit ratio of w and c^2, i.e.

$$ds^2 = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 - 2\left(1 - \frac{\mathrm{w}}{c^2}\right) v_i\, dx^i dt + g_{ik}\, dx^i dx^k. \qquad (1.217)$$

Having substituted $\mathrm{w} = c^2$ into this formula, we obtain the space-time metric on the surface of the gravitational collapsar

$$ds^2 = g_{ik}\, dx^i dx^k. \qquad (1.218)$$

From here we see that gravitational collapse in the four-dimensional space-time can be correctly determined only if the the space-time is holonomic, i.e. the three-dimensional space of the observer is free of rotation (his spatial section is everywhere orthogonal to the lines of time).

Since in the absence of the space rotation the interval of the physically observable time is $d\tau = \sqrt{g_{00}}\, dt = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt$, the observable time stops ($d\tau = 0$) on the surface of a gravitational collapsar.

As a matter of fact that the denominator of the linear velocity of the space rotation

$$v_i = -\, c\, \frac{g_{0i}}{\sqrt{g_{00}}} = -\, c\, \frac{g_{0i}}{1 - \frac{\mathrm{w}}{c^2}} \qquad (1.219)$$

goes over to zero in the case of collapse ($\mathrm{w} = c^2$) and v_i becomes infinite. To avoid this, we assume $g_{0i} = 0$. Then metric (1.217) takes the form

$$ds^2 = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\, dx^i dx^k, \qquad (1.220)$$

so the problem of a singular state of the space-time becomes automatically removed. Proceeding from this, the metric on the surface of a gravitational collapsar (1.218) is

$$ds^2 = -\, d\sigma^2 = -\, h_{ik}\, dx^i dx^k, \qquad h_{ik} = -\, g_{ik}\,. \qquad (1.221)$$

From here we see that the four-dimensional interval on the surface of a gravitational collapsar is space-like: the elementary distance between two point on the surface of a gravitational collapsar is imaginary

$$ds = i d\sigma = i\sqrt{h_{ik}\, dx^i dx^k}\,. \qquad (1.222)$$

If $ds = 0$, the observable three-dimensional distance $d\sigma$ between two points on the surface of a gravitational collapsar also becomes zero.

Now we are going to look at gravitational collapse in different regions of the four-dimensional space-time.

COLLAPSE IN A SUBLUMINAL REGION Within this region, $ds^2 > 0$. This is the habitat of regular, real particles which move at subluminal velocities. Hence a gravitational collapsar in this region is filled with collapsed substance (*substantional collapsar*). On the surface of such a collapsar, the metric is space-like: here $ds^2 < 0$, so all particles there bear imaginary relativistic masses. Of course, the metric on the surface of such a gravitational collapsar is non-degenerate.

COLLAPSE IN A LIGHT-LIKE REGION Within this region $ds^2 = 0$. This is an isotropic space of light-like (massless) particles. A gravitational collapsar in this region is filled with light-like matter (*light-like collapsar*). The metric (1.221) on its surface is $d\sigma^2 = -\,g_{ik}\,dx^i dx^k = 0$. This can be true provided that:

a) The surface of the light-like collapsar shrinks into a point (in other word, all $dx^i = 0$), or
b) The three-dimensional spatial metric is degenerate ($\det ||\,g_{ik}|| = 0$). Because the four-dimensional metric is degenerate too, such a light-like collapsar is a zero-space in this case.

COLLAPSE IN A DEGENERATE REGION (ZERO-SPACE) As obvious the matter of a fully degenerate space-time region (zero-space) can collapse too. We will call such gravitational collapsars *degenerate collapsars*. As a matter of fact, from the condition of degeneration

$$\mathrm{w} + v_i u^i = c^2, \qquad g_{ik}\,dx^i dx^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2, \tag{1.223}$$

we see that in the case of collapse ($\mathrm{w} = c^2$) there is

$$v_i u^i = 0\,, \qquad g_{ik}\,dx^i dx^k = 0\,. \tag{1.224}$$

Hence gravitational collapse in a zero-space region also occurs in the absence of the space rotation ($v_i = 0$), and, because the conditions (1.224) are true, at the same time the surface of a degenerate collapsar is shrunk into a point.

Chapter 2 Motion of Particles as a Result of Motion of Space Itself

§2.1 Problem statement

Having substituted the gravitational potential w and the linear velocity of the space rotation v_i into the definition of the interval of the physically observable time $d\tau$ (1.21), we obtain the formula (1.21) as

$$\Big(1+\frac{1}{c^2}\,v_i\mathrm{v}^i\Big)d\tau=\Big(1-\frac{\mathrm{w}}{c^2}\Big)\,dt\,. \tag{2.1}$$

From here we see that a significant difference between $d\tau$ and dt may result from either a strong gravitational field or the velocities comparable to the velocity of light. Hence, in everyday life the difference between $d\tau$ and dt is not significantly great.

The physically observable time coincides with the coordinate time $dt=d\tau$ only under the condition

$$\mathrm{w}=-\,v_i\mathrm{v}^i. \tag{2.2}$$

Actually, such a condition means that the gravitational attraction of a particle by the reference body of the observer is fully compensated by the rotation of the space of the reference body (the reference space) and the motion of the particle itself. That is, (2.2) is the mathematical formulation of the *weightless condition*. Having the gravitational potential plugged in according to Newton' formula, we obtain

$$\frac{GM}{r}=v_i\mathrm{v}^i. \tag{2.3}$$

If the orbital velocity of the particle is equal to the linear velocity of rotation of the gravitating body in this orbit, the weightless condition for the particle takes the form

$$\frac{GM}{r}=v^2, \tag{2.4}$$

i.e. the more distant the orbit from the attracting body, the lesser the velocity of a satellite in this orbit.

Is this statement met by experimental data? The Table below gives the orbital velocities of the Moon and the planets measured in astronomical observations and those calculated from the weightless condition.

Planet	Orbital velocity, km/sec	
	Measured	Calculated
Mercury	47.9	47.9
Venus	35.0	35.0
Earth	29.8	29.8
Mars	24.1	24.1
Jupiter	13.1	13.1
Saturn	9.6	9.6
Uranus	6.8	6.8
Neptune	5.4	5.4
Pluto	4.7	4.7
Moon	1.0	1.0

From the Table we see that the weightless condition we have obtained is true for any satellite which orbits a gravitating body. Note that the condition is true, if the orbital velocity of a planet is equal (or is very close) to the linear velocity of rotation of the space of the gravitating body in this orbit (2.4). This means that the rotating space of the gravitating body *carries* all bodies around it, generating their rotation.

If the space of the gravitating body would rotate like a solid body i.e. without any deformation, its angular velocity was constant ($\omega = const$), while the orbital velocities $\mathrm{v} = \omega r$ of the accompanying satellites were grow along with the radii of their orbits. However as we have just seen from the example of the planets in the solar system, the linear velocity of the orbital rotation decreases along with the distance from the Sun. This means that in reality the space of a gravitating body (the space of reference) does not rotate like a solid body, but rather like a viscous and deformable medium, wherein the layers distant from the centre do not rotate as far as those closer to the centre. As a result, the space of the gravitating body is *twisted* and the profile of the orbital velocities simply repeats the structure of the twisted space.

Hence we see that the orbital motion of particles in a gravitational field results from the *rotation of the space of the attracting body itself.*

What are the possible sequels for our present mathematical theory of the motion of particles following the conclusions we have just arrived at? We are going to find out shortly in what follows.

Assume a metric space. As obvious, the motion of the space itself allows us to match any of the points of this space to the vector of motion of such a point Q^α. It is also obvious that all points of the space will experience the same motion as the space itself. Hence Q^α can be regarded as the vector of motion of the space itself (at a given point). As a result, we obtain a vector field which describes the motion of the whole space.

Of course if the length of the vector Q^α remains constant in the motion, such a space moves so that its metric remains unchanged too. Hence if in such a space the vector of motion Q^α is set at a given point, the metric of the space can be found proceeding from the motion of the point (along with the motion of the space).

A way to solve this problem was paved in the late 19th century by Sophus Lie [16]. He obtained equations for the exterior derivative from the fundamental metric tensor $g_{\alpha\beta}$ of the space with respect to the trajectory of motion of the vector Q^α, where the components of Q^α were present as fixed coefficients. The number of the equations is equal to the number of the components of the metric tensor. Hence having the vector Q^α fixed, i.e. having the motion of the space set, we can solve the equations to find the components of the metric tensor $g_{\alpha\beta}$ proceeding from the components Q^α.

Later Van Danzig suggested calling such a derivative of the metric *Lie's derivative*.

Now we are going to look at a particular case of motion of the space which leaves its metric constant. This case was studied by Walter Killing [17]. As obvious, such a motion amounts to making Lie's derivative equal to zero (*Killing's equations*). Hence if the motion of the space leaves its metric the same and we know the vector Q^α for any of its points (the motion of the space at this point is set), the motion of the point(s) can be used to obtain the metric of the space from Killing's equations.

On the other hand, the motion of particles is described by the equations of motion. On the contrary, these equations leave the metric of the space fixed and the problem here is to find the dynamical vector of the particle Q^α. The fixed metric in the equations of motion makes the Christoffel symbols, which are functions of the metric components $g_{\alpha\beta}$, appear in the equations as fixed coefficients. Hence as soon as a particular metric of space is set, we can use the equations of motion to obtain the vector Q^α for the particle in such a space.

Therefore we now arrive at the following. Because $g_{\alpha\beta}$ is a symmetric tensor ($g_{\alpha\beta} = g_{\beta\alpha}$), only 10 components, out of 16, have different

numerical values. In Killing's equations (10 equations), the vector of motion of a point in the space is fixed, while the components of the metric tensor are unknown (10 unknowns). The equations of motion of a free particle (4 equations), on the contrary, leave the metric fixed, but the components of the vector of motion of the particle (4 components) are unknown. Then as soon as we look at the free motion of a particle as the motion of any of the points in the space carried by the motion of the space itself, we can create a system of 10 Killing equations (the equations of motion of the space) and of 4 equations of motion of the particle. The system of 14 equations will have 14 unknowns, 10 out of which are unknown components of the metric and 4 are unknown components of the dynamical vector of the particle. Hence, having this system solved, we obtain the motion of the particle in the space and the metric of the space at the same time.

In particular, while solving the system of the equations, we can find the motions of particles which result from the motion of the space itself. For this type of motion the knowledge of the motion of a certain particle can manifestly produce the metric of the space itself.

For instance, having Killing's equations and the dynamical equations of motion solved for a satellite (or a planet) we can use its motion to find the metric of the space of the gravitating body.

In the next paragraph we shall proceed to obtain Killing's equations in the chr.inv.-form.

§2.2 Equations of motion and Killing's equations

Assume a moving space (not necessarily a metric one). As obvious, the vector of motion Q^α of any point of the space is the vector of motion of the space itself at this point. The motion of a metric space is described by Lie's derivative

$$\underset{\text{L}}{\delta}\, g_{\alpha\beta} = Q^\alpha \frac{\partial g_{\alpha\beta}}{\partial x^\sigma} + g_{\alpha\sigma}\frac{\partial Q^\sigma}{\partial x^\beta} + g_{\beta\sigma}\frac{\partial Q^\sigma}{\partial x^\alpha}\,, \tag{2.5}$$

which is the derivative of the fundamental metric tensor of the space with respect to the direction of parallel transfer of the vector Q^α (the direction of motion of the space itself).

We will now be looking at the picture as follows. We assume a point in the space. If the space moves, the point is a subject to the action of the accompanying vector Q^α which is the vector of motion of the space itself. For the point itself, the space rests and only the "wind" produced by the motion of the space as the vector Q^α will disclose the motion of the whole space.

In a general case Lie's derivative is not zero. That is, the motion of the space alters its metric. But in a Riemannian space the metric is fixed by definition, so the length of a vector parallel-transferred to itself remains constant. This means that the parallel transport of a vector across a "non-smooth" structure in a Riemannian space will alter the vector along with the configuration of the space. As a result, Lie's derivative of the metric in a Riemannian space should be zero

$$\underset{\text{L}}{\delta}\, g_{\alpha\beta} = 0\,. \tag{2.6}$$

Lie's equations in a Riemannian space were first studied by Killing and, as we mentioned above, are known as Killing's equations. Later Petrov showed [18] that Killing's equations for any point are the necessary and sufficient condition for the motion of the point to be the motion of a Riemannian space itself. In other words, if a point is carried by the motion of a Riemannian space and moves along with it, Killing's equations must be true for that point.

As obvious, to obtain the components of the metric tensor out of Killing's equations we need to invoke a particular vector Q^α of motion of a point. Then we will have 10 Killing equations versus 10 unknown metric components, so we able to solve the system.

Generally speaking, there may be different kinds of motion in a Riemannian space. We will set up the vector of motion Q^α so as to fit the needs of our problem.

There exists free (geodesic) motion in which a point moves along a geodesic trajectory (the shortest one among the others that between two points). We assume that any point of the Riemannian space carried by the motion of the space itself moves along a geodesic trajectory. Hence the motion of the entire Riemannian space will be geodesic as well. Then we can match the motion of a point carried by the motion of the space to the motion of a free particle.

We call a motion the *geodesic motion of a space* if the free motion of particles results from their being carried by the moving space.

Let us look at the following system of the dynamical equations of motion of free particles and Killing's equations

$$\left.\begin{array}{l} \dfrac{\mathrm{D}Q^\alpha}{d\rho} = 0 \\[2ex] \underset{\text{L}}{\delta}\, g_{\alpha\beta} = 0 \end{array}\right\}, \tag{2.7}$$

where Q^α stands for the dynamical vector of motion of the particle, ρ stands for the derivation parameter along to the trajectory of motion,

while Lie's derivative can be expressed through *Lie's differential* as

$$\underset{\text{L}}{\delta}\, g_{\alpha\beta} = \frac{\underset{\text{L}}{\text{D}}\, g_{\alpha\beta}}{d\rho}\,. \tag{2.8}$$

Actually, the system of equations (2.7) means that the motion of a free particle is a geodesic one and, at the same time, results from the particle being carried by the motion of the space. The system solves as a set of the components of the dynamical vector Q^α as well as the components of the metric tensor $g_{\alpha\beta}$ for which the geodesic motion of particles results from the geodesic motion of the space itself.

To solve the problem in correct way, we need to present Killing's equations in the chr.inv.-form, thus presenting them through the physical properties (standards) of the space. It is especially interesting to know what physical standards result from the motion of the space itself.

According to the theory of chronometric invariants, the physically observable quantities produced from Killing's equations should be the chr.inv.-projections of the equations onto the time line (1 component), the mixed projection (3 components), and the spatial projection (6 components)

$$\left.\begin{aligned} &\frac{\underset{\text{L}}{\delta}\, g_{00}}{g_{00}} = 0 \\ &\frac{\underset{\text{L}}{\delta}\, g_0^i}{\sqrt{g_{00}}} = \frac{g^{i\alpha}\underset{\text{L}}{\delta}\, g_{0\alpha}}{\sqrt{g_{00}}} = 0 \\ &\underset{\text{L}}{\delta}\, g^{ik} = g^{i\alpha}g^{k\beta}\underset{\text{L}}{\delta}\, g_{\alpha\beta} = 0 \end{aligned}\right\}. \tag{2.9}$$

Here we are looking at the motion of the space and particles from the viewpoint of a regular subluminal observer.

Having presented the derivatives of the metric in Lie's derivative through the chr.inv.-differential operators, and substituted a short notation for the chr.inv.-projections of the dynamical vector of a particle Q^α as $\varphi = \frac{Q_0}{\sqrt{g_{00}}}$ and $q^i = Q^i$, we arrive at the *chr.inv.-Killing equations*

$$\left.\begin{aligned} &\frac{{}^*\partial\varphi}{\partial t} - \frac{1}{c}F_i q^i = 0 \\ &\frac{1}{c}\frac{{}^*\partial q^i}{\partial t} - h^{im}\frac{{}^*\partial\varphi}{\partial x^m} - \frac{\varphi}{c^2}F^i + \frac{2}{c}A^{\cdot i}_{k\cdot}q^k = 0 \\ &\frac{2\varphi}{c}D^{ik} + h^{im}h^{kn}q^l\frac{{}^*\partial h_{mn}}{\partial x^l} + h^{im}\frac{{}^*\partial q^k}{\partial x^m} + h^{km}\frac{{}^*\partial q^i}{\partial x^m} = 0 \end{aligned}\right\}. \tag{2.10}$$

If the vector Q^α at the same time complies with the chr.inv.-Killing equations and the dynamical chr.inv.-equations of motion of the particle, this particle is said to be in motion, carried by the geodesic motion of the space.

The joint solution of the equations in a general form is problematic and so we will limit ourselves to a single particular case, which is still of great importance. Let the dynamical vector of motion of the space Q^α be the dynamical vector of motion of a mass-bearing particle

$$Q^\alpha = m_0 \frac{dx^\alpha}{ds} = \frac{m}{c}\frac{dx^\alpha}{d\tau}\,, \tag{2.11}$$

and the observer accompanies the particle ($\mathrm{v}^i = 0$). In such a case

$$\varphi = m_0 = const, \qquad q^i = \frac{m}{c}\,\mathrm{v}^i, \tag{2.12}$$

and the chr.inv.-Killing equations (2.10) are simplified to

$$\left.\begin{aligned} F^i &= 0 \\ D^{ik} &= 0 \end{aligned}\right\}. \tag{2.13}$$

According to (1.42), $D^{ik} = 0$ means a stationary state of the observable metric: $h^{ik} = const$. The condition $F^i = 0$ is meant for the following equalities to be true only by transformation of the time coordinate

$$g_{00} = 1\,, \qquad \frac{\partial g_{0i}}{\partial t} = 0\,. \tag{2.14}$$

Besides, the quantities F^i and A_{ik} are linked through Zelmanov's identity (see formula 1.37 in §1.2)

$$\frac{1}{2}\left(\frac{{}^*\partial F_k}{\partial x^i} - \frac{{}^*\partial F_i}{\partial x^k}\right) + \frac{{}^*\partial A_{ik}}{\partial t} = 0\,, \tag{2.15}$$

from which we see that $F^i = 0$ means also

$$\frac{{}^*\partial A_{ik}}{\partial t} = 0\,, \tag{2.16}$$

so the space motion in such a case is a *stationary rotation*.

Further, as seen from the Killing equations (2.13), the tensor of the rates of deformation of the space is zero, hence stationary rotation does not alter the structure of the space. The vanishing of the gravitational inertial force in the Killing equations means that, from the viewpoint of an observer linked to a particle being carried in motion of the space

($\mathrm{v}^i = 0$), this particle weighs nothing and is not attracted to anything (the weightless state). This does not contradict the weightless condition $\mathrm{w} = -v_i \mathrm{v}^i$ obtained earlier, because from the viewpoint of such an observer the gravitational potential of the body of reference satisfies $\mathrm{w} = 0$ and $F^i = 0$ as well.

Hence if Q^α is the vector of motion of a mass-bearing particle in a Riemannian space, the geodesic motion of the space along to this vector is stationary rotation.

As seen, the geodesic motion of mass-bearing particles is stationary rotation. Such a stationary rotation results from the carrying of the gravitating body (the body of reference) by the space of reference surrounding it. At the same time we know that the basic type of motion in the Universe is the orbiting. Hence the basic motion in the Universe is a geodesic motion which results from the carrying of objects by the stationary (geodesic) rotation of the spaces of the gravitating bodies.

§2.3 Conclusions

So what is a space, which bears a gravitational potential w, can be deformed, and, in rotation, behaves like a viscous media? It is worth noting that if we place a particle in the space, the moving space will carry it just like an oceanic stream carries a tiny boat and a giant iceberg.

The answer is as follows: according to the results we have obtained in the above, the space of reference of a body and its gravitational field are the same thing. Physically speaking, points of the space of reference can be considered as particles in the gravitational field of the body of reference.

If the space of reference does not rotate, a satellite will fall on the body of reference under the action of the gravitational force. But in the presence of the space rotation, the satellite will be under the action of the carrying force. This force acts like a wind or an oceanic stream, pushing the satellite forward, not allowing it to fall down and making it orbiting the gravitating body along with the rotating space (of course, an additional velocity given to the satellite will make it move faster than the rotating space).

Chapter 3 Deviation of World-Lines. Detecting Gravitational Waves

§3.1 Gravitational wave detectors

Consider two particles of a rest-mass m_0 each one connected by a force Φ^α not of gravitational nature. Such particles move along neighbouring non-geodesic world-lines with the same four-dimensional velocity U^α, according to the non-geodesic equations of motion

$$\frac{dU^\alpha}{ds} + \Gamma^\alpha_{\mu\nu} U^\mu U^\nu = \frac{\Phi^\alpha}{m_0 c^2}\,, \tag{3.1}$$

while relative deviations of the world-lines (particles) are given by the Synge-Weber equation [19]

$$\frac{\mathrm{D}^2\eta^\alpha}{ds^2} + R^\alpha_{\cdot\beta\gamma\delta}\, U^\beta U^\delta \eta^\gamma = \frac{1}{m_0 c^2}\frac{\mathrm{D}\Phi^\alpha}{dv}\, dv\,, \tag{3.2}$$

where $\mathrm{D}\eta^\alpha = d\eta^\alpha + \Gamma^\alpha_{\mu\nu}\,\eta^\mu dx^\nu$ is the absolute differential, $\eta^\alpha = \frac{\partial x^\alpha}{\partial v}\, dv$ is the vector of the relative deviation of the particles, v is a parameter having the same numerical value along a world-line and different as dv in the neighbouring world-lines.

If two neighbouring particles are free ($\Phi^\alpha = 0$), they move along neighbouring geodesics, according to the geodesic equations of motion

$$\frac{dU^\alpha}{ds} + \Gamma^\alpha_{\mu\nu} U^\mu U^\nu = 0\,, \tag{3.3}$$

while relative deviations of the geodesics (particles) are given by the Synge equations [20]

$$\frac{\mathrm{D}^2\eta^\alpha}{ds^2} + R^\alpha_{\cdot\beta\gamma\delta}\, U^\beta U^\delta \eta^\gamma = 0\,. \tag{3.4}$$

A gravitational wave as a wave of the space metric deforming the space should produce some effect in a two-particle system. The effect could be found as a solution of the deviation equations in the gravitational wave metric. Therefore two kinds of gravitational wave detectors were presumed in 1960's by Joseph Weber, who pioneered experimental research on gravitational waves:

a) Solid-body detector — a freely suspended cylindrical pig, approximated by two masses connected by a spring. Such a detector should be deformed under the action of a gravitational wave. This deformation should lead to a piezoelectric effect therein;

b) Free-mass detector — a system, consisting of two freely suspended mirrors, distantly separated within the visibility, and fitted with a laser range-finder. Supposed deviations of the mirrors, derived from a gravitational wave, should be registered by the laser beam.

§3.2 A brief history of the measurements

Initial interest in gravitational waves arose in 1968–1970 when Joseph Weber, professor at Maryland University (USA), carried out his first experiments with solid-body gravitational wave detectors. He registered a few weak signals, in common with all his independent detectors, which were as distant located from each other as up to 1000 km [21–23]. He supposed that some processes at the centre of the Galaxy were the origin of the registered signals.

The experiments were continued in the next decades by many groups of researchers working at laboratories and research institutes throughout the world. The registering systems used in these attempts were more sensitive than those of Weber. In his pioneering observations of 1968–1970 Weber used very simple detectors in room-temperature conditions. To amplify the effect in measurements, the level of noise in all solid-body detectors of the second generation was lowered by cooling the cylinder pigs down to temperature close to 0 K. Besides gravitational antennae of the solid-body kind, many antennae based on free masses were constructed. But even the second generation of gravitational wave detectors have not led scientists to the expected results. None registered something similar to the Weber effect.

Nonetheless it is accepted by most physicists that the discovery of gravitational waves should be expected as one of the main effects of the General Theory of Relativity. The main arguments in support of this thesis are [15]: 1) gravitational fields bear an energy described by the energy-momentum pseudotensor; 2) a linearized form of Einstein's equations permits a solution describing weak plane gravitational waves, which are transverse; 3) an energy flux, radiated by gravitational waves, can be calculated through the energy-momentum pseudotensor of a gravitational field.

Therefore no doubt that gravitational radiation will have been discovered in the future.

The corner-stone of the problem was the fact that Weber's conclusions on the construction of the gravitational wave detectors were not based on an exact solution to the deviation equations, but on an *approximate analysis* of what could be expected: Weber expected that a plane weak wave of the space metric (gravitational wave) may displace two particles at rest with respect to each other.

Here we deduce *exact solutions* to both the Synge equation and the Synge-Weber equation (the exact theory to free-mass and solid-body detectors). The exact solutions show instead Weber's supposition that gravitational waves cannot displace resting particles; some effect may be produced only if the particles are in motion. According to the exact solutions we may alter the construction of both solid-body and free-mass detectors so that they may register oscillations produced by gravitational waves. Weber most probably detected them as claimed by him in 1968–1970, as his room-temperature solid-body pigs may have their own relative oscillations of the butt-ends, whereas the oscillations are inadvertently suppressed as noise in the detectors developed by his all followers, who have had no positive result in over 35 years.

§3.3 Weber's approach and criticism thereof

Weber proposed the relative displacement of the particles η^α consisting of a constant distance r^α and an infinitely small displacement ζ^α caused by a gravitational wave

$$\eta^\alpha = r^\alpha + \zeta^\alpha, \qquad \zeta^\alpha \ll r^\alpha, \qquad \frac{\mathrm{D}\, r^\alpha}{ds} = 0\,. \tag{3.5}$$

Thus the non-geodesic deviation equation (3.2) is

$$\frac{\mathrm{D}^2\zeta^\alpha}{ds^2} + R^\alpha_{\cdot\beta\gamma\delta}\, U^\beta U^\delta \left(r^\gamma + \zeta^\gamma\right) = \frac{\Phi^\alpha}{m_0 c^2}\,, \tag{3.6}$$

Then he takes Φ^α as the sum of the returning elastic force $k^\alpha_\sigma \zeta^\sigma$ and the damping factor $d^\alpha_\sigma \frac{\mathrm{D}\zeta^\sigma}{ds}$, while k^α_σ and d^α_σ describe the properties of the spring. As a result the equation (3.6) becomes

$$\frac{\mathrm{D}^2\zeta^\alpha}{ds^2} + \frac{d^\alpha_\sigma}{m_0 c^2}\frac{\mathrm{D}\zeta^\sigma}{ds} + \frac{k^\alpha_\sigma}{m_0 c^2}\,\zeta^\sigma = -\,R^\alpha_{\cdot\beta\gamma\delta}\left(r^\gamma + \zeta^\gamma\right) \tag{3.7}$$

that is the equation of forced oscillations, where the curvature tensor $R^\alpha_{\cdot 0\sigma 0}$ is a forcing factor. After some simplifications, he transformed the non-geodesic deviation equation (3.7) to

$$\frac{d^2\zeta^\alpha}{dt^2} + \frac{d^\alpha_\sigma}{m_0}\frac{d\zeta^\sigma}{dt} + \frac{k^\alpha_\sigma}{m_0}\,\zeta^\sigma = -\,c^2 R^\alpha_{\cdot 0\sigma 0}\, r^\sigma. \tag{3.8}$$

Weber didn't solve his equation (3.8). He limited himself by using the curvature tensor as a forcing factor in his calculations of expected resonant oscillations in solid-body detectors [19].

Solution of Weber's equation (3.8) with all his simplifications was obtained in 1978 by Borissova [24], in the field of a weak plane gravitational wave. Assuming, according to Weber, r^α and its length $r=\sqrt{g_{\mu\nu}r^\mu r^\nu}$ to be covariantly constant $\frac{\mathrm{D}\,r^\alpha}{ds}=0$, Borissova showed that in the case of a gravitational wave linearly polarized in the x^2 direction, and propagating along x^1, the equation $\frac{\mathrm{D}\,r^\alpha}{ds}=0$ gives $r^2=r^2_{(0)}\left[1-A\sin\frac{\omega}{c}\left(ct+x^1\right)\right]$ (in a case where the detector is oriented along x^2).

From this result, she obtained Weber's equation (3.8) in the form

$$\frac{d^2\zeta^2}{dt^2}+2\lambda\frac{d\zeta^2}{dt}+\Omega_0^2\zeta^2=-A\,\omega^2 r^2_{(0)}\sin\frac{\omega}{c}\left(ct+x^1\right), \qquad (3.9)$$

i.e. an equation of forced oscillations, where the forcing factor is the relative displacement of the particles caused by the gravitational wave. Here $2\lambda=\frac{b}{m_0}$ and $\Omega^2_{(0)}=\frac{k}{m_0}$ come from the formula for the non-gravitational force, acting along x^2 in this case: $\Phi^2=-k\zeta^2-b\dot{\zeta}^2$. The elastic coefficient of the "spring" is k, the friction coefficient is b.

She then obtained the exact solution of the equation: the relative displacement $\eta^2=\eta_y$ of the butt-ends transverse to the falling gravitational wave is

$$\eta^2=r^2_{(0)}\left[1-A\sin\frac{\omega}{c}\left(ct+x^1\right)\right]+Me^{-\lambda t}\sin\left(\Omega t+\alpha\right)-$$
$$-\frac{A\,\omega^2 r^2_{(0)}}{(\Omega_0^2-\omega^2)^2}\cos\left(\omega t+\delta+\frac{\omega}{c}\,x^1\right), \qquad (3.10)$$

where $\Omega=\sqrt{\Omega_0^2-\omega^2}$, $\delta=\arctan\frac{2\lambda\omega}{\omega^2-\Omega_0^2}$, while M and α are constants.

In this solution the relative oscillations consist of the "basic" harmonic oscillations and relaxing oscillations (first two terms), and also the resonant oscillations (third term).

As was shown by Borissova [24], Weber's final equation (3.8) can only be obtained under his simplifications:

a) He has actually two detectors in one: a big pig having the constant length r and a small pig which length ζ changes under the same gravitational wave. However in actual experiments a solid-body pig reacts as a whole to external influences;

b) Christoffel's symbols $\Gamma^\alpha_{\mu\nu}$ are all zero. However, since the curvature tensor is different from zero, $\Gamma^\alpha_{\mu\nu}$ cannot be reduced to zero in a finite region [18]. So in the neighbouring particle $\Gamma^\alpha_{\mu\nu}\neq 0$;

c) The butt-ends of the pig are at rest with respect to the observer ($U^i = 0$) all the time before a gravitational wave passes. Therefore *only resonant oscillations* can be registered by such a detector. Parametric oscillations cannot appear there.

Because the same assumptions were applied to the geodesic deviation equation, all that has been said is applicable to a free-mass detector.

Thus, by his simplified equation (3.8), Weber actually postulated that gravitational waves force rest-particles to undergo relative resonant oscillations. His assumptions led to a specific construction of the solid-body and free-mass detectors, where parametric oscillations are obviated.

§3.4 The main equations

Here we solve the deviation equations in conjunction with the equations of motion in the general case where both particles in the pair move initially with respect to the observer ($U^i \neq 0$), and without Weber's simplifications. We solve the equations in the terms of physically observable quantities [2–4], which are the chr.inv.-projections of four-dimensional quantities onto the line of time and onto the spatial section of an observer. For instance (see §1.2 of Chapter 1), any vector Q^α has two chr.inv.-projections: $\frac{Q_0}{\sqrt{g_{00}}}$ and Q^i. We denote

$$\sigma = \frac{\Phi_0}{\sqrt{g_{00}}}\,, \qquad f^i = \Phi^i \tag{3.11}$$

for the connecting force Φ^α, and also

$$\varphi = \frac{\eta_0}{\sqrt{g_{00}}}\,, \qquad \eta^i \equiv \eta^i \tag{3.12}$$

for the deviation vector η^α.

We consider the deviating non-geodesics as a common case, where the right side is non-zero.

The general covariant non-geodesic equations of motion (3.1) have two chr.inv.-projections

$$\left.\begin{aligned} &\frac{dm}{d\tau} - \frac{m}{c^2}\,F_i \mathrm{v}^i + \frac{m}{c^2}\,D_{ik}\mathrm{v}^i\mathrm{v}^k = \frac{\sigma}{c} \\ &\frac{d}{d\tau}(m\mathrm{v}^i) - mF^i + 2m\big(D^i_k + A^{\cdot i}_{k\cdot}\big) + m\Delta^i_{kn}\mathrm{v}^k\mathrm{v}^n = f^i \end{aligned}\right\}, \tag{3.13}$$

where m is the relativistic mass of the particle, v^i is its physically observable chr.inv.-velocity, $d\tau$ is the interval of the physically observable

time, F_i is the chr.inv.-vector of the gravitational inertial force, A_{ik} is the chr.inv.-tensor of the angular velocities of the space rotation, D_{ik} is the tensor of the space deformations, Δ^i_{kn} are the chr.inv.-Christoffel symbols, built like Christoffel's usual symbols $\Gamma^\alpha_{\mu\nu}$ using h_{ik} instead $g_{\alpha\beta}$ (see definitions of the chr.inv.-quantities in §1.2 of Chapter 1).

We write the Synge-Weber equation of the deviating non-geodesics (3.2) in the expanded form

$$\frac{d^2\eta^\alpha}{ds^2} + 2\Gamma^\alpha_{\mu\nu}\,\frac{d\eta^\mu}{ds}\,U^\nu + \frac{\partial\Gamma^\alpha_{\beta\delta}}{\partial x^\gamma}\,U^\beta U^\delta\eta^\gamma = \frac{1}{m_0c^2}\,\frac{\partial\Phi^\alpha}{\partial x^\gamma}\,\eta^\gamma, \tag{3.14}$$

where ds^2 can be expressed through the observable time interval $d\tau$ according to (1.29) as $ds^2 = c^2d\tau^2 - d\sigma^2 = c^2d\tau^2\big(1 - \frac{\mathrm{v}^2}{c^2}\big)$.

We consider the well-known metric of the field of weak plane gravitational waves

$$\begin{aligned} ds^2 = c^2dt^2 - (dx^1)^2 - (1+a)(dx^2)^2 + \\ + 2b\,dx^2dx^3 - (1-a)(dx^3)^2, \end{aligned} \tag{3.15}$$

where a and b are functions of $ct + x^1$ (if propagation is along x^1), and are small values so the squares and products of their derivatives vanish.

The velocity of both particles (butt-ends) in a detector is obviously small. In such a case, in the gravitational wave metric (3.15),

$$\left.\begin{aligned} &d\tau = dt\,, && \eta^0 = \eta_0 = \varphi\,, && \Phi^0 = \Phi_0 = \sigma \\ &\Gamma^0_{kn} = \frac{1}{c}\,D_{kn}\,, && \Gamma^i_{0k} = \frac{1}{c}\,D^i_k\,, && \Gamma^i_{kn} = \Delta^i_{kn} \end{aligned}\right\}. \tag{3.16}$$

With these, after algebra we obtain the chr.inv.-projections of the Synge-Weber equation (3.14)

$$\left.\begin{aligned} &\frac{d^2\varphi}{dt^2} + \frac{2}{c}\,D_{kn}\frac{d\eta^k}{dt}\,\mathrm{v}^n + \left(\varphi\,\frac{\partial D_{kn}}{\partial t} + c\,\frac{\partial D_{kn}}{\partial x^m}\,\eta^m\right)\frac{\mathrm{v}^k\mathrm{v}^n}{c^2} = \\ &= \frac{1}{m_0}\left(\frac{\varphi}{c}\,\frac{\partial\sigma}{\partial t} + \frac{\partial\sigma}{\partial x^m}\,\eta^m\right) \\ &\frac{d^2\eta^i}{dt^2} + \frac{2}{c}\,D^i_k\left(\frac{d\varphi}{dt}\,\mathrm{v}^k + c\,\frac{d\eta^k}{dt}\right) + 2\Delta^i_{kn}\frac{d\eta^k}{dt}\,\mathrm{v}^n + \\ &+ 2\left(\frac{\varphi}{c}\,\frac{\partial D^i_k}{\partial t} + \frac{\partial D^i_k}{\partial x^m}\,\eta^m\right)\mathrm{v}^k + \left(\frac{\varphi}{c}\,\frac{\partial\Delta^i_{kn}}{\partial t} + \frac{\partial\Delta^i_{kn}}{\partial x^m}\,\eta^m\right)\mathrm{v}^k\mathrm{v}^n = \\ &= \frac{1}{m_0}\left(\frac{\varphi}{c}\,\frac{\partial f^i}{\partial t} + \frac{\partial f^i}{\partial x^m}\,\eta^m\right) \end{aligned}\right\}. \tag{3.17}$$

In component notation, the obtained chr.inv.-deviation equations (3.17) are a system of four 2nd order differential equations with respect to φ, η^1, η^2, η^3, where the variable coefficients of the functions are the quantities $\dot{a}$, $\ddot{a}$, v^1, v^2, v^3. To solve this system we will get a from the given gravitational wave metric (3.15), while v^i come as the solutions to the non-geodesic equations of motion (3.13).

§3.5 Exact solution for a free-mass detector

We first solve the chr.inv.-deviation equations (3.17) for a free-mass detector, where two particles don't interact with each other ($\Phi^\alpha = 0$) — the right side is zero in the equations.

We find the solution in the field of a gravitational wave falling along x^1 and linearly polarized in the x^2 direction ($b = 0$). With these the gravitational wave metric (3.15) gives

$$\left.\begin{aligned} &D_{22} = -\,D_{33} = \frac{1}{2}\,\dot{a}\,, && \frac{d}{dx^1} = \frac{1}{c}\,\frac{d}{dt} \\ &\Delta^1_{22} = -\,\Delta^1_{33} = -\,\frac{1}{2\,c}\,\dot{a}\,, && \Delta^2_{12} = -\,\Delta^3_{13} = \frac{1}{2\,c}\,\dot{a} \end{aligned}\right\}. \tag{3.18}$$

In such a case, and since $\Phi^\alpha = 0$, the chr.inv.-equations of motion (3.13) take the form

$$\left.\begin{aligned} &(\mathrm{v}^2)^2 - (\mathrm{v}^3)^2 = 0 \\ &\frac{d\mathrm{v}^1}{dt} = 0\,, \quad \frac{d\mathrm{v}^2}{dt} + \dot{a}\,\mathrm{v}^2 = 0\,, \quad \frac{d\mathrm{v}^3}{dt} + \dot{a}\,\mathrm{v}^3 = 0 \end{aligned}\right\}. \tag{3.19}$$

As seen $\mathrm{v}^1 = \mathrm{v}^1_{(0)} = const$, so a transverse gravitational wave does not move a single particle in the longitudinal direction. Henceforth,

$$\mathrm{v}^1 = \mathrm{v}^1_{(0)} = 0\,. \tag{3.20}$$

The rest two spatial equations of (3.19) are also simple to integrate. We obtain

$$\mathrm{v}^2 = \mathrm{v}^2_{(0)}\,e^{-a}, \qquad \mathrm{v}^3 = \mathrm{v}^3_{(0)}\,e^{+a}. \tag{3.21}$$

Assuming the wave simple harmonic, $\omega = const$, with a constant amplitude $A = const$, i.e. $a = A\sin\frac{\omega}{c}(ct + x^1)$, and expanding the exponent into series (with high order terms withheld), we obtain

$$\mathrm{v}^2 = \mathrm{v}^2_{(0)}\left[1 - A\sin\frac{\omega}{c}(ct + x^1)\right], \tag{3.22}$$

$$\mathrm{v}^3 = \mathrm{v}^3_{(0)}\left[1 + A\sin\frac{\omega}{c}(ct + x^1)\right]. \tag{3.23}$$

Substituting these solutions into the chr.inv.-equations of deviating non-geodesics (3.17) and setting the right side to zero as for geodesics, we obtain

$$\frac{d^2\varphi}{dt^2} + \frac{\dot a}{c}\left(\frac{d\eta^2}{dt}\,\mathrm{v}^2_{(0)} - \frac{d\eta^3}{dt}\,\mathrm{v}^3_{(0)}\right) = 0\,, \tag{3.24}$$

$$\frac{d^2\eta^1}{dt^2} - \frac{\dot a}{c}\left(\frac{d\eta^2}{dt}\,\mathrm{v}^2_{(0)} - \frac{d\eta^3}{dt}\,\mathrm{v}^3_{(0)}\right) = 0\,, \tag{3.25}$$

$$\frac{d^2\eta^2}{dt^2} + \dot a\,\frac{d\eta^2}{dt} + \frac{\dot a}{c}\left(\frac{d\varphi}{dt} + \frac{d\eta^1}{dt}\right)\mathrm{v}^2_{(0)} + \frac{\ddot a}{c}\,(\varphi+\eta^1)\,\mathrm{v}^2_{(0)} = 0\,, \tag{3.26}$$

$$\frac{d^2\eta^3}{dt^2} - \dot a\,\frac{d\eta^3}{dt} - \frac{\dot a}{c}\left(\frac{d\varphi}{dt} + \frac{d\eta^1}{dt}\right)\mathrm{v}^2_{(0)} - \frac{\ddot a}{c}\,(\varphi+\eta^1)\,\mathrm{v}^2_{(0)} = 0\,. \tag{3.27}$$

Summing the first two equations then integrating the sum, we obtain

$$\varphi + \eta^1 = B_1\,t + B_2\,, \tag{3.28}$$

where B_1 and B_2 are integration constants. Substituting these into the other two, we obtain two equations which are different solely in the sign of a, and can therefore be solved in the same way

$$\frac{d^2\eta^2}{dt^2} + \dot a\,\frac{d\eta^2}{dt} + \frac{\dot a}{c}\,B_1\,\mathrm{v}^2_{(0)} + \frac{\ddot a}{c}\,(B_1 t + B_2)\,\mathrm{v}^2_{(0)} = 0\,, \tag{3.29}$$

$$\frac{d^2\eta^3}{dt^2} - \dot a\,\frac{d\eta^2}{dt} - \frac{\dot a}{c}\,B_1\,\mathrm{v}^3_{(0)} - \frac{\ddot a}{c}\,\big(B_1 t + B_2\big)\,\mathrm{v}^3_{(0)} = 0\,. \tag{3.30}$$

We introduce a new variable $y = \frac{d\eta^2}{dt}$. Then we have a linear uniform equation of the 1st order with respect to y

$$\dot y + \dot a y = -\,\frac{\dot a}{c}\,B_1\,\mathrm{v}^2_{(0)} - \frac{\ddot a}{c}\,\big(B_1 t + B_2\big)\,\mathrm{v}^2_{(0)}\,, \tag{3.31}$$

which has the solution

$$y = e^{-F}\left(y_0 + \int_0^t g\,(t)\,e^F\,dt\right), \quad F(t) = \int_0^t f\,(t)\,dt\,, \tag{3.32}$$

where, in the given case, $F(t) = \dot a$, $g(t) = -\frac{\dot a}{c}B_1\mathrm{v}^2_{(0)} - (B_1 t + B_2)\mathrm{v}^2_{(0)}$. Expanding the exponent into series in y (3.32), and then integrating, we obtain

$$y = \dot\eta^2 = \dot\eta^2_{(0)}\left[1 - A\sin\frac{\omega}{c}\,(ct + x^1)\right] - \frac{A\omega}{c}\,\mathrm{v}^2_{(0)}\times$$
$$\times\,\big(B_1 t + B_2\big)\cos\frac{\omega}{c}\,(ct + x^1) + \frac{A\omega}{c}\,B_2\,\mathrm{v}^2_{(0)}\,. \tag{3.33}$$

We integrate this equation, then apply the same method for η^3. As a result, we obtain the physically observable relative displacements η^2 and η^3 in a free-mass detector

$$\eta^2 = \eta^2_{(0)} + \left(\dot\eta^2_{(0)} + \frac{A\,\omega B_2\,\mathrm{v}^2_{(0)}}{c}\right) t + \frac{A}{\omega}\left(\dot\eta^2_{(0)} - \frac{\mathrm{v}^2_{(0)}}{c}\,B_1\right)\times$$
$$\times\left[\cos\frac{\omega}{c}\,(ct+x^1) - 1\right] - \frac{A\mathrm{v}^2_{(0)}}{c}\,(B_1 t + B_2)\sin\frac{\omega}{c}\,(ct+x^1)\,, \quad (3.34)$$

$$\eta^3 = \eta^3_{(0)} + \left(\dot\eta^3_{(0)} - \frac{A\,\omega B_2\,\mathrm{v}^3_{(0)}}{c}\right) t - \frac{A}{\omega}\left(\dot\eta^3_{(0)} - \frac{\mathrm{v}^3_{(0)}}{c}\,B_1\right)\times$$
$$\times\left[\cos\frac{\omega}{c}\,(ct+x^1) - 1\right] + \frac{A\mathrm{v}^3_{(0)}}{c}\,(B_1 t + B_2)\sin\frac{\omega}{c}\,(ct+x^1)\,. \quad (3.35)$$

With $\dot\eta^2$ and $\dot\eta^3$, we get the physically observable relative displacement η^1 (3.25) in a free-mass detector and the physically observable time shift φ (3.24) at its ends

$$\eta^1 = \dot\eta^1_{(0)}\,t - \frac{A}{\omega c}\left(\mathrm{v}^2_{(0)}\,\dot\eta^2_{(0)} - \mathrm{v}^3_{(0)}\,\dot\eta^3_{(0)}\right)\left[1 - \cos\frac{\omega}{c}\,(ct+x^1)\right] + \eta^1_{(0)}\,, \quad (3.36)$$

$$\varphi = \dot\varphi_{(0)}\,t + \frac{A}{\omega c}\left(\mathrm{v}^2_{(0)}\,\dot\eta^2_{(0)} - \mathrm{v}^3_{(0)}\,\dot\eta^3_{(0)}\right)\left[1 - \cos\frac{\omega}{c}\,(ct+x^1)\right] + \eta^1_{(0)}\,. \quad (3.37)$$

Finally, we substitute φ and η^1 into $\varphi + \eta^1 = B_1\,t + B_2$ (3.28) to fix the integration constants. We obtain

$$B_1 = \dot\varphi_{(0)} + \dot\eta^1_{(0)}\,, \qquad B_2 = \varphi_{(0)} + \eta^1_{(0)}\,. \quad (3.38)$$

Thus, we have obtained the exact solutions φ, η^1, η^2, η^3 to the chr. inv.-equations of the deviating geodesics in a gravitational wave field.

Proceeding from the exact solutions we arrive at the next conclusions on a free-mass detector:

1) As seen from the solutions η^2 (3.34) and η^3 (3.35), gravitational waves may force the ends of a free-mass detector to undergo relative oscillations in the directions x^2 and x^3, transverse to that of the wave propagation. At the same time, this effect is permitted only if the detector initially moves with respect to the local space ($\mathrm{v}^2_{(0)} \neq 0$ or $\mathrm{v}^3_{(0)} \neq 0$) or, alternatively, its ends initially move with respect to each other ($\dot\eta^2_{(0)} \neq 0$ or $\dot\eta^3_{(0)} \neq 0$). For instance, if the ends of a free-mass detector are at rest with respect to x^2, an x^1-propagating gravitational wave cannot displace them in the x^2 direction;

2) The solution η^1 (3.36) manifests that gravitational waves may oscillatory bounce the ends of a free-mass detector even in the same direction of the wave propagation, if they initially move both with respect to the local space and each other in at least one of the transverse directions x^2 and x^3;
3) The solution φ (3.37) is the time shift in the clocks located at the ends of a free-mass detector, caused by a gravitational wave. From (3.37), this effect is permitted if the ends initially move both with respect to the local space and each other in at least one of the transverse directions x^2 and x^3.

In view of these results we have obtained, we propose a new experimental statement, based on a free-mass detector:

> New experiment (free-mass detector): A free-mass detector, where two mirrors, distantly separated, are suspended and vibrating so that they have free oscillations with respect to each other ($\dot\eta^i_{(0)}\neq 0$) or common oscillations along parallel lines ($\mathrm{v}^i_{(0)}\neq 0$). According to the exact solution for a free-mass detector given above, a falling gravitational wave produces a parametric effect in the basic oscillations of the mirrors, that may be registered with a laser range-finder. Besides, as the solution predicts, a time shift is produced in the mirrors, that may be registered by synchronized clocks located with each of the mirrors: their de-synchronization means a gravitational wave detection.

§3.6 Exact solution for a solid-body detector

We assume the elastic force $\Phi^\alpha = -k^\alpha_\sigma x^\sigma$ connecting two particles in a solid-body detector to be independent of time ($k^0_\sigma = 0$). In such a case the chr.inv.-equations of motion (3.13) take the form

$$(\mathrm{v}^2)^2 - (\mathrm{v}^3)^2 = 0\,, \tag{3.39}$$

$$\frac{d\mathrm{v}^1}{dt} = -\frac{k^1_\sigma}{m_0}\,x^\sigma, \tag{3.40}$$

$$\frac{d\mathrm{v}^2}{dt} + \dot a\,\mathrm{v}^2 = -\frac{k^2_\sigma}{m_0}\,x^\sigma, \tag{3.41}$$

$$\frac{d\mathrm{v}^3}{dt} - \dot a\,\mathrm{v}^3 = -\frac{k^3_\sigma}{m_0}\,x^\sigma, \tag{3.42}$$

where (3.40) means $\mathrm{v}^1 = \mathrm{v}^1_{(0)} = const$. Henceforth, in the detector,

$$\mathrm{v}^1 = \mathrm{v}^1_{(0)} = 0\,, \qquad k^1_\sigma = 0\,. \tag{3.43}$$

Only two equations, (3.41) and (3.42), are essential. They differ solely by the sign of $\dot{a}$, so we solve only (3.41).

Let the solid-body detector be elastic in only two directions transverse to the direction x^1 of the propagation of the gravitational wave, which falls onto the detector. In such a case the elastic coefficient is $k_\sigma^2 = k_\sigma^3 = k = const$. With that, since $a = A \sin \frac{\omega}{c}(ct + x^1)$ as previously, and denoting $x^2 \equiv x$, $\frac{k}{m_0} = \Omega^2$, $A\omega = -\mu$, we reduce (3.41) to

$$\ddot{x} + \Omega^2 x = \mu \cos \frac{\omega}{c}(ct + x^1)\,\dot{x}\,, \tag{3.44}$$

where μ is the so-called "small parameter". We solve this equation using the small parameter method of Poincaré, known also as the perturbation method: we consider the right side as a forcing perturbation of a harmonic oscillation described by the left side. The Poincaré method is related to exact solution methods, because a solution produced with it is a power series expanded by the small parameter (see Chapter XII, §2 in Lefschetz [25]).

We introduce a new variable $t' = \Omega t$ in order to make it dimensionless as according to Lefschetz, and $\mu' = \frac{\mu}{\Omega}$

$$\ddot{x} + x = \mu' \cos \frac{\omega}{\Omega c}(ct' + \Omega x^1)\,\dot{x}\,. \tag{3.45}$$

A general solution of this equation, representable as

$$\dot{x} = y\,, \quad \dot{y} = -x + \mu' \cos \frac{\omega}{\Omega c}(ct' + \Omega x^1)\,y \tag{3.46}$$

with the initial data $x_{(0)}$ and $y_{(0)}$ at $t' = 0$, is determined by the series pair (Lefschetz)

$$\left.\begin{aligned} x &= P_0(x_{(0)}, y_{(0)}, t') + \mu' P_1(x_{(0)}, y_{(0)}, t') + \dots \\ y &= \dot{P}_0(x_{(0)}, y_{(0)}, t') + \mu' \dot{P}_1(x_{(0)}, y_{(0)}, t') + \dots \end{aligned}\right\}. \tag{3.47}$$

We substitute these into (3.46) and, equating coefficients in the same orders of μ', obtain

$$\left.\begin{aligned} &\ddot{P}_0 + P_0 = 0 \\ &\ddot{P}_1 + P_1 = \dot{P}_0 \cos \frac{\omega}{\Omega c}(ct' + \Omega x^1) \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{aligned}\right\} \tag{3.48}$$

with the initial data $P_0(0) = \xi$, $\dot{P}_0(0) = \vartheta$, $P_1(0) = \dot{P}_1(0) = 0$ (where $n > 0$) at $t' = 0$. Because the amplitude A (we have it in the variable $\mu' = -\frac{\omega}{\Omega} A$)

is small, this problem takes only the first two equations into account. The first of them is a harmonic oscillation equation, with the solution

$$P_0 = \xi \cos t' + \vartheta \sin t', \tag{3.49}$$

while the second equation, with this solution, is

$$\ddot{P}_1 + P_1 = (-\xi \sin t' + \vartheta \cos t') \cos \frac{\omega}{\Omega c} (c\, t' + \Omega x^1) \,. \tag{3.50}$$

This is a linear uniform equation. The solution, according to Kamke (see Part III, Chapter II, §2.5 in [26]), is

$$\begin{aligned} P_1 = \frac{\vartheta \Omega^2}{2} \left\{ \frac{\cos\left[(\Omega-\omega)\, t - \frac{\omega}{c}\, x^1\right]}{\Omega^2 - (\Omega-\omega)^2} + \frac{\cos\left[(\Omega+\omega)\, t + \frac{\omega}{c}\, x^1\right]}{\Omega^2 - (\Omega+\omega)^2} \right\} - \\ - \frac{i\xi\Omega^2}{2} \left\{ \frac{\sin\left[(\Omega-\omega)\, t - \frac{\omega}{c}\, x^1\right]}{\Omega^2 - (\Omega-\omega)^2} + \frac{\sin\left[(\Omega+\omega)\, t + \frac{\omega}{c}\, x^1\right]}{\Omega^2 - (\Omega+\omega)^2} \right\}, \end{aligned} \tag{3.51}$$

where the brackets contain the real and imaginary parts of the sum $e^{i(\Omega-\omega)\, t - \frac{\omega}{c}\, x^1} + e^{i(\Omega+\omega)\, t + \frac{\omega}{c}\, x^1}$. Substituting these into (3.47), and going back to $x = x^2$, we obtain the final solution in the reals

$$\begin{aligned} x^2 = \xi \cos \Omega t + \vartheta \sin \Omega t - \frac{A\omega\Omega\vartheta}{2} \times \\ \times \left\{ \frac{\cos\left[(\Omega-\omega)\, t - \frac{\omega}{c}\, x^1\right]}{\Omega^2 - (\Omega-\omega)^2} + \frac{\cos\left[(\Omega+\omega)\, t + \frac{\omega}{c}\, x^1\right]}{\Omega^2 - (\Omega+\omega)^2} \right\}, \end{aligned} \tag{3.52}$$

while the solution for x^3 will differ solely in the sign of A.

With this result we solve the chr.inv.-equations of the deviating non-geodesics (3.17).

For the cylindrical pig under consideration we assume $\mathrm{v}^1 = 0$, $\mathrm{v}^2 = \mathrm{v}^3$, $\Phi^1 = 0$, $\Phi^2 = -\frac{k}{m_0}\eta^2$, $\Phi^3 = -\frac{k}{m_0}\eta^3$, where $\mathrm{v}^2 = \mathrm{v}^3$ means that the initial conditions ξ and ϑ are the same in both x^2 and x^3 directions. So the deviation equations along $x^0 = ct$ and x^1 are

$$\frac{d^2\varphi}{dt^2} = 0\,, \qquad \frac{d^2\eta^1}{dt^2} = 0\,, \tag{3.53}$$

so we may put their solutions as $\varphi = 0$ and $\eta^1 = 0$.

With all these, the deviation equation along x^2 (it differs to that along x^3 by the sign of A) is

$$\frac{d^2\eta^2}{dt^2} + \frac{k}{m_0}\,\eta^2 = -\,A\omega \cos \frac{\omega}{c}\,(ct + x^1)\,\frac{d\eta^2}{dt}\,, \tag{3.54}$$

which is like (3.44). So the solutions η^2 and η^3 should be like (3.52). As a result we obtain

$$\eta^2 = \xi\cos\Omega t + \vartheta\sin\Omega t - \frac{A\omega\Omega\vartheta}{2}\times \\ \times\left\{\frac{\cos\left[(\Omega-\omega)t-\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega-\omega)^2}+\frac{\cos\left[(\Omega+\omega)t+\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega+\omega)^2}\right\}, \qquad (3.55)$$

$$\eta^3 = \xi\cos\Omega t + \vartheta\sin\Omega t + \frac{A\omega\Omega\vartheta}{2}\times \\ \times\left\{\frac{\cos\left[(\Omega-\omega)t-\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega-\omega)^2}+\frac{\cos\left[(\Omega+\omega)t+\frac{\omega}{c}x^1\right]}{\Omega^2-(\Omega+\omega)^2}\right\}. \qquad (3.56)$$

These are the exact solutions to the chr.inv.-equations of the deviating non-geodesics in a gravitational wave field. The solutions lead us to the conclusions:

1) The solutions $\varphi = const$ and $\eta^1 = const$ manifest that a gravitational wave falling down from upstarts onto a horizontally suspended solid-body pig does not change both the vertical size η^1 of the pig and the time shift φ at its butt-ends;
2) As seen from the solutions η^2 (3.55) and η^3 (3.56), gravitational waves may force the butt-ends of a solid-body pig to undergo relative oscillations, transverse to the wave propagation: a) forced relative oscillations at a frequency ω of the gravitational waves; b) resonant oscillations which occur as soon as the gravitational wave's frequency becomes double the frequency of the butt-ends' basic oscillation ($\omega = 2\Omega$). Both effects have parametric origin: they are permitted only if the butt-ends of the pig have an initial relative oscillation ($\Omega \neq 0$). If there is no initial oscillation, such a solid-body detector does not react on gravitational waves.

Owing to the theoretical results we have obtained, we propose a new experimental statement for a solid-body detector:

> New experiment (solid-body detector): Use a solid-body detector (cylindrical pig), horizontally suspended and having a laboratory induced oscillation of its body so that there are relative oscillations of its butt-ends ($\Omega \neq 0$). Such a system, according to the exact solution for a solid-body detector, may have a parametric effect in the basic oscillations of its butt-ends due to a falling gravitational wave that may be measured as a piezo-effect in the pig.

§3.7 Conclusions

The experimental statement on gravitational waves proceeds from the equation for deviating geodesic lines and the equation for deviating non-geodesics. Weber's result was not based on an exact solution to the equations, but on an approximate analysis of what could be expected: he expected that a plane weak wave of the space metric may displace two resting particles with respect to each other. Exact solutions have been obtained here for the deviation equation of both free and spring-connected particles. The solutions show that a gravitational wave may displace particles in a two-particle system only if they are in motion with respect to each other or the local space (there is no effect if they are at rest). Thus, gravitational waves produce a parametric effect on a two-particle system. According to the solutions, an altered detector construction can be proposed such that it might interact with gravitational waves: a) a free-mass detector where suspended mirrors have laboratory induced basic oscillations relative to each other; b) a horizontally suspended cylindrical pig, whose butt-ends have basic relative oscillations induced by a laboratory source.

Chapter 4 Instant Displacement World-Lines. Teleporting Particles

§4.1 Trajectories for instant displacement. Zero-space: the way for non-quantum teleportation of photons

As well-known, the basic space-time of the General Theory of Relativity is a four-dimensional pseudo-Riemannian space, which is, in general, curved, inhomogeneous, anisotropic, non-holonomic (rotating), and deformed. Therein, the space-time interval $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$, being expressed in the terms of physically observable quantities [2–4], is

$$ds^2 = c^2 d\tau^2 - d\sigma^2. \tag{4.1}$$

where the quantity

$$d\tau = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt - \frac{1}{c^2}\, v_i dx^i \tag{4.2}$$

is the interval of the physically observable time, $\mathrm{w} = c^2(1 - \sqrt{g_{00}})$ is the gravitational potential, v_i is the linear velocity of the space rotation, $d\sigma^2 = h_{ik}\, dx^i dx^k$ is the square of the spatial physically observable interval, h_{ik} is the physically observable chr.inv.-metric tensor.

Following the form (4.2), we consider a particle displaced by ds in the space-time. We write ds^2 as follows

$$ds^2 = c^2 d\tau^2 \left(1 - \frac{\mathrm{v}^2}{c^2}\right), \tag{4.3}$$

where $\mathrm{v}^2 = h_{ik}\, \mathrm{v}^i \mathrm{v}^k$, while $\mathrm{v}^i = \frac{dx^i}{d\tau}$ is the observable three-dimensional velocity of the particle. So the numerical value of the space-time interval ds is: a) a substantial number under $\mathrm{v} < c$; b) zero under $\mathrm{v} = c$; c) an imaginary number under $\mathrm{v} > c$.

According to the formula for ds^2, particles with non-zero rest-masses ($m_0 \neq 0$) can be moved: a) along real world-trajectories ($c\,d\tau > d\sigma$), having real relativistic masses; b) along imaginary world-trajectories ($c\,d\tau < d\sigma$), having imaginary relativistic masses (tachyons). The world-lines of both kinds are non-isotropic. In both cases relativistic masses are not zero ($m \neq 0$). These are the particles of substance.

Massless particles — particles with zero rest-masses ($m_0 = 0$), having non-zero relativistic masses ($m \neq 0$), move along world-trajectories of zero four-dimensional length ($ds = 0$, $c\,d\tau = d\sigma \neq 0$) at the velocity of light. These are isotropic trajectories. Massless particles are related to light-like particles — the quanta of an electromagnetic field (photons).

A condition under which a particle may realize an instantaneous displacement (*teleportation*) is the vanishing of the observable time interval $d\tau = 0$ so that the *teleportation condition* is

$$\mathrm{w} + v_i u^i = c^2, \tag{4.4}$$

where $u^i = \frac{dx^i}{dt}$ is its three-dimensional coordinate velocity. Hence the space-time interval by which this particle is instantaneously displaced takes the form

$$ds^2 = -\,d\sigma^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\,dx^i dx^k \neq 0\,, \tag{4.5}$$

where $1 - \frac{\mathrm{w}}{c^2} = \frac{v_i u^i}{c^2}$ in this case, because $d\tau = 0$.

In such a case the signature $(+---)$ in the space-time region of a regular observer becomes $(-+++)$ in the space-time region where particles may be teleported. So the terms "time" and "three-dimensional space" are interchanged in such a region. "Time" of teleporting particles is "space" of the regular observer, and vice versa "space" of teleporting particles is "time" of the regular observer.

Let us first consider substantial particles. As it easy to see, instant displacement (teleportation) of such particles manifests along world-trajectories in which $ds^2 = -\,d\sigma^2 \neq 0$ is true. So these trajectories represented in the terms of physically observable quantities are purely spatial lines of imaginary three-dimensional lengths $d\sigma$, although when taken in the ideal world-coordinates t and x^i the trajectories are four-dimensional. In a particular case, where the space is free of rotation ($v_i = 0$) or the linear velocity of its rotation v_i is orthogonal to the coordinate velocity u^i of the teleporting particle (their scalar product is $v_i u^i = |v_i||u^i| \cos(v_i; u^i) = 0$), substantial particles may be teleported only if gravitational collapse occurs ($\mathrm{w} = c^2$). In this case, the world-trajectories of teleportation taken in the ideal world-coordinates also become purely spatial $ds^2 = g_{ik}\,dx^i dx^k$.

The second case, massless light-like particles (e.g. photons). Such particles may be teleported along world-trajectories located in a space possessing the metric

$$ds^2 = -\,d\sigma^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\,dx^i dx^k = 0\,, \tag{4.6}$$

because for photons $ds^2 = 0$ by definition. As a result we see that the space of photon teleportation characterizes itself by the conditions $ds^2 = 0$ and $d\sigma^2 = c^2 d\tau^2 = 0$.

The obtained condition of photon teleportation (4.6) is like the light cone equation $c^2 d\tau^2 - d\sigma^2 = 0$, where $d\sigma \neq 0$ and $d\tau \neq 0$. This equation describes the light cone, elements of which are the world-trajectories of massless (light-like) particles. In contrast to the light cone equation, the obtained equation (4.6) is constituted by the ideal world-coordinates t and x^i, i.e. not this equation in the terms of physically observable quantities. So teleporting photons actually move along trajectories, which are the elements of the world-cone (like the light cone) in the region of the space-time, where substantial particles may be teleported as well (the metric inside such a region has been obtained above).

Considering the condition of photon teleportation (4.6) from the viewpoint of a regular observer, we can see the obvious fact that, in such a case, the observable spatial metric $d\sigma^2 = h_{ik}\, dx^i dx^k$ becomes degenerate: $h = \det ||\, h_{ik}|| = 0$. This case means actually the degenerate light cone. Taking the relationship $g = -\, h\, g_{00}$ [2–4] into account, we conclude that the four-dimensional metric $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ becomes degenerate there as well: $g = \det ||\, g_{\alpha\beta}|| = 0$. The last fact means that the signature conditions, which determine a pseudo-Riemannian space, are broken, so photon teleportation manifests outside the basic space-time of the General Theory of Relativity. Such a fully degenerate space-time, considered earlier in §1.4 and §1.5 of Chapter 1 in this book, is referred to as *zero-space* since, from the viewpoint of a regular observer, all spatial and time intervals are zero therein.

Once $d\tau = 0$ and $d\sigma = 0$, the observable relativistic mass m and the frequency ω become zero. Thus, from the viewpoint of a regular observer, any particle located in a zero-space (in particular, a teleporting photon) having zero rest-mass $m_0 = 0$ appear as zero relativistic mass $m = 0$ and frequency $\omega = 0$. Therefore particles of this kind may be assumed to be the ultimate case of massless (light-like) particles.

In §1.4 we have introduced a term *zero-particles* for all particles located in a zero-space.

According to the wave-particle duality each particle can be resent as a wave. In the framework of this concept each mass-bearing particle is given by its own four-dimensional wave vector $K_\alpha = \frac{\partial \psi}{\partial x^\alpha}$, where ψ is the wave phase known also as eikonal. The eikonal equation $K_\alpha K^\alpha = 0$ [15], setting forth the fact that the length of a four-dimensional vector remains unchanged in the four-dimensional pseudo-Riemannian space, for regular massless light-like particles (regular photons) it becomes a trav-

elling wave equation (see §1.3 for detail)

$$\frac{1}{c^2}\left(\frac{{}^*\partial\psi}{\partial t}\right)^2 - h^{ik}\frac{{}^*\partial\psi}{\partial x^i}\frac{{}^*\partial\psi}{\partial x^k} = 0\,. \tag{4.7}$$

The eikonal equation in a zero-space region takes the form (see §1.5)

$$h^{ik}\frac{{}^*\partial\psi}{\partial x^i}\frac{{}^*\partial\psi}{\partial x^k} = 0\,, \tag{4.8}$$

because there $\omega = \frac{{}^*\partial\psi}{\partial t} = 0$, putting the time term of the equation to zero. It is a standing wave equation. So, from the viewpoint of a regular observer, in the framework of the wave-particle concept, all particles located in a zero-space region manifest as *standing light waves*, so that the entire zero-space appears filled with a system of standing light waves — light-like holograms. This means that an experiment for discovering non-quantum teleportation of photons should be linked to stationary (stopped) light.

At the end, we conclude that instant displacements of particles are naturally permitted in the space-time of the General Theory of Relativity. As it has been shown, teleportation of substantial particles and photons realizes itself in different space-time regions. But it would be a mistake to think that teleportation requires accelerating a substantial particle to superluminal velocities (into tachyonic regime), while a photon needs to be accelerated to infinite velocity. No — as it is easy to see from the teleportation condition $\mathrm{w} + v_i u^i = c^2$, if the gravitational potential is essential and if the space rotates at a velocity close to the velocity of light, substantial particles may be teleported at regular subluminal velocities. Photons can reach the teleportation condition easier, because they move at the velocity of light. From the viewpoint of a regular observer, as soon as the teleportation condition realizes itself in the neighbourhood around a moving particle, the particle "disappears" although it continues its motion at a subluminal (or light) coordinate velocity u^i in another space-time region invisible to us. Then, having its velocity lowered or if something else breaks the teleportation condition (such as lowering the gravitational potential or the linear velocity of rotation of the space), it "appears" at the same observable moment at another point in our observable space at that distance and in the direction which it has got itself.

There is no problem with photon teleportation being realized along fully degenerate world-trajectories ($g = 0$) outside the basic pseudo-Riemannian space ($g < 0$), while teleportation trajectories of substantial particles are strictly non-degenerate ($g < 0$) so these world-lines are lo-

cated in the pseudo-Riemannian space*. It presents no problem because at any point in the pseudo-Riemannian space we can place a tangent space of $g \leqslant 0$ consisting of the regular pseudo-Riemannian space ($g < 0$) and the zero-space ($g = 0$) as two different regions of the same manifold. A space of $g \leqslant 0$ is a natural generalization of the basic space-time of the General Theory of Relativity, permitting non-quantum ways for teleportation of both photons and substantial particles (previously achieved only in the strict quantum fashion — quantum teleportation of photons in 1998 [27] and of atoms in 2004 [28, 29]).

Until now teleportation has had an explanation given only by Quantum Mechanics [30]. Now the situation changes: with our theory we can find physical conditions for the realization of teleportation of both photons and substantial particles in a non-quantum way (*non-quantum teleportation*), in the framework of the General Theory of Relativity.

The only difference is that from the viewpoint of a regular observer the length of any parallel transported vector remains unchanged. It is also an "observable truth" for vectors in a zero-space region, because the observer reasons the standards (properties) of his pseudo-Riemannian space anyway. The eikonal equation in a zero-space region, expressed in his observable world-coordinates, is $K_\alpha K^\alpha = 0$. However in the ideal world-coordinates t and x^i the metric inside zero-space, $ds^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\, dx^i dx^k = 0$, degenerates into a three-dimensional $d\mu^2$ which, depending on the gravitational potential w uncompensated by something else, is not invariant

$$d\mu^2 = g_{ik}\, dx^i dx^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 \neq inv\,. \tag{4.9}$$

As a result, within a zero-space the length of a transported vector, the four-dimensional vector of a coordinate velocity U^α for instance, being degenerate into a spatial U^i, does not remain unchanged

$$U_i U^i = g_{ik}\, U^i U^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 \neq const\,, \tag{4.10}$$

so that although the observed geometry inside the zero-space is Riemannian for a regular observer, the real geometry of the zero-space within the space itself is non-Riemannian.

*Any space in Riemannian geometry has strictly non-degenerate metric nature $g < 0$ by definition. Pseudo-Riemannian spaces are a particular case of Riemannian spaces, where the metric is sign-alternating. Einstein set forth a four-dimensional pseudo-Riemannian space of the signature $(+---)$ or $(-+++)$ as the base of the General Theory of Relativity. So the basic space-time of the General Theory of Relativity is as well of strictly non-degenerate metric ($g < 0$).

In connexion with the results, it is important to remember the "Infinite Relativity Principle", introduced by Abraham Zelmanov. Proceeding from his studies on relativistic cosmology, he concluded that [31,32]:

> Zelmanov's "Infinite Relativity Principle": In homogeneous isotropic cosmological models spatial infinity of the Universe depends on our choice of that reference frame from which we observe the Universe (the observer's reference frame). If the three-dimensional space of the Universe, being observed in one reference frame, is infinite, it may be finite in another reference frame. The same is just as well true for the time during which the Universe evolves.

We have come to the "Finite Relativity Principle" here. As we have showed, because of a difference between the physically observable world-coordinates and the ideal world-coordinates, the same space-time regions may be very different, being determined in each of the frames. Thus, in the observable world-coordinates, a zero-space region is a point ($d\tau = 0$, $d\sigma = 0$), while $d\tau = 0$ and $d\sigma = 0$ taken in the ideal world-coordinates become $-\left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\,dx^i dx^k = 0$, which is a four-dimensional cone equation like the light cone equation $c^2 d\tau^2 - d\sigma^2 = 0$. Actually, here is the "Finite Relativity Principle" for observed objects — an observed point is the whole space taken in the ideal coordinates.

Moreover, this research is currently the sole theoretical explanation of the observed phenomenon of teleportation [27–29] given by the General Theory of Relativity.

§4.2 The geometric structure of zero-space

As we have obtained, a regular real observer perceives the entire zero-space as a region determined by the observable conditions of degeneration, which are $d\tau = 0$ and $d\sigma^2 = h_{ik}\,dx^i dx^k = 0$ (see §1.4 for detail).

The physical sense of the first condition $d\tau = 0$ is that the real observer perceives any two events in the zero-space as simultaneous, at whatever distance from them they are. Such a way of instantaneous spread of information is referred to as the *long-range action*.

The second condition $d\sigma^2 = 0$ means the absence of observable distance between the event and the observer. Such "superposition" of observer and observed object is only possible if we assume that our regular four-dimensional pseudo-Riemannian space meets the entire zero-space at each point (as is "stuffed" with the zero-space).

Let us now turn to the mathematical interpretation of the conditions of degeneration.

The quantity $c\,d\tau$ is a chr.inv.-projection of the four-dimensional coordinate interval dx^α onto the line of time: $c\,d\tau = b_\alpha dx^\alpha$. The proper world-vector of the observer b^α by definition is not zero and dx^α is not zero as well. Then $d\tau = 0$ is true at $d\sigma^2 = 0$ only if the space-time metric $ds^2 = c^2 d\tau^2 - d\sigma^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ is degenerate, i.e. the determinant of the fundamental metric tensor is zero

$$g = \det ||\, g_{\alpha\beta}|| = 0\,. \tag{4.11}$$

Similarly, the condition $d\sigma^2 = h_{ik}\, dx^i dx^k = 0$ in a region means that, in this region, the observable three-dimensional metric is degenerate

$$h = \det ||\, h_{ik}|| = 0\,. \tag{4.12}$$

Having the conditions of degeneration of space-time, $\mathrm{w} + v_i u^i = c^2$ and $g_{ik}\, dx^i dx^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2$, substituted into $d\sigma^2 = h_{ik}\, dx^i dx^k = 0$ we obtain the zero-space metric

$$ds^2 = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 - g_{ik}\, dx^i dx^k = 0\,. \tag{4.13}$$

Hence inside a zero-space (from the viewpoint of an "inner" observer) the three-dimensional space is holonomic, while rotation of the zero-space is present in the time component of its metric

$$\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 = \left(\frac{v_i u^i}{c^2}\right)^2 c^2 dt^2. \tag{4.14}$$

If $\mathrm{w} = c^2$ (the condition for gravitational collapse), the zero-space metric (4.13) takes the form

$$ds^2 = -\, g_{ik} dx^i dx^k = 0\,, \tag{4.15}$$

i.e. it becomes purely three-dimensional and the three-dimensional space becomes degenerate as well

$$g_{(3\mathrm{D})} = \det ||\, g_{ik}|| = 0\,. \tag{4.16}$$

Here the condition $g_{(3\mathrm{D})} = 0$ originates in the fact that $g_{ik}\, dx^i dx^k$ is sign-definite, so it can only become zero provided the determinant of the three-dimensional metric tensor g_{ik} is zero.

Because in any zero-space $\mathrm{w} + v_i u^i = c^2$, in the case of gravitational collapse the condition $v_i u^i = 0$ also becomes true.

The quantity $v_i u^i = vu \cos(v_i; u^i)$, which is the scalar product of the linear velocity of the space rotation, and the coordinate velocity of a zero-particle will be referred to as the *chirality* of the zero-particle.

In the case where $v_i u^i > 0$, the angle α between v_i and u^i is within the range of $\frac{3\pi}{2} < \alpha < \frac{\pi}{2}$. Because $g_{ik} u^i u^k = c^2 \big(1 - \frac{\mathrm{w}}{c^2}\big)^2$, i.e. the second condition of degeneration, means that $u = c\big(1 - \frac{\mathrm{w}}{c^2}\big)$, in this case the gravitational potential is $\mathrm{w} < c^2$ (a regular gravitational field).

In the case where $v_i u^i < 0$, the angle α is within $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$, so we obtain $\mathrm{w} > c^2$ (a super-strong gravitational field).

The condition $v_i u^i = 0$ is only true if $\alpha = \{\frac{\pi}{2}; \frac{3\pi}{2}\}$ or if $\mathrm{w} = c^2$ (gravitational collapse).

Hence the chirality of a zero-particle is zero if either the velocity of the particle is orthogonal to the linear velocity of the space rotation, or if gravitational collapse occurs*.

Because $\mathrm{w} = c^2(1 - e_{(0)})$ and $v_i = -c\, e_{(i)} \cos(x^0; x^i)$ in the basis form, the condition of degeneration $\mathrm{w} + v_i u^i = c^2$ can be written as

$$c\, e_{(0)} = -\, e_{(i)} u^i \cos(x^0; x^i)\,. \tag{4.17}$$

The dimension of a space is determined by the number of the linearly independent basis vectors in it. In our formula (4.17), which is the basic notation for the condition $\mathrm{w} + v_i u^i = c^2$, the time basis vector is linearly dependent on all spatial basis vectors. This means actual degeneration of the space-time. Hence our formula (4.17) can be regarded as the *geometric condition of degeneration*.

In the case of gravitational collapse ($\mathrm{w} = c^2$) the length of the time basis vector $e_{(0)} = 1 - \frac{\mathrm{w}}{c^2}$ becomes zero. In the absence of gravitational fields ($\mathrm{w} = 0$), or in a weak gravitational field ($\mathrm{w} \to 0$), the quantity $e_{(0)}$ takes its maximum length equal to 1. In intermediate cases, $e_{(0)}$ becomes shorter as the gravitational field becomes stronger.

As known, at any point in the four-dimensional space-time, there exists an *isotropic cone* — a hyper-surface whose metric is

$$g_{\alpha\beta}\, dx^\alpha dx^\beta = 0\,. \tag{4.18}$$

Geometrically speaking, this is a region of the space-time which hosts light-like particles. Because the space-time interval in such a region is zero, all directions inside it are equal (in other word, they are isotropic). Therefore the region which hosts light-like particles is commonly referred to as the *isotropic cone* or the *light cone*.

Because in a zero-space the metric is also equal to zero (4.13) an isotropic cone can be constructed at any of its points. Such an isotropic

*This is because, under the condition of gravitational collapse, the modulus of the coordinate velocity of the particle equals zero ($u = 0$).

cone is described by a somewhat different equation

$$\left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2dt^2-g_{ik}\,dx^i dx^k=0\,. \tag{4.19}$$

The difference between such an isotropic cone and the light cone is that it satisfies the condition

$$1-\frac{\mathrm{w}}{c^2}=\frac{v_i u^i}{c^2}\,, \tag{4.20}$$

which is only typical for a degenerate space-time (zero-space). We therefore will call it the *degenerate isotropic cone*. Because the specific term (4.20) is function of the space rotation, the degenerate isotropic cone is a cone of rotation.

Under gravitational collapse ($\mathrm{w}=c^2$) the first term in (4.20) becomes zero (the stopping point of the coordinate time), while the remaining second term $g_{ik}\,dx^i dx^k=0$ describes a three-dimensional degenerate hyper-surface. However if $\mathrm{w}=0$, $v_i u^i=0$ and the equation of the degenerate isotropic cone (4.20) becomes

$$c^2dt^2-g_{ik}\,dx^i dx^k=0\,, \tag{4.21}$$

i.e. the coordinate time flows evenly.

The greater the gravitational potential w, the more severe the degenerate cone becomes and the closer it is to the spatial section. In the ultimate case where $\mathrm{w}=c^2$ the degenerate cone becomes flattened over the three-dimensional space (collapses). The "lightest" cone $\mathrm{w}=0$ is the most distant one from the spatial section.

Hence a gravitational collapsar in a zero-space region is similar to the zero-space observed by a regular observer like us. In other words, the entire zero-space for us is a degenerate state of the regular space-time, while for a zero-observer a gravitational collapsar is the degenerate state of the zero-space. This means that an isotropic light cone contains a degenerate isotropic cone of the entire zero-space, which, in turn, contains a particular collapsed degenerate isotropic zero-space cone of a collapsar inside the zero-space. This is an illustration of the fractal structure of the world presented here as a system of the isotropic cones found inside each other.

§4.3 Zero-space as home space for virtual particles. Geometric interpretation of Feynman diagrams

As well-known, the Feynman diagrams are a graphical description of the various interactions between elementary particles. The diagrams clearly

show that the actual carriers of interactions are virtual particles. In other words, almost all physical processes rely on the emission and the absorption of virtual particles (e.g. virtual photons) by real particles of our world.

Another notable property of the Feynman diagrams is that they are capable of describing particles (e.g. electrons) and antiparticles (e.g. positrons) at the same time. In this example, a positron is represented as an electron which moves back in time.

According to Quantum Electrodynamics, the interaction of particles at every branching point of the Feynman diagrams conserves four-dimensional momentum. This suggests a possible geometric interpretation of the Feynman diagrams in the General Theory of Relativity.

As a matter of fact, in the four-dimensional pseudo-Riemannian space, which is the basic space-time of the General Theory of Relativity, the following objects can get correct, formal definitions:

1) Mass-bearing particle — a particle, whose rest-mass is not zero ($m_0 \neq 0$) and allowed trajectories are non-isotropic ($ds \neq 0$). These are subluminal mass-bearing particles (real particles) and superluminal mass-bearing particles (tachyons). Mass-bearing particles include both particle and antiparticle options, realizing motion from the past into the future and from the future into the past, respectively;
2) Massless particle — a particle with zero rest-mass ($m_0 = 0$), but a non-zero relativistic mass ($m \neq 0$), which moves along isotropic trajectories ($ds = 0$) at the velocity of light. These are light-like particles, e.g. photons. Massless particles include both particle and antiparticle options as well;
3) Zero-particle — a particle with zero rest-mass and zero relativistic mass, which moves along trajectories in the fully degenerate space-time (zero-space). From the viewpoint of a regular observer, whose location is our world, the physicall observable time stops on zero-particles. So both particle and anti-particle options become senseless for zero-particles.

Hence to translate Feynman diagrams into the space-time geometry of the General Theory of Relativity we only need a formal definition for virtual particles. The way to do it is as follows.

In Quantum Electrodynamics, virtual particles are particles for which, contrary to regular ones, the relationship between energy and momentum

$$E^2 - c^2 p^2 = E_0^2 \,, \tag{4.22}$$

where $E = mc^2$, $p^2 = m^2\mathrm{v}^2$, $E_0 = m_0c^2$, is not true. In other word, for virtual particles

$$E^2 - c^2p^2 \neq E_0^2\,. \tag{4.23}$$

In a pseudo-Riemannian space, the relationship (4.22) in the chr.inv.-form has a similar representation

$$p^2 = h_{ik}p^ip^k, \tag{4.24}$$

where $p^i = m\mathrm{v}^i$ stands for the physically observable chr.inv.-vector of the momentum of the particle. Dividing (4.24) by c^4, we obtain

$$m^2 - \frac{p^2}{c^2} = m_0^2\,, \tag{4.25}$$

i.e. the chr.inv.-formulation for the conservation of the four-dimensional momentum of a mass-bearing real particle

$$P_\alpha P^\alpha = g_{\alpha\beta}\,P^\alpha P^\beta = m_0^2\,g_{\alpha\beta}\,\frac{dx^\alpha}{ds}\,\frac{dx^\beta}{ds} = m_0^2 \tag{4.26}$$

in parallel transfer along to the entire trajectory of this particle, where $ds^2 > 0$, i.e. along a subluminal trajectory. For a superluminal particle (tachyon), whose four-dimensional momentum is

$$P^\alpha = m_0\frac{dx^\alpha}{|ds|}\,, \tag{4.27}$$

the chr.inv.-relationship between mass and momentum (4.25) becomes

$$\frac{p^2}{c^2} - m^2 = (im_0)^2\,. \tag{4.28}$$

For massless (light-like) particles, e.g. photons, the rest-mass is zero and the relationship between mass and momentum transforms as

$$m^2 = \frac{p^2}{c^2}\,, \tag{4.29}$$

where the relativistic mass m is determined from the mass-energy equivalence $E = mc^2$, while the physically observable momentum $p^i = mc^i$ is expressed through the chr.inv.-vector of the velocity of light.

Thus the obtained equations (4.25), (4.28), (4.29) characterize the chr.inv.-relationship between mass and momentum for regular particles which inhabit the space-time of the General Theory of Relativity. Interactions between regular particles are carried out by virtual particles. Given this fact, in order to geometrically interpret the Feynman diagrams we need a geometric description of virtual particles.

By definition, the chr.inv.-formula (4.25) presenting the observable relationship between mass and momentum should not be true for virtual particles. From the geometric viewpoint, this means that the length of the four-dimensional vector of momentum of any virtual particle does not conserve in parallel transfer of it along to the world-trajectory of the virtual particle. In a Riemannian space, particularly in the four-dimensional pseudo-Riemannian space (the basic space of the General Theory of Relativity), the length of any vector remains unchanged in parallel transfer of it, by definition of Riemannian geometry. This means that world-trajectories of virtual particles lie in a space with non-Riemannian geometry, i.e. outside the four-dimensional pseudo-Riemannian space.

In §1.4 unrelated to virtual particles, we have showed that trajectories, along which the square of a tangential vector being transferred in parallel to itself does not conserve, are located in a zero-space we called a fully degenerate space-time ($g = \det ||\, g_{\alpha\beta} ||\, = 0$). In a pseudo-Riemannian space $g < 0$ is always true by definition of the Riemannian metric. Hence the entire zero-space is located beyond the four-dimensional pseudo-Riemannian space and its geometry is not Riemannian. Besides, as we have showed, the relativistic masses of particles which the zero-space hosts (zero-particles) is zero and, from the viewpoint of an observer whose location is our world, their motion expects to be observed as instantaneous displacement (long-range action).

Analysis of the above facts brings us into the conclusion that zero-particles can be equated to virtual particles in the *extended space-time*, wherein $g \leqslant 0$ (we introduced such a space in §1.5). Such an extended space (space-time) permits degeneration of the metric and considering not only the motion of regular mass-bearing and massless particles, but also their interaction by means of their exchange with virtual particles (zero-particles) in the zero-space. In fact, this is the geometric interpretation of the Feynman diagrams in the General Theory of Relativity.

In particular, because the zero-space metric $d\mu^2$ (4.9) is not invariant $d\mu^2 = g_{ik}\, dx^i dx^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 \neq inv$, the length of a degenerate four-dimensional vector being transferred in parallel to itself in the zero-space does not conserve. For instance, for a degenerate world-vector $U^\alpha = \frac{dx^\alpha}{dt}$ we have

$$U_\alpha U^\alpha = g_{ik}\, u^i u^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 \neq const. \tag{4.30}$$

Applying the theory of physically observable quantities to this situation again shows us the way out. Since we consider all quantities from the viewpoint of an observer, all quantities, including those in a zero-

space, can be expressed through the physical observable characteristics (physical standards) of his space of reference. Therefore a zero-particle from the viewpoint of a regular observer possesses a four-dimensional vector of momentum P^α, whose length remains unchanged in parallel transfer in the host space of it (zero-space)

$$P^\alpha = m_0 \frac{dx^\alpha}{ds} = \frac{M}{c} \frac{dx^\alpha}{dt}\,, \qquad P_\alpha P^\alpha = \frac{M^2}{c^2} \frac{ds^2}{dt^2} = 0\,, \tag{4.31}$$

because in a zero-space, by definition, $ds^2 = 0$. On the other hand, once we turn out to the frame of reference of a hypothetical observer whose location is the zero-space, i.e. to the space with the metric $d\mu^2$ (4.9), the length of the transferred vector does not conserve any longer.

Now we are going to look what kinds of particles are hosted by the zero-space. First we look at the degeneration conditions (1.98) in the absence of the gravitational potential ($\mathrm{w} = 0$). These are

$$v_i u^i = c^2, \qquad g_{ik}\, u^i u^k = c^2, \tag{4.32}$$

i.e. in the absence of gravitation zero-particles move in the zero-space at a coordinate velocity, which is equal to the velocity of light

$$u = \sqrt{g_{ik} u^i u^k} = c\,, \tag{4.33}$$

despite the fact that their motion seems to be instantaneous displacement from the viewpoint of a regular observer like us, located in the strictly non-degenerate pseudo-Riemannian space.

The first condition of degeneration is the scalar product between the linear velocity of the space rotation and the three-dimensional coordinate velocity of the particle

$$v_i u^i = v u \cos\left(v_i; u^i\right) = c^2. \tag{4.34}$$

Since $u = c$, this condition is true for the vectors v_i and u^i which are co-directed (or coincide with each other, like in this case). Hence in the absence of gravitation zero-particles move in the zero-space which hosts them at the velocity of light, while, at the same time, the zero-space rotates with the velocity of light as well. We will refer to such zero-particles as *virtual photons*. The zero-space metric along their trajectories is

$$d\mu^2 = g_{ik}\, dx^i dx^k = c^2 dt^2 \neq 0\,, \tag{4.35}$$

similar to the metric $d\sigma^2 = c^2 d\tau^2 \neq 0$ along the trajectories of regular photons in the pseudo-Riemannian space.

Now we will look what kinds of particles are hosted by the zero-space in the presence of the gravitational potential ($\mathrm{w} \neq 0$). In such a case the degeneration conditions (1.98) are

$$v_i u^i = c^2 - \mathrm{w}\,, \qquad u^2 = g_{ik}\, dx^i dx^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2, \tag{4.36}$$

so the scalar product $v_i u^i = c^2 - \mathrm{w}$ can be represented as

$$v_i u^i = v u \cos\left(v_i; u^i\right) = v c \left(1 - \frac{\mathrm{w}}{c^2}\right) \cos\left(v_i; u^i\right) = \left(1 - \frac{\mathrm{w}}{c^2}\right) c^2. \tag{4.37}$$

This equation is true given that the vectors v_i and u^i are co-directed, and also $v = c$, i.e. in the presence of gravitation zero-particles move in the zero-space which hosts them at a velocity of the magnitude

$$u = c \left(1 - \frac{\mathrm{w}}{c^2}\right), \tag{4.38}$$

while the zero-space itself rotates at the velocity of light $v = c$.

Just we turn to the zero-space metric in the presence of gravitation

$$d\mu^2 = g_{ik}\, dx^i dx^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2, \tag{4.39}$$

we see that the real time parameter here is not the coordinate time t, but the following variable (it can be called the *gravitational time*, because dependent on the potential)

$$t_* = \left(1 - \frac{\mathrm{w}}{c^2}\right) t\,, \tag{4.40}$$

i.e. the real coordinate velocity of zero-particles along such trajectories depends on the gravitational potential

$$u^i_* = \frac{dx^i}{dt_*} = \frac{u^i}{1 - \frac{\mathrm{w}}{c^2}}\,. \tag{4.41}$$

Due to the second degeneration condition of (1.98), the real coordinate velocity of these zero-particles equals the velocity of light

$$u^2_* = g_{ik}\, u^i_* u^k_* = \frac{g_{ik}\, dx^i dx^k}{\left(1 - \frac{\mathrm{w}}{c^2}\right)^2} = c^2, \tag{4.42}$$

i.e. they are *virtual photons* as well. Due to the first degeneration condition of (1.98), we see that in the presence of gravitation the real linear velocity of rotation of the zero-space which hosts virtual photons is as well the velocity of light

$$v_i u^i_* = c^2. \tag{4.43}$$

It is worth noting that considering virtual mass-bearing particles is senseless, because all particles hosted by a zero-space by definition possess zero rest-mass, and therefore are not mass-bearing particles. Therefore only virtual photons and their varieties are virtual particles.

Now we are going to define virtual particles in a collapsed zero-space area ($\mathrm{w} = c^2$). We will refer to virtual particles as *virtual collapsars*. For them, the degeneration conditions (1.98) become

$$v_i u^i = 0\,, \qquad g_{ik}\, dx^i dx^k = 0\,, \tag{4.44}$$

i.e. zero-collapsars either rest with respect to the space of reference of an observer whose location is the zero-space, or the world around him shrinks into a point (all $dx^i = 0$), or the three-dimensional metric of the collapsed zero-space area is degenerate $g_{(3\mathrm{D})} = \det ||\, g_{ik}|| = 0$. The zero-space metric along the trajectories of virtual collapsars is

$$d\mu^2 = g_{ik}\, dx^i dx^k = 0\,. \tag{4.45}$$

So we have obtained that virtual particles of two kinds can exist in a zero-space, which is a degenerate four-dimensional space-time:

1) Virtual photons — they possess forward motion at the velocity of light, while the zero-space which hosts them rotates at the velocity of light as well;
2) Virtual collapsars which rest with respect to the zero-space.

As a result we arrive at a conclusion that all interactions between regular mass-bearing and massless particles in the basic space-time of the General Theory of Relativity (the four-dimensional pseudo-Riemannian space), are affected through an *exchange buffer*, in whose capacity the zero-space acts. Material carriers of interactions within such a buffer are virtual particles of the two aforementioned kinds.

In §1.5 of Chapter 1, on considering particles in the framework of the wave-particle duality, we have obtained that the eikonal equation for zero-particles is a standing wave equation of stopped light (1.122). Hence virtual particles are actually *standing light waves*, and interaction between regular particles in our regular space-time is transmitted through a system of standing light-like waves (*standing-light holograms*), which fills the exchange buffer (zero-space).

This research currently is the sole explanation of virtual particles and virtual interaction given by the geometrical methods of Einstein's General Theory of Relativity.

Conclusions

With the foregoing results, we can now draw the general picture of the kinds of particles, which are theoretically conceivable in the four-dimensional space-time of the General Theory of Relativity.

We solved this problem with use of the mathematical apparatus of physically observable quantities (chronometric invariants). The essence of this method, developed in 1944 by Abraham Zelmanov, is simple. As known, the components of a tensor quantity are determined in a system of the orthogonal ideal (straight and uniform) axes, which are tangential to the real (curved and non-uniform) axes at the origin of the coordinates. Real space-time can be imagined as a set of the curved and non-uniform spatial sections (three-dimensional spaces), "pierced" in each point by the non-uniform axes of time. Projecting a four-dimensional quantity onto the line of time and onto the spatial section of an observer, we obtain quantities really registered by him. Because projection is done in the real space, the result depends on the properties of the space such as its rotation, deformation, curvature, etc. Numerous experiments, which have been done since 1950's, showed significant impact of the properties of space on the measured length and time. The most tremendous out of those experiments were no-landing flights around the terrestrial globe in the 1970's (the Hafele-Keating experiment).

As we found, the mathematical method of chronometric invariants presents two cases, which could not be studied using the general covariant method: a) "splitting" the space-time into a region, where time flows from the past into the future (our world) and a region, where time flows into the opposite direction (the mirror world); b) a region, where the four-dimensional interval, the observable three-dimensional interval, and the interval of observable time are zeroes (zero-space).

Let us discuss the first case first. The method of chronometric invariants manifests that relativistic mass m is the scalar observable projection of the four-dimensional vector of the momentum of a mass-bearing particle, while relativistic frequency ω is the scalar observable projection of the four-dimensional wave vector of a massless (light-like) particle. According to this result, mass-bearing particles with positive relativistic masses $m>0$ inhabit our world wherein they move from the past into the future with respect to a regular observer, realizing the direct

flow of time. Particles with negative relativistic masses $m<0$ inhabit the mirror world wherein they move from the future into the past, from our point of view, so we see that there time flows in the opposite direction. All these events occur in the "internal" region of the light cone. Inside the "walls" of the light cone, the condition $c^2d\tau^2=d\sigma^2\neq 0$ is true, i.e. the time and spatial projections of the four-dimensional coordinates are equal and non-zero, while the space-time interval is degenerate $ds^2=c^2d\tau^2-d\sigma^2=0$. This is the habitat of massless (light-like) particles, e.g. photons. Light-like particles of our world bear positive frequencies $\omega>0$; they move from the past into the future. In the mirror world, light-like particles bear negative frequencies $\omega<0$ and move from the future into the past, from our point of view.

Further, we found that the chronometrically invariant (observable) equations of motion for particles of our world and for those of the mirror world are asymmetric, i.e. for particles observable motion either into the past or into the future is not the same. This fact means that the physical conditions of motion into the past or into the future differ from each other. Such an asymmetry depends on only the properties of space-time such as the gravitational inertial force, the space rotation, and the space deformation.

If the physically observable time τ was not different from the coordinate time t (they differ due to the gravitational potential and the rotation of space), the very statement of a problem of the space-time regions with either the direct or reverse flow of time would be impossible.

Here we come to an important question. Assume four independent coordinate axes — one time axis and three spatial axes. From the geometric viewpoint both directions along the time axis are absolutely equal. But what asymmetry are we speaking about and isn't it a sort of mistake? No, it isn't a mistake. Of course, if the spatial section (three-dimensional space) is uniform and isotropic, both directions into the past and into the future are equal. But as soon as the spatial section becomes rotated or deformed (this is like a crumpled paper sheet set upon an axis and rotated around it), the space-time becomes anisotropic with respect to the line of time. This anisotropy leads to different physical conditions of motion into the past and into the future.

Furthermore, looking at the motion of particles as the propagation of waves (within de Broglie's wave-particle duality), we observe no asymmetry: the propagation of waves is observed to be the same in both directions in time, while the motion of "particle-balls" is not.

As a result, in our real space-time we should have two different four-dimensional regions: our world with the direct flow of time and the

mirror world wherein, from our point of view, time flows in the opposite direction. These regions are separated with a space-time membrane, on which, from the viewpoint of an "external observer" whose location is our world or the mirror world, the observable time stops $d\tau = 0$.

What sort of membrane is that and isn't it merely a border surface between our world and the mirror world? Our study of the question using the method of physical observable quantities gave the following result. Inside the membrane, which separates our world from the mirror world, a somewhat stricter condition is true $d\tau = 0$, i.e. the observable time is degenerate. This fact manifest two cases on the four-dimensional interval $ds^2 = c^2 d\tau^2 - d\sigma^2$ in the space-time region occupied by such a membrane: a) $d\tau = 0$, while $d\sigma \neq 0$ and $ds^2 = -d\sigma^2 \neq 0$, so this part of the space-time membrane should be observed by us a three-dimensional region inhabited by mass-bearing particles all physical processes on whom have been stopped; b) $d\tau = 0$, while $d\sigma = 0$ and $ds^2 = 0$ as well. The second case manifests both physically observable time $d\tau$, four-dimensional metric $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ and observable three-dimensional metric $d\sigma^2 = h_{ik}\, dx^i dx^k$ to be degenerate. Mathematically this means *full degeneration* of the space-time region. This part of the space-time membrane should be observed as an entire three-dimensional region shrunk into a single point, despite the fact that the coordinate time interval dt and the coordinate three-dimensional metric $d\mu^2 = g_{ik}\, dx^i dx^k$ are non-degenerate inside such a region.

What is a fully degenerate space-time and does it contain any particles? According to the general covariant method, which isn't related to any specific frame of reference, in such a case we have absolute zero and the very statement of the problem is nonsense. We therefore called a fully degenerate space-time or any fully degenerate region of the regular, non-degenerate space-time *zero-space*. But the method of physical observable quantities, linked to a real frame of reference and its properties, allows an observer to "look" inside a zero-space so that we see what is going on therein. As a result we found that any zero-space contained an entire world with its own coordinates, trajectories and particles (*zero-particles*). On the other hand, due to the geometric structure of the four-dimensional space-time a regular observer on the Earth sees an entire zero-space shrunk into a single point where the observable time stops. But this fact doesn't mean that the only way to enter the zero-space from our world is through a single special point. Quite the contrary, the entrance is permitted at any point. What is necessary is to create the physical condition of degeneration in the local space of the entering object. This condition means a special combination of the

gravitational potential w, of the linear velocity of the space rotation v_i, and of the penetrating object's linear velocity u^i, which finally takes the form $w + v_i u^i = c^2$. In a particular case, in the absence of the rotation of the object's local space or if this object rests, the degeneration condition meets the condition of gravitational collapse $w = c^2$: the entering a zero-space is possible also through the state of gravitational collapse.

Because the interval of observable time and the observable spatial interval in a zero-space are observed from our world as zeroes, any displacements of zero-particles are instantaneous from the viewpoint of a regular observer. We call such way of interaction *long-range action*. Because particles of our world can not move in instant, they cannot carry long-range action. But if interaction between two particles of our world is transmitted through a zero-space region (by means of the exchange of zero-particles), long-range action becomes possible: in such a case the observed time between the emission and the reception of a signal becomes zero.

Further studies showed that zero-particles also bear a mass and frequency, but to see them we must enter the zero-space themselves.

How do zero-particles look like from the viewpoint of an observer who is located in our world? Can we detect zero-particles in experiments? We have looked at this problem within de Broglie's wave-particle concept. We have found that the wave phase equation (eikonal equation) of zero-particles is a standing wave equation. In other words, from our point of view zero-particles should be observed as light-like standing waves — the waves of "stopped" light. So all zero-space is filled with standing light waves, or, in other word, standing light holograms. It is possible that the "stop-light experiments" done in Harvard by Lene Hau's group and independently by Lukin and Walsworth may be an experimental "foreword" to discovery of zero-particles.

In up-to-date science the one and only type of particles is known for which the relationship between the energy and the momentum is not true. These are virtual particles. According to the contemporary views based on experimental data, virtual particles carry interaction between any two observable particles (either mass-bearing or light-like ones). This fact allows unambiguous interpretation of zero-particles and zero-space: a) zero-particles are virtual particles that carry interaction between any regular particles; b) zero-space is a space-time region inhabited by virtual particles, and, at the same time, this is the membrane between our world and the mirror world.

Gravitational collapse is also allowed in a zero-space. As long as the gravitational potential w grows, we "descend" into the funnel of the

zero-space deeper and deeper until w finally becomes equal to c^2 and we shall find ourselves in a gravitational collapsar. From the viewpoint of a hypothetical observer whose location is a zero-space, the surface of a gravitational collapsar in the zero-space shrinks into a single point $g_{ik}\,dx^i dx^k = 0$. This is the matter of degenerate gravitational collapsars which, contrary to the regular gravitational collapsars, are located in a zero-space.

There is another interesting fact. A zero-space can only exist in the presence of the space rotation under the condition $\mathrm{w} + v_i u^i = c^2$. In the absence of the rotation, the zero-space always collapses: in such a case the gravitational collapsar expands to occupy the whole zero-space. If both gravitational field and space rotation are absent, the entering into the space-time membrane becomes impossible and any connexion between our world and the mirror world is lost.

In general, such a purely geometric approach allowed us to see the fact that all properties of the particles which inhabit the substantional world, the light-like world, and the zero-world are a sequel of the geometrical structure of those regions of the space-time.

All of the aforementioned results have been obtained exclusively thanks to Zelmanov's method of physically observable quantities (chronometric invariants). The regular generally covariant method has been, and will be, of no use here.

As a result, we can see that not all physical effects in the General Theory of Relativity are yet known in contemporary science. Further developments in experimental physics and observational astronomy will discover new phenomena, related, in particular, to the acceleration, rotation, deformation, and curvature of the local (laboratory) space of reference considered here.

Epilogue

In *Far Rainbow*, written by Arcady and Boris Strugatsky over 40 years ago a character recalls that...

> "...Being a schoolboy he was surprised by the problem: move things across vast spaces in no time. The goal was set to contradict any existing views of absolute space, space-time, kappa-space... At that time they called it "punch of Riemannian fold". Later it would be dubbed "hyper-infiltration", "sigma-infiltration", or "zero-contraction". At length it was named zero-transportation or "zero-T" for short. This produced "zero-T-equipment", "zero-T-problems", "zero-T-tester", "zero-T-physicist".
>
> — What do you do?
>
> — I'm a zero-physicist.
>
> A look full of surprise and admiration.
>
> — Excuse me, could you explain what zero-physics is? I don't understand a bit of it.
>
> — Well... Neither I do".

This passage might be a good afterword to our study. In the early 1960's words like "zero-space" or "zero-transportation" sounded science-fiction or at least something to be brought to (real) life generations from now.

But science is progressing faster then we think. The results obtained in this book suggest that the variety of existing particles, along with the types of their interactions, is not limited to those known to contemporary physics. We should expect that further advancements in experimental technique will discover zero-particles, which inhabit degenerate space-time (zero-space) and can be observed as waves of "stopped light" (standing light waves). From the viewpoint of a regular observer, zero-particles move in instant (despite they move in zero-space at the velocity of light), thus they can realize zero-transportation.

Here, we think it's a mistake to believe or take for granted that most Laws of Nature have already been discovered by contemporary science. What seems more likely is that we are just at the very beginning of a long, long road to the Unknown World.

Appendix A Notations of tensor algebra and analysis

Ordinary differential of a vector:

$$dA^\alpha = \frac{\partial A^\alpha}{\partial x^\sigma}\, dx^\sigma .$$

Absolute differential of a contravariant vector:

$$\mathrm{D}A^\alpha = \nabla_\beta A^\alpha dx^\beta = dA^\alpha + \Gamma^\alpha_{\beta\mu} A^\mu dx^\beta .$$

Absolute differential of a covariant vector:

$$\mathrm{D}A_\alpha = \nabla_\beta A_\alpha dx^\beta = dA_\alpha - \Gamma^\mu_{\alpha\beta} A_\mu dx^\beta .$$

Absolute derivative of a contravariant vector:

$$\nabla_\beta A^\alpha = \frac{\mathrm{D}A^\alpha}{dx^\beta} = \frac{\partial A^\alpha}{\partial x^\beta} + \Gamma^\alpha_{\beta\mu} A^\mu .$$

Absolute derivative of a covariant vector:

$$\nabla_\beta A_\alpha = \frac{\mathrm{D}A_\alpha}{dx^\beta} = \frac{\partial A_\alpha}{\partial x^\beta} - \Gamma^\mu_{\alpha\beta} A_\mu \,.$$

Absolute derivative of a 2nd rank contravariant tensor:

$$\nabla_\beta F^{\sigma\alpha} = \frac{\partial F^{\sigma\alpha}}{\partial x^\beta} + \Gamma^\alpha_{\beta\mu} F^{\sigma\mu} + \Gamma^\sigma_{\beta\mu} F^{\alpha\mu} .$$

Absolute derivative of a 2nd rank covariant tensor:

$$\nabla_\beta F_{\sigma\alpha} = \frac{\partial F_{\sigma\alpha}}{\partial x^\beta} - \Gamma^\mu_{\alpha\beta} F_{\sigma\mu} - \Gamma^\mu_{\sigma\beta} F_{\alpha\mu} \,.$$

Absolute divergence of a vector:

$$\nabla_\alpha A^\alpha = \frac{\partial A^\alpha}{\partial x^\alpha} + \Gamma^\alpha_{\alpha\sigma} A^\sigma .$$

Chr.inv.-divergence of a chr.inv.-vector:

$${}^*\nabla_i q^i = \frac{{}^*\partial q^i}{\partial x^i} + q^i \frac{{}^*\partial \ln\sqrt{h}}{\partial x^i} = \frac{{}^*\partial q^i}{\partial x^i} + q^i \Delta^j_{ji} \,.$$

Physical chr.inv.-divergence:

$$^*\widetilde{\nabla}_i\, q^i = {}^*\nabla_i\, q^i - \frac{1}{c^2}\, F_i\, q^i.$$

D'Alembert's general covariant operator:

$$\Box = g^{\alpha\beta}\, \nabla_\alpha \nabla_\beta\,.$$

Laplace's ordinary operator:

$$\Delta = -\, g^{ik}\, \nabla_i \nabla_k\,.$$

Chr.inv.-Laplace operator:

$$^*\Delta = h^{ik}\, {}^*\nabla_i\, {}^*\nabla_k\,.$$

Chr.inv.-derivative with respect to the time coordinate and that with respect to the spatial coordinates:

$$\frac{^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}\,. \qquad \frac{^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2}\, v_i\, \frac{^*\partial}{\partial t}\,.$$

The square of the physically observable velocity:

$$\mathrm{v}^2 = \mathrm{v}_i\, \mathrm{v}^i = h_{ik}\, \mathrm{v}^i \mathrm{v}^k.$$

The linear velocity of the space rotation:

$$v_i = -\, c\, \frac{g_{0i}}{\sqrt{g_{00}}}\,, \quad v^i = -\, c g^{0i} \sqrt{g_{00}}\,, \quad v_i = h_{ik}\, v^k.$$

The square of v_i. This is the proof: because of $g_{\alpha\sigma} g^{\sigma\beta} = g^\beta_\alpha$, then under $\alpha = \beta = 0$ we have $g_{0\sigma} g^{\sigma 0} = \delta^0_0 = 1$, hence $v^2 = v_k v^k = c^2(1 - g_{00}\, g^{00})$, i.e.:

$$v^2 = h_{ik} v^i v^k.$$

The determinants of the metric tensors $g_{\alpha\beta}$ and $h_{\alpha\beta}$ are connected as:

$$\sqrt{-\, g} = \sqrt{h}\, \sqrt{g_{00}}\,.$$

Derivative with respect to the physically observable time:

$$\frac{d}{d\tau} = \frac{^*\partial}{\partial t} + \mathrm{v}^k\, \frac{^*\partial}{\partial x^k}\,.$$

The 1st derivative with respect to the space-time interval:

$$\frac{d}{ds} = \frac{1}{c\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}\, \frac{d}{d\tau}\,.$$

The 2nd derivative with respect to the space-time interval:

$$\frac{d^2}{ds^2} = \frac{1}{c^2 - \mathrm{v}^2}\frac{d^2}{d\tau^2} + \frac{1}{(c^2-\mathrm{v}^2)^2}\left(D_{ik}\mathrm{v}^i\mathrm{v}^k + \mathrm{v}_i\frac{d\mathrm{v}^i}{d\tau} + \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial x^m}\,\mathrm{v}^i\mathrm{v}^k\mathrm{v}^m\right)\frac{d}{d\tau}\,.$$

The chr.inv.-metric tensor:

$$h_{ik} = -\,g_{ik} + \frac{1}{c^2}\,v_i v_k\,, \quad h^{ik} = -\,g^{ik}, \quad h^k_i = \delta^k_i\,.$$

Zelmanov's relations between the Christoffel regular symbols and the chr.inv.-characteristics of the space of reference:

$$D^i_k + A^{\cdot i}_{k\cdot} = \frac{c}{\sqrt{g_{00}}}\left(\Gamma^i_{0k} - \frac{g_{0k}\Gamma^i_{00}}{g_{00}}\right),$$

$$g^{i\alpha}g^{k\beta}\Gamma^m_{\alpha\beta} = h^{iq}h^{ks}\Delta^m_{qs}\,, \quad F^k = -\,\frac{c^2\,\Gamma^k_{00}}{g_{00}}\,.$$

Zelmanov's 1st identity and 2nd identity:

$$\frac{{}^*\partial A_{ik}}{\partial t} + \frac{1}{2}\left(\frac{{}^*\partial F_k}{\partial x^i} - \frac{{}^*\partial F_i}{\partial x^k}\right) = 0\,,$$

$$\frac{{}^*\partial A_{km}}{\partial x^i} + \frac{{}^*\partial A_{mi}}{\partial x^k} + \frac{{}^*\partial A_{ik}}{\partial x^m} + \frac{1}{2}\left(F_i A_{km} + F_k A_{mi} + F_m A_{ik}\right) = 0\,.$$

Derivative from v^2 with respect to the physically observable time:

$$\frac{d}{d\tau}\left(\mathrm{v}^2\right) = \frac{d}{d\tau}\left(h_{ik}\mathrm{v}^i\mathrm{v}^k\right) = 2D_{ik}\mathrm{v}^i\mathrm{v}^k + \frac{{}^*\partial h_{ik}}{\partial x^m}\,\mathrm{v}^i\mathrm{v}^k\mathrm{v}^m + 2\,\mathrm{v}_k\frac{d\mathrm{v}^k}{d\tau}\,.$$

The completely antisymmetric chr.inv.-tensor:

$$\varepsilon^{ikm} = \sqrt{g_{00}}\,E^{0ikm} = \frac{e^{0ikm}}{\sqrt{h}}\,, \quad \varepsilon_{ikm} = \frac{E_{0ikm}}{\sqrt{g_{00}}} = e_{0ikm}\sqrt{h}\,.$$

Appendix B A thesis presented at the April Meeting 2008 of the American Physical Society

2008 APS April Meeting
April 12–15, 2008, St. Louis, Missouri

Exact Solution for a Gravitational Wave Detector — *by Dmitri Rabounski and Larissa Borissova* — The experimental statement on gravitational waves proceeds from the equation for deviating geodesic lines and the equation for deviating non-geodesics. Weber's result was not based upon an exact solution to the equations, but on an approximate analysis of what could be expected: he expected that a plane weak wave of the space metric may displace two resting particles with respect to each other. In this work, exact solutions are presented for the deviation equation of both free and spring-connected particles. The solutions show that a gravitational wave may displace particles in a two-particle system only if they are in motion with respect to each other or the local space (there is no effect if they are at rest). Thus, gravitational waves produce a parametric effect on a two-particle system. According to the solutions, an altered detector construction can be proposed such that it might interact with gravitational waves: 1) a horizontally suspended cylindrical pig, whose butt-ends have basic relative oscillations induced by a laboratory source; 2) a free-mass detector where suspended mirrors have laboratory induced basic oscillations relative to each other.

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Svenska fysikarkivet books

A Revised Electromagnetic Theory with Fundamental Applications *by Bo Lehnert*

Svenska fysikarkivet, 2008, 158 pages. ISBN 978-91-85917-00-6

Summary: There are important areas within which the conventional electromagnetic theory of Maxwell's equations and its combination with quantum mechanics does not provide fully adequate descriptions of physical reality. As earlier pointed out by Feynman, these difficulties are not removed by and are not directly associated with quantum mechanics. Instead the analysis has to become modified in the form of revised quantum electrodynamics, for instance as described in this book by a Lorentz and gauge invariant theory. The latter is based on a nonzero electric charge density and electric field divergence in the vacuum state, as supported by the quantum mechanical vacuum fluctuations of the zero-point energy. This theory leads to new solutions of a number of fundamental problems, with their applications to leptons and photon physics. They include a model of the electron with its point-charge-like nature, the associated self-energy, the radial force balance in presence of its self-charge, and the quantized minimum value of the free elementary charge. Further there are applications on the individual photon and on light beams, in respect to the angular momentum, the spatially limited geometry with an associated needle-like radiation, and the wave-particle nature in the photoelectric effect and in two-slit experiments.

Spin-Curvature and the Unification of Fields in a Twisted Space *by Indranu Suhendro*

Svenska fysikarkivet, 2008, 78 pages. ISBN 978-91-85917-01-3

Summary: The book draws theoretical findings for spin-curvature and the unification of fields in a twisted space. A space twist, represented through the appropriate formalism, is related to the anti-symmetric metric tensor. Kaluza's theory is extended and given an appropriate integrability condition. Both matter and the isotropic electromagnetic field are geometrized through common field equations: trace-free field equations giving the energy-momentum tensor for such an electromagnetic field solely via the (generalized) Ricci curvature tensor and scalar are obtained. In the absence of electromagnetic fields the theory goes to Einstein's 1928 theory of distant parallelism where only matter field is geometrized (through the twist of space-time). The above results in common with respective wave equations are joined into a "unified field theory of semi-classical gravoelectrodynamics".

Particles Here and Beyond the Mirror *by D. Rabounski and L. Borissova*

This is a research on all kinds of particles, which could be conceivable in the space-time of General Relativity. In addition to mass-bearing particles and light-like particles, zero-particles are predicted: such particles can exist in a fully degenerate space-time region (zero-space). Zero-particles seems as standing light waves, which travel in instant (non-quantum teleportation of photons); they might be observed in a further development of the "stopped light experiment" which was first conducted in 2001, at Harvard, USA. The theoretical existence of two separate regions in the space-time is also shown, where the observable time flows into the future and into the past (our world and the mirror world). These regions are separated by a space-time membrane wherein the observable time stops.

A few other certain problems are considered. It is shown, through Killing's equations, that geodesic motion of particles is a result of stationary geodesic rotation of the space which hosts them. Concerning the theory of gravitational wave detectors, it is shown that both free-mass detector and solid-body detector may register a gravitational wave only if such a detector bears an oscillation of the butt-ends.

Particlar här och bakom spegeln *av D. Rabounski och L. Borissova*

Denna forskning avser alla slag av partiklar, vilka kan vara tänkbara i allmänna relativitetsteorins rumtid. Förutom partiklar med restmassa och ljusliknande partiklar förutsäges noll-partiklar: sådana partiklar kan existera i en fullt degenererad region av rumtiden (noll-rummet). Noll-partiklar uppträder härvid som stående ljusvågor vilka utbredes utan fördröjning (icke-kvantiserad teleportation av fotoner): de skulle kunna observeras i en vidare utveckling av "experimentet med stoppat ljus" vilket först utfördes år 2001 i Harvard, USA. Den teoretiska existensen av två separata regioner hos rumtiden visas också, där den observerbara tiden förlöper in i framtiden och in i det förflutna (vår värld och spegelvärlden). Dessa regioner separeras av ett membran hos rumtiden inom vilket den observerbara tiden står stilla.

Några andra speciella problem betraktas även. Med Killings ekvationer visas att partiklars geodesiska rörelse är resultatet av stationär geodesisk rotation hos rummet som inhyser dem. När det gäller teorin för gravitationsvågornas detektorer, visas vidare att både en detektor med fri massa och en sådan med fast massa bara kan registrera gravitationsvågor då detektorn påverkas av en oscillation vid sina ändar.

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